Worksheet # 19: The Shape of a Graph

1. Explain how to use the second derivative test to identify and classify local extrema of a twice differentiable function $f(x)$. Does the test always work? What should you do if it fails?

2. Suppose that $g(x)$ is differentiable for all x and that $-5 \leq g'(x) \leq 2$ for all x. Assume also that $g(0) = 2$. Based on this information, use the Mean Value Theorem to determine the largest and smallest possible values for $g(2)$.

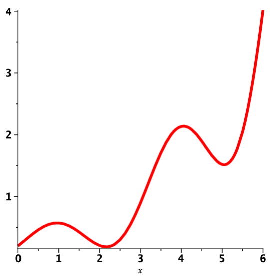
3. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why did she deserve the ticket?

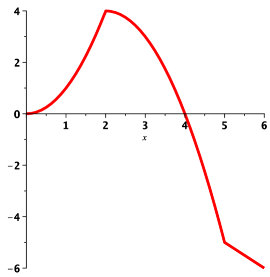
4. (a) Consider the function $f(x) = x^4 - 4x^3 - 8x^2$.
   i. Find the intervals on which the graph of $f(x)$ is increasing or decreasing.
   ii. Find the intervals of concavity of $f(x)$.
   iii. Find the points of inflection of $f(x)$.

   (b) Repeat with the function $f(x) = 2x + \sin(x)$ on $\left(-\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$.

   (c) Repeat with the function $f(x) = x + \dfrac{4}{x}$

5. For each of the following graphs:





   (a) Find the intervals where the function is increasing and decreasing respectively.
   (b) Find the intervals of concavity for each function.
   (c) Identify all local extrema and inflection points on the interval (0,6).

6. Find the local extrema of the following functions using the second derivative test:
   (a) $f(x) = x^5 - 5x + 4$
   (b) $g(x) = 5x - 10\ln(2x)$

7. Find the local extrema of $f(x) = 3x^5 - 5x^3 + 10$ using the second derivative where possible.

8. Sketch a graph of a continuous function $f(x)$ with the following properties:
   - f is increasing on $(-\infty, -3) \cup (1, 7) \cup (7, \infty)$
   - f is decreasing on $(-3, 1)$
   - f is concave up on $(0, 3) \cup (7, \infty)$
   - f is concave down on $(-\infty, 0) \cup (3, 7)$