

## Worksheet # 9: Limits at infinity and Intermediate Value Theorem

- Describe the behavior of the function  $f(x)$  if  $\lim_{x \rightarrow \infty} f(x) = L$  and  $\lim_{x \rightarrow -\infty} f(x) = M$ .
  - Explain the difference between " $\lim_{x \rightarrow -3} f(x) = \infty$ " and " $\lim_{x \rightarrow \infty} f(x) = -3$ ".
- Evaluate the following limits, or explain why the limit does not exist:
  - $\lim_{x \rightarrow \infty} \frac{3x^2 - 7x}{x - 8}$
  - $\lim_{x \rightarrow \infty} \frac{2x^2 - 6}{x^4 - 8x + 9}$
  - $\lim_{x \rightarrow -\infty} \frac{x}{x^6 - 4x^2}$
  - $\lim_{x \rightarrow -\infty} 3$
  - $\lim_{x \rightarrow \pm\infty} \frac{5x^3 - 7x^2 + 9}{x^2 - 8x^3 - 8999}$
  - $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^{10} + 2x}}{x^5}$
- Find the limits  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$  if  $f(x) = \left( \frac{x^2}{x+1} - \frac{x^2}{x-1} \right)$ .
- Sketch a graph with all of the following properties:
  - $\lim_{t \rightarrow \infty} f(t) = 2$
  - $\lim_{t \rightarrow -\infty} f(t) = 0$
  - $\lim_{t \rightarrow 0^+} f(t) = \infty$
  - $\lim_{t \rightarrow 0^-} f(t) = -\infty$
  - $\lim_{t \rightarrow 4} f(t) = 3$
  - $f(4) = 6$
- Find the following limits;
  - $\lim_{x \rightarrow \infty} \frac{3x + 2\sqrt{x}}{1 - x}$
  - $\lim_{x \rightarrow -\infty} \frac{2x - 5}{|3x + 2|}$
  - $\lim_{x \rightarrow \infty} \frac{5x^2 + \sin x}{3x^2 + \cos x}$
- State the Intermediate Value Theorem.
  - Show that  $f(x) = x^3 + x - 1$  has a zero in the interval  $[0, 1]$ .
- Use the Intermediate Value Theorem to find an interval of length 1 in which a solution to the equation  $2x^3 + x = 5$  must exist.
- Show that there is some  $a$  with  $0 < a < 2$  such that  $a^2 + \cos(\pi a) = 4$ .
- Show that the equation  $\ln(x) = e^{-x}$  has a solution between 1 and 2.
- Let  $f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$  be a piecewise function.

Although  $f(-1) = 0$  and  $f(1) = 1$ ,  $f(x) \neq 1/2$  for all  $x$  in its domain. Why doesn't this contradict to the Intermediate Value Theorem?
- Prove that  $x^4 = -1$  has no solution.