

Worksheet # 4: Basic Limit Laws

1. Given $\lim_{x \rightarrow 2} f(x) = 5$ and $\lim_{x \rightarrow 2} g(x) = 2$, use limit laws (justify your work) to compute the following limits. Note when working through a limit problem that your answers should be a chain of equalities. Make sure to keep the $\lim_{x \rightarrow a}$ operator until the very last step.

- (a) $\lim_{x \rightarrow 2} 2f(x) - g(x)$
- (b) $\lim_{x \rightarrow 2} \frac{f(x)g(x)}{x}$
- (c) $\lim_{x \rightarrow 2} f(x)^2 + x \cdot g(x)^2$
- (d) $\lim_{x \rightarrow 2} [f(x)]^{\frac{3}{2}}$

2. Calculate the following limits if they exist or explain why the limit does not exist.

- (a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$
- (b) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 2}$
- (c) $\lim_{x \rightarrow 2^+} \frac{x^2 - 1}{x - 2}$
- (d) $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

3. Can the quotient law of limits be applied to evaluate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$?

4. Find the value of c such that $\lim_{x \rightarrow 2} \frac{x^2 + 3x + c}{x - 2}$ exists. What is the limit?

5. find the value of c such that $\lim_{x \rightarrow 5} \begin{cases} 2x + c & \text{if } x \leq 5 \\ -3x & \text{if } x > 5 \end{cases}$ exists. What is the limit?

6. Show that $\lim_{h \rightarrow 0} \frac{|h|}{h}$ does not exist by examining one-sided limits. Then sketch the graph of $\frac{|h|}{h}$ and check your reasoning.

7. True or False:

- (a) Let $f(x) = \frac{(x+2)(x-1)}{x-1}$ and $g(x) = x+2$. Then $f(x) = g(x)$.
- (b) Let $f(x) = \frac{(x+2)(x-1)}{x-1}$ and $g(x) = x+2$. Then $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} g(x)$.
- (c) If both the one-sided limits of $f(x)$ exist as x approaches a , then $\lim_{x \rightarrow a} f(x)$ exists.
- (d) If $\lim_{x \rightarrow a} f(x)$ exists then $\lim_{x \rightarrow a} f(x) = f(a)$.

8. Draw a graph of two functions $f(x)$ and $g(x)$ such that $\lim_{x \rightarrow 0} (f(x) + g(x))$ exist but neither $\lim_{x \rightarrow 0} f(x)$ nor $\lim_{x \rightarrow 0} g(x)$ exist.