Worksheet # 12: Higher Derivatives and Trigonometric Functions

1. Given that $G(t) = 4t^2 - 3t + 42$ find the instantaneous rate of change when $t = 3$.

2. An object which is thrown upward with an initial velocity of 5 m/s from an initial height of 40 m. Find the velocity of the object when it hits the ground.

3. Find the instantaneous rate of change in the area with respect to base b and height b of a triangle whose base equals its height when $b = 7$.

4. Calculate the indicated derivative:

   (a) $f^{(4)}(1)$, $\quad f(x) = x^4$

   (b) $g^{(3)}(5)$, $\quad g(x) = 2x^2 - x + 4$

   (c) $h^{(3)}(t)$, $\quad h(t) = 4e^t - t^3$

   (d) $s^{(2)}(w)$, $\quad s(w) = \sqrt{w}e^w$

5. Calculate the first three derivatives of $f(x) = xe^x$ and use these to guess a general formula for $f^{(n)}(x)$, the n-th derivative of f.

6. Differentiate each of the following functions:

   (a) $f(t) = \cos(t)$

   (b) $g(u) = \dfrac{1}{\cos(u)}$

   (c) $r(\theta) = \theta^3 \sin(\theta)$

   (d) $s(t) = \tan(t) + \csc(t)$

   (e) $h(x) = \sin(x)\csc(x)$

   (f) $f(x) = x^2 \sin^2(x)$

   (g) $g(x) = \sec(x) + \cot(x)$

7. Calculate the first five derivatives of $f(x) = \sin(x)$. Then determine $f^{(8)}$ and $f^{(37)}$

8. A particle's distance from the origin (in meters) along the x-axis is modeled by $p(t) = 2\sin(t) - \cos(t)$, where t is measured in seconds.

   (a) Determine the particle's speed (speed is defined as the absolute value of velocity) at π seconds.

   (b) Is the particle moving towards or away from the origin at π seconds? Explain.

   (c) Now, find the velocity of the particle at time $t = \dfrac{3\pi}{2}$. Is the particle moving toward the origin or away from the origin?

   (d) Is the particle speeding up at $\frac{\pi}{2}$ seconds?

9. Find an equation of the tangent line at the point specified:

   (a) $y = x^3 + \cos(x)$, $\quad x = 0$

   (b) $y = \csc(x) - \cot(x)$, $\quad x = \frac{\pi}{4}$

   (c) $y = e^\theta \sec(\theta)$, $\quad \theta = \frac{\pi}{4}$