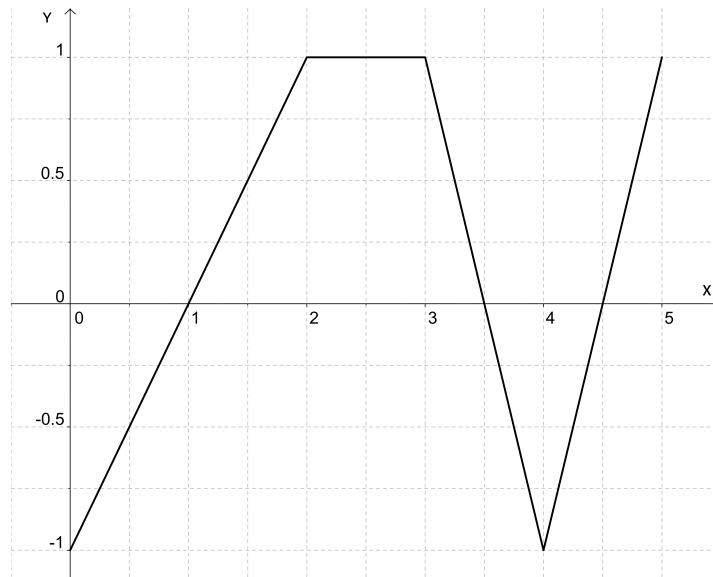


## Worksheet # 10: Derivatives

1. Comprehension check:

- (a) Are differentiable functions also continuous? Are continuous functions also differentiable?
- (b) What does the derivative of  $f(x)$  at  $x = a$  describe graphically?
- (c) True or false: If  $f'(x) = g'(x)$  then  $f(x) = g(x)$ ?
- (d) True or false:  $(f(x) + g(x))' = f'(x) + g'(x)$
- (e) How is the number  $e$  defined?

2. Consider the graph below of the function  $f(x)$  on the interval  $[0, 5]$ .



- (a) For which  $x$  values would the derivative  $f'(x)$  not be defined?
  - (b) Sketch the graph of the derivative function  $f'$ .
3. Find  $f'(a)$  using either form of the definition for the derivative:
- (a)  $f(x) = 3x^2 - 2x + 1$ ,  $a = 2$ .
  - (b)  $f(x) = \frac{1}{x+3}$ ,  $a = -1$ .
  - (c)  $f(x) = \sqrt{x}$ ,  $a = 9$ .
4. Let

$$h(t) = \begin{cases} at + b & \text{if } t \leq 0 \\ t^3 + 1 & \text{if } t > 0 \end{cases}.$$

Find  $a$  and  $b$  so that  $h$  is differentiable at  $t = 0$ .

5. Compute the derivative of the following functions:

- (a)  $f(x) = \frac{9}{4}x^8$
- (b)  $h(x) = 3e^x + x^2 + 1$
- (c)  $k(x) = \frac{A}{x^4} + Bx^2 + Cx + D$
- (d)  $l(x) = \left(x + \frac{1}{x}\right)^2$

6. Find an equation for the tangent line to the curve  $y = x^{3/2} + 2$  at  $x = 3$ .
7. Find the equation of each tangent line to the parabola  $y = x^2$  which pass through the point  $(0, -1)$ . First sketch the graph of the parabola and the desired tangent line(s).
8. Consider the function  $f(x) = x^4 - x^3 - 8x^2 + 25x + 10$ . Use the Intermediate Value Theorem to show that the graph of  $f$  has a horizontal tangent line between  $x = -3$  and  $x = -2$ .
9. Find a function  $f$  and a number  $a$  so that the following limit represents a derivative  $f'(a)$ .

$$\lim_{h \rightarrow 0} \frac{(4+h)^3 - 64}{h}$$