MAT 221 **Name (Print):** _____
Fall 2019
Exam 1
9/30/19
Time Limit: 50 Minutes Student ID _____

This exam contains 5 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	6	
3	10	
4	6	
5	24	
6	10	
7	10	
8	15	
9	15	
Total:	100	

1. (4 points) Find the domain of the following function

$$f(x) = \frac{1}{x^2 - 25}$$

2. (6 points) Given $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 2} g(x) = 2$, use limit laws to compute the following limits or explain why we cannot find the limit.

a). $\lim_{x\to 2} \frac{f(x)}{g(x)}$, b). $\lim_{x\to 2}[f(x) + 3g(x)]$,

3. (10 points) Find parameters a and b so that the following function is continuous.

$$f(x) = \begin{cases} 2x^2 + 3x, & x \leq -4 \\ ax + b & -4 < x < 3 \\ -x^3 + 4x^2 - 5, & 3 \leq x \end{cases}$$

4. (6 points) Let

$$f(x) = \begin{cases} -x - 8, & x \leq 1 \\ -x^2 - 4x - 4, & x > -1 \end{cases}$$

Find each limit (if it exists).

$$\lim_{x \to 1^-} f(x), \qquad \lim_{x \to 1^+} f(x), \qquad \lim_{x \to 1} f(x)$$

5. (24 points) Find the value of the limit, and, when applicable, indicate the limit theorems being used.

a). $\lim_{x \to 5} \dfrac{x^2 - 25}{x^2 - 4x - 5}$

b). $\lim_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$

c). $\lim_{x \to \infty} \dfrac{x^4 - 10}{4x^3 + x}$

d). $\lim_{x \to 2} \dfrac{x - 2}{\frac{1}{x} - \frac{1}{2}}$

6. (10 points) use the Squeeze Theorem to find

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

7. (10 points) If $f(x) = -2x + 1$ and $g(x) = \sqrt{x^2 - 5}$, find and simplify $[f \circ g](x)$ and $[g \circ f](x)$

8. (15 points) Let

$$f(x) = \frac{|x-3|}{x^2 - x - 6}$$

Find each limit (if it exists).

$$\lim_{x \to 3^-} f(x), \qquad \lim_{x \to 3^+} f(x), \qquad \lim_{x \to 3} f(x)$$

9. (15 points) Show that $f(x) = x^3 + x - 1$ has a zero in the interval $(0, 1)$ with the Intermediate Value Theorem.