Worksheet # 8: Review for Exam I

1. Find all real numbers of the constant a and b for which the function $f(x) = ax + b$ satisfies:
   (a) $f \circ f(x) = f(x)$ for all x.
   (b) $f \circ f(x) = x$ for all x.
2. Simplify the following expressions.
   (a) $\log_5 125$
   (b) $(\log_4 16)(\log_4 2)$
   (c) $\log_{15} 75 + \log_{15} 3$
   (d) $\log_x (x(\log_y y^x))$
   (e) $\log_\pi (1 - \cos x) + \log_\pi (1 + \cos x) - 2 \log_\pi \sin x$
3. (a) Solve the equation $3^{2x+5} = 4$ for x. Show each step in the computation.
   (b) Express the quantitiy $\log_2(x^3 - 2) + \dfrac{1}{3} \log_2(x) - \log_2(5x)$ as a single logarithm.
4. Calculate the following limits using the limit laws. Carefully show your work and use only one limit law per step.
   (a) $\lim\limits_{x \to 0} (2x - 1)$
   (b) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$
5. Calculate the following limits if they exist or explain why the limit does not exist.
   (a) $\lim\limits_{x \to 1} \dfrac{x - 2}{\frac{1}{x} - \frac{1}{2}}$
   (b) $\lim\limits_{x \to 2} \dfrac{x - 2}{\frac{1}{x} - \frac{1}{2}}$
   (c) $\lim\limits_{x \to 2} \dfrac{x^2}{x - 2}$
   (d) $\lim\limits_{x \to 4} \dfrac{\sqrt{x} - 2}{x^2 - 16}$
   (e) $\lim\limits_{x \to 2} \dfrac{x + 1}{x - 2}$
6. Use the fact that $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$ to find $\lim\limits_{x \to 0} \dfrac{x}{\sin(3x)}$.
7. (a) State the Squeeze Theorem.
   (b) Use the Squeeze Theorem to find the limit $\lim\limits_{x \to 0} x \sin \dfrac{1}{x^2}$
8. Use the Squeeze Theorem to find $\lim\limits_{x \to \frac{\pi}{2}} \cos x \cos(\tan x)$
9. If $f(x) = \dfrac{|x - 3|}{x^2 - x - 6}$, find $\lim\limits_{x \to 3^+} f(x)$, $\lim\limits_{x \to 3^-} f(x)$ and $\lim\limits_{x \to 3} f(x)$.
10. (a) State the definition of the continuity of a function $f(x)$ at $x = a$.

(b) Find the constant a so that the function is continuous on the entire real line.

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 8 & \text{if } x = a \end{cases}$$

11. Complete the following statements:

(a) A function $f(x)$ passes the horizontal line test, if the function f is ....

(b) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{provided ....}$$

(c) $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$ if and only if ....

(d) Let $g(x) = \begin{cases} x & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ be a piecewise function.
The function $g(x)$ is NOT continuous at $x = 2$ since ....

(e) Let $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$ be a piecewise function.
The function $f(x)$ is NOT continuous at $x = 0$ since ....