

Worksheet # 22 and 23: Newton's Method, Antiderivatives, and Area

Due to election day on Tuesday, we have combined two worksheets into one and deleted about half the problems. The full worksheets are available at <http://www.math.uky.edu/~ma113/>

1. Use Newton's method to find an approximation to $\sqrt[3]{2}$. You may do this by finding a solution of $x^3 - 2 = 0$.
2. Use Newton's method to approximate the critical points of the function $f(x) = x^5 - 7x^2 + x$.
3. (a) Let $f(x) = \frac{x^3}{3} + 1$. Calculate the derivative $f'(x)$. What is an anti-derivative of $f'(x)$?
(b) Let $g(x) = x^2 + 1$. Let $G(x)$ be any anti-derivative of g . What is $G'(x)$?

4. Find f given that

$$f'(x) = \sin(x) - \sec(x) \tan(x)$$

$$f(\pi) = 1.$$

5. Find g given that

$$g''(t) = -9.8, \quad g'(0) = 1, \quad g(0) = 2.$$

On the surface of the earth, the acceleration of an object due to gravity is approximately -9.8 m/s^2 . What situation could we describe using the function g ? Be sure to specify what g and its first two derivatives represent.

6. Write each of the following in summation notation:

(a) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

(b) $2 + 4 + 6 + 8 + 10 + 12 + 14$

(c) $2 + 4 + 8 + 16 + 32 + 64 + 128$.

7. Compute $\sum_{i=1}^4 \left(\sum_{j=1}^3 (i+j) \right)$.

The following formulae will be useful below.

$$\sum_{j=1}^n j = \frac{n(n+1)}{2}, \quad \sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$$

8. Find the number n such that $\sum_{i=1}^n i = 78$.

9. Give the value of the following sums.

(a) $\sum_{j=1}^{20} (2k^2 + 3)$

(b) $\sum_{j=11}^{20} (3k + 2)$

10. Let $f(x) = \sqrt{1 - x^2}$. Divide the interval $[0, 1]$ into four equal subintervals and compute L_4 and R_4 , the left and right-endpoint approximations to the area under the graph of f . Is R_4 larger or smaller than the true area? Is L_4 larger or smaller than the true area? What can you conclude about the value π ?
11. Let $f(x) = x^2$.
- (a) If we divide the interval $[0, 2]$ into n equal intervals of equal length, how long is each interval?
 - (b) Write a sum which gives the right-endpoint approximation R_n to the the area under the graph of f on $[0, 2]$.
 - (c) Use one of the formulae for the sums of powers of k to find a closed form expression for R_n .
 - (d) Take the limit of R_n as n tends to infinity to find an exact value for the area.