Worksheet # 20: L'Hôpital's Rule & Optimization

1. Carefully state l'Hôpital's Rule.

2. Compute the following limits. Use l'Hôpital's Rule where appropriate but first check that no easier method will solve the problem.

   (a) $\lim_{x\to 1} \dfrac{x^9-1}{x^5-1}$

   (b) $\lim_{x\to 0} \dfrac{\sin(4x)}{\tan(5x)}$

   (c) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$

   (d) $\lim_{x\to 1} \dfrac{x^2+2x-2}{x^2-2x+2}$

3. Find the dimensions of x and y of the rectangle of maximum area that can be formed using 3 meters of wire.

   (a) What is the constraint equation relating x and y?

   (b) Find a formula for the area in terms of x alone.

   (c) Solve the optimization problem.

4. A flexible tube of length 4 m is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?

5. A rancher will use 600 m of fencing to build a corral in the shape of a semicircle on top of a rectangle. Find the dimensions that maximize the area of the corral. (Hint: draw a picture)

6. Find the value A for which we can use l'Hôpital's rule to evaluate the limit

$$\lim_{x\to 2} \frac{x^2+Ax-2}{x-2}.$$

   For this value of A, give the value of the limit.

7. Compute the following limits. Use l'Hôpital's Rule where appropriate but first check that no easier method will solve the problem.

   (a) $\lim_{x\to -\infty} x^2e^x$

   (b) $\lim_{x\to \infty} x^3e^{-x^2}$

   (c) $\lim_{x\to \pi} \dfrac{\cos(x)+1}{x^2-\pi^2}$

   (d) $\lim_{x\to \infty} x\cdot\left(\arctan(x)-\dfrac{\pi}{2}\right)$

8. Find the dimensions x and y of the rectangle inscribed in a circle of radius r that maximizes the quantity xy^2.

9. Find the point on the line $y=x$ closest to the point $(1,0)$. Find the point on the line $y=x$ closest to the point $(r,1-r)$. What do these points look like graphically?