Worksheet # 24: Definite Integrals and The Fundamental Theorem of Calculus

1. Suppose
$$\int_0^1 f(x) dx = 2$$
, $\int_1^2 f(x) dx = 3$, $\int_0^1 g(x) dx = -1$, and $\int_0^2 g(x) dx = 4$.

Compute the following using the properties of definite integrals:

(a)
$$\int_{1}^{2} g(x) dx$$

(b)
$$\int_0^2 [2f(x) - 3g(x)] dx$$

(c)
$$\int_{1}^{1} g(x) dx$$

(d)
$$\int_{1}^{2} f(x) dx + \int_{2}^{0} g(x) dx$$

(e)
$$\int_0^2 f(x) dx + \int_2^1 g(x) dx$$

2. Simplify
$$\int_0^2 3f(x) dx + \int_1^3 3f(x) dx - \int_0^3 2f(x) dx - \int_1^2 3f(x) dx$$

3. Find
$$\int_0^5 f(x) dx$$
 where $f(x) = \begin{cases} 3 & \text{if } x < 3 \\ x & \text{if } x \ge 3 \end{cases}$

(b) Consider the function
$$f(x)$$
 on $[1,\infty)$ defined by $f(x) = \int_1^x \sqrt{t^5 - 1} dt$. Argue that f is increasing.

(c) Find the derivative of the function
$$g(x) = \int_1^{x^3} \sqrt{t^5 - 1} dt$$
 on $(1, \infty)$.

(a)
$$g(x) = \int_{1}^{x} (2 + t^{4})^{5} dt$$

(b)
$$F(x) = \int_x^4 \cos\left(t^5\right) dt$$

(c)
$$h(x) = \int_0^{x^2} \sqrt[3]{1+r^3} dr$$

(d)
$$y(x) = \int_{\frac{1}{x^2}}^0 \sin^3(t) dt$$

(e)
$$G(x) = \int_{\sqrt{x}}^{x^2} \sqrt{t} \sin(t) dt$$

(a)
$$\int_{-2}^{5} 6x \, dx$$

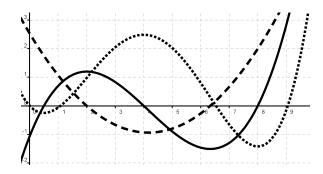
(b)
$$\int_{-2}^{7} \frac{1}{x^5} dx$$

(c)
$$\int_{-1}^{1} e^{u+1} du$$

(d)
$$\int_0^{\frac{\pi}{4}} \sec^2(t) \, dt$$

(e)
$$\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sin(2x)}{\sin(x)} dx$$

7. Below is pictured the graph of the function f(x), its derivative f'(x), and an antiderivative $\int f(x) dx$. Identify f(x), f'(x) and $\int f(x) dx$.



8. Evaluate the following limits by first recognizing the sum as a Riemann sum:

(a)
$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$$

(b)
$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sqrt{3 + \frac{i}{n}}}{n}$$

(c)
$$\lim_{n \to \infty} \sum_{i=1}^{n} 2 \frac{(2 + \frac{2i}{n})^2}{n}$$