MAT 221
Fall 2019
Final Exam
12/10/2019
Time Limit: 120 Minutes

Name (Print): _____

Student ID _____

This exam contains 7 pages (including this cover page) and 11 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	8	
2	20	
3	15	
4	15	
5	20	
6	32	
7	15	
8	15	
9	15	
10	20	
11	25	
Total:	200	

1. (8 points) Let

$$f(x) = \begin{cases} x^2, & x \le 0 \\ x - 1, & x > 0 \text{ and } x \ne 2. \\ -3, & x = 2 \end{cases}$$

Find each limit (if it exists).

1). $\lim_{x \to 0^-} f(x) =$ 2). $\lim_{x \to 0^+} f(x) =$ 3). $\lim_{x \to 0} f(x) =$ 4). $f(0) =$

5). $\lim_{x \to 2^-} f(x) =$ 6). $\lim_{x \to 2^+} f(x) =$ 7). $\lim_{x \to 2} f(x) =$ 8). $f(2) =$

2. (20 points) Find the value of the limit, and, when applicable, indicate the limit theorems being used.

a). $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$ b). $\lim_{t \to 6^-} \dfrac{t^2 - t - 6}{|t - 3|}$

c). $\lim_{x \to -3^+} \dfrac{x + 2}{x + 3}$ d). $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin x}$

3. (15 points) Find the value of a and k that makes $f(x)$ continuous at $x = -2$, where

$$f(x) = \begin{cases} 2x + 2, & x \leq -2 \\ a, & x = -2 \\ kx, & x > -2 \end{cases}$$

4. (15 points) Given that $\lim_{x \to 2} f(x) = 5$, $\lim_{x \to 2} g(x) = 2$, find the following limit.

(a) $\lim_{x \to 2} (2f(x) - g(x))$

(b) $\lim_{x \to 2} \frac{f(x)g(x)}{x}$

5. (20 points) Let $g(x) = \sqrt{x}$, use the **Limit Definition** of the Derivative to find $g'(x)$

6. (32 points) Find the following derivatives.

(a) $f(x) = 2e^{5x} + \frac{1}{\sqrt[5]{x^3}} - \pi$

(b) $f(x) = e^x \sec x$

(c) $f(x) = \frac{(x^2+1)(x^3+2)}{x^5}$

(d) $f(x) = \sqrt[3]{2x^3 + 7x + 3}$

7. (15 points) Consider the function

$$f(x) = \sin^2 x + \cos x$$

Write an equation for the slope of the tangent line at the point $x = \frac{\pi}{2}$.

8. (15 points) Consider the given equation $4x^2 + e^{xy} + 7y^3 = 13$. Assume that it determines an implicit differentiable function f such that $y = f(x)$. Find $\frac{dy}{dx}$

9. (15 points) Find the absolute minimum and maximum values of the function on given interval

$$f(t) = 2t^3 + 3t^2 - 12t + 4, \quad [-4, 2]$$

10. (20 points) We need to enclose a rectangular field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so wont need any fencing. Determine the dimensions of the field that will enclose the largest area.

11. (25 points) Sketch the graph of $y = f(x)$, where $f(x) = x^4 - 4x^3$ in the usual xy-plane.

(a) Find the domain of function f and x-intercepts, y-intercepts.

(b) Find all critical numbers of f and show where f is increasing or decreasing

(c) Find all possible inflection number (if any) and show where the graph is concave up or concave down.

(d) Classify all points at critical numbers as local maximum points, local minimum points, or neither.

(e) Sketch the graph of function f. Show all steps, as we have done in class.