Worksheet # 22: Newton's Method and Antiderivatives

1. Use Newton's method to find an approximation to $\sqrt[3]{2}$. You may do this by finding a solution of $x^3 - 2 = 0$.

2. Use Newton's method to approximate the critical points of the function $f(x) = x^5 - 7x^2 + x$.

3. Let $f(x) = \dfrac{x}{1+x^2}$.

   (a) Solve $f(x) = 0$ without using Newton's method.

   (b) Use Newton's method to solve $f(x) = 0$ beginning with the starting point $x_1 = 2$. Does something interesting happen?

   (c) Make a sketch of the graph of f and explain what you observed in part b).

4. (a) Let $f(x) = \dfrac{x^3}{3} + 1$. Calculate the derivative $f'(x)$. What is an anti-derivative of $f'(x)$?

   (b) Let $g(x) = x^2 + 1$. Let $G(x)$ be any anti-derivative of g. What is $G'(x)$?

5. Find f given that
$$f'(x) = \sin(x) - \sec(x)\tan(x), \qquad f(\pi) = 1.$$

6. Find g given that
$$g''(t) = -9.8, \qquad g'(0) = 1, \qquad g(0) = 2.$$
On the surface of the earth, the acceleration of an object due to gravity is approximately $-9.8\,\mathrm{m/s^2}$. What situation could we describe using the functino g? Be sure to specify what g and its first two derivatives represent.

7. A small rock is dropped from a bridge and the splash is heard 3 seconds later. How high is the bridge?

8. Let f be a function on the domain $(-\infty, \infty)$ that satisfies $(f')^2 = 1$. This is an example of a *differential equation*. Suppose also that we are given an *initial value condition* $f(0) = 1$.

   (a) Show that this does not have a unique solution by finding two different functions that satisfy both conditions.

   (b) What does the fact that there are multiple solutions say about this as a model for physical phenomena?

9. Find a function $f(x)$ such that $f'(x) = f(x)$. Find the solution, given initial condition $f(0) = \pi$.

10. Let $f(x) = 1/x$, $F(x) = \ln(|x|)$, and
$$G(x) = \begin{cases} \ln(x), & x > 0 \\ \ln(-x) + 8, & x < 0. \end{cases}$$

    (a) Is F an anti-derivative of f? Is G an anti-derivative of f? Is $F - G$ equal to a constant?

    (b) Does Theorem 1 on page 275 imply that $F - G$ is constant? Is the theorem wrong?