MAT 221
Fall 2018
Final Exam
12/11/2018
Time Limit: 120 Minutes

Name (Print): ____________________

Student ID ____________________

This exam contains 8 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	6	
2	20	
3	10	
4	12	
5	20	
6	28	
7	14	
8	15	
9	20	
10	15	
11	20	
12	20	
Total:	200	

1. (6 points) Let

$$f(x) = \begin{cases} 3x+5, & x \leq 4 \\ 4x-2, & x > 4 \end{cases}$$

Find each limit (if it exists).

a). $\lim\limits_{x \to 4^-} f(x)$, b). $\lim\limits_{x \to 4^+} f(x)$, c). $\lim\limits_{x \to 4} f(x)$

2. (20 points) Find the value of the limit, and, when applicable, indicate the limit theorems being used.

a). $\lim\limits_{x \to 16} \dfrac{4-\sqrt{x}}{x-16}$ b). $\lim\limits_{t \to 0^-} \dfrac{2t}{|t|}$

c). $\lim\limits_{x \to 0} \dfrac{x^2+3x}{xe^{2x}}$ d). $\lim\limits_{x \to 0} \dfrac{\sin x(1-\cos x)}{x^2}$

3. (10 points) Find the value of c that makes $f(x)$ continuous at $x = 4$, where

$$f(x) = \begin{cases} 5 - 2x, & x \leq 4 \\ cx + 7 & x > 4 \end{cases}$$

4. (12 points) Given that $\lim_{x \to a} f(x) = -3$, $\lim_{x \to a} g(x) = 0$, $\lim_{x \to a} h(x) = 8$, for some constant a, find the following limit.

   (a) $\lim_{x \to a} \frac{f(x)}{g(x)}$

   (b) $\lim_{x \to a} \frac{2f(x)}{h(x) - f(x)}$

5. (20 points) Let $g(x) = \frac{6}{x}$, use the **Limit Definition** of the Derivative to find $g'(x)$

6. (28 points) Find the following derivatives.

   (a) $f(x) = 2e^{5x} + \frac{1}{\sqrt[5]{x^3}} - \pi$

   (b) $f(x) = (x+2)^5 \cos(5x^2)$

   (c) $R(w) = \frac{\cot(w)+3}{w^2-1}$

   (d) $f(x) = \tan\left(\frac{3x}{4}\right)$

7. (14 points) Consider the function

$$f(x) = \frac{1}{x+2}$$

Write an equation for the slope of the tangent line at the point $x = -1$.

8. (15 points) Consider the given equation $4x^2 + e^{xy} + 7y^3 = 13$. Assume that it determines an implicit differentiable function f such that $y = f(x)$. Find $\frac{dy}{dx}$

9. (20 points) Given function f with $f'(x) = \frac{4-4x^2}{(x^2+1)^2}$ and $f''(x) = \frac{8x^3-24x}{(x^2+1)^3}$.

   (a) Give the interval(s) on which the graph of $y = f(x)$ is concave up. Write your answer in interval form.

   (b) Give the interval(s) on which the graph of $y = f(x)$ is increasing or decreasing. Write your answer in interval form.

10. (15 points) Find the absolute minimum and maximum values of the function on given interval

$$f(x) = 3x^2 - 24x - 1, \quad [-1, 5]$$

11. (20 points) (Closed box problem). We need a closed rectangular cardboard box with a square top, a square bottom, and a volume of 32 m^3. Find the dimensions of the valid box that requires the least amount of cardboard, and find the amount of cardboard needed.

12. (20 points) Sketch the graph of $y = f(x)$, where $f(x) = x^4 - 4x^3$ in the usual xy-plane.

   (a) Find the domain of function f and x-intercepts, y-intercepts.

   (b) Find all critical numbers of f and show where f is increasing or decreasing

   (c) Find all possible inflection number (if any) and show where the graph is concave up or concave down.

   (d) Classify all points at critical numbers as local maximum points, local minimum points, or neither.

   (e) Sketch the graph of function f. Show all steps, as we have done in class.