

## Worksheet # 23: Approximating Area

1. Write each of following in summation notation:

(a)  $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

(b)  $2 + 4 + 6 + 8 + 10 + 12 + 14$

(c)  $2 + 4 + 8 + 16 + 32 + 64 + 128$ .

2. Compute  $\sum_{i=1}^4 \left( \sum_{j=1}^3 (i+j) \right)$ .

The following summation formulas will be useful below.

$$\sum_{j=1}^n j = \frac{n(n+1)}{2}, \quad \sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Find the number  $n$  such that  $\sum_{i=1}^n i = 78$ .

4. Give the value of the following sums.

(a)  $\sum_{j=1}^{20} (2k^2 + 3)$

(b)  $\sum_{j=11}^{20} (3k + 2)$

5. The velocity of a train at several times is shown in the table below. Assume that the velocity changes linearly between each time given.

t=time in minutes	0	3	6	9
v(t)=velocity in Km/h	20	80	100	140

- (a) Plot the velocity of the train versus time.
- (b) Compute the left and right-endpoint approximations to the area under the graph of  $v$ .
- (c) Explain why these approximate areas are also an approximation to the distance that the train travels.
6. Let  $f(x) = 1/x$ . Divide the interval  $[1, 3]$  into five subintervals of equal length and compute  $R_5$  and  $L_5$ , the left and right endpoint approximations to the area under the graph of  $f$  in the interval  $[1, 3]$ . Is  $R_5$  larger or smaller than the true area? Is  $L_5$  larger or smaller than the true area?
7. Let  $f(x) = \sqrt{1-x^2}$ . Divide the interval  $[0, 1]$  into four equal subintervals and compute  $L_4$  and  $R_4$ , the left and right-endpoint approximations to the area under the graph of  $f$ . Is  $R_4$  larger or smaller than the true area? Is  $L_4$  larger or smaller than the true area? What can you conclude about the value  $\pi$ ?
8. Let  $f(x) = x^2$ .
- (a) If we divide the interval  $[0, 2]$  into  $n$  equal intervals of equal length, how long is each interval?
- (b) Write a sum which gives the right-endpoint approximation  $R_n$  to the the area under the graph of  $f$  on  $[0, 2]$ .
- (c) Use one of the formulae for the sums of powers of  $k$  to find a closed form expression for  $R_n$ .
- (d) Take the limit of  $R_n$  as  $n$  tends to infinity to find an exact value for the area.