DISCRIMINATIVE GROUP COLLABORATIVE COMPETITIVE REPRESENTATION FOR VISUAL CLASSIFICATION

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ABSTRACT

In pattern recognition, the representation-based classification (RBC) has attracted much attention recently. As a representative one of RBC, collaborative representation-based classification (CRC) and its variants have achieved promising classification performance in many visual classification tasks. However, most of the CRC method cannot directly consider the class discrimination information of data that is very important for classification. To fully use the class discrimination information, we propose a novel discriminative group collaborative competitive representation based-classification method (DGCCR) in this paper. In the designed DGCCR model, the discriminative competitive relationships of classes, the discriminative decorrelations among classes and the weighted class-specific group constraints are simultaneously taken into account for strengthening the power of pattern discrimination. Experiments on three visual classification data sets demonstrate that the proposed DGCCR outperforms state-of-the-art RBC methods.

Index Terms— Collaboration representation-based classification, Collaborative-competitive representation-based classification, Pattern recognition

1. INTRODUCTION

Representation-based classification (RBC) has been widely studied in the fields of computer vision and pattern recognition. It has been well applied in many visual classification tasks, such as face recognition [1, 2, 3, 4], scene classification [5, 6] and object recognition [7, 8]. Recently, RBC mainly includes two types: sparse representation-based classification (SRC) with l_1-norm regularization [1] and collaborative representation-based classification (CRC) with l_2-norm regularization [2].

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The benchmark SRC method was proposed for robust face recognition in [1]. It represents a query image sample as a linear combination of a few training image samples, and has outstanding classification performance for image recognition. SRC tends to find a representative sample from a group of the correlative training samples instead of all the training sample from the true group of the query sample. To avoid the issue, the group sparsity classification (GSC) method was proposed by l_2-norm regularization with the class information to promote the group sparsity in [9]. With an efficient closed-form solution, a simple and robust discriminative sparse representation-based classification method (DSRC) was proposed in [3]. In DSRC, the l_2-norm regularization of the class-specific reconstruction was designed to obtain the similar sparsity as SRC.

In contrast to SRC, CRC is more efficient in pattern recognition because of its closed-form solution. As argued in [2], the collaboration of l_2-norm rather than the sparsity of l_1-norm makes SRC discriminative for classification. CRC collaboratively reconstructs and classifies the query sample using all the training samples. The mechanism of CRC was analyzed from the perspective of probability in [7], and accordingly a probabilistic collaborative representation based classifier (ProCRC) was proposed. ProCRC holds the promising classification performance on some challenging visual classification data sets, compared to deep CNN models [10]. The extended ProCRC (EProCRC) was proposed by considering the priori inherent structure information of the subspace of each class in [8]. Collaborative representation optimized classifier (CROC) [11] was proposed to make a balance between CRC and nearest subspace classifier (NSC) [12] with the weighted integration of the reconstruction errors of CRC and NSC. Through the combination the collaboration of l_1-norm and the sparsity of l_2-norm, sparsity augmented collaborative representation-based classification method (SA-CRC) was proposed for improving classification performance [5]. To address the negative influence from the misleading coefficients from the wrong class, the superposed linear representation classifier (SLRC) was proposed and the discriminant nature of CRC was analyzed in [13].

Currently, two competitive CRC models as the latest extensions of CRC were proposed in [6, 14]. Both of them can make each class to competitively reconstruct the query sample. One is the competitive and collaborative representation-based classification (Co-CRC) method [6]. In the Co-CRC, the reconstruction contribution of each class is competitively enhanced by a discriminative regularization term, in order to make the correct class has more contribution to representing the query sample. The other one is the collaborative-competitive representation-based classification (CCRC) method [14]. In the CCRC, the categorical reconstruction errors as a regularization term were added into CRC to strengthen the power of competitions among all classes.

In this paper, with the aim of further improving the power of pattern discrimination for CRC, we propose a novel discriminative group collaborative competitive representation-based classification (DGCCR) method for visual classification. The proposed DGCCR not only considers the class-specific competitive representation, but also enhances the discriminative ability of each class by discriminatively degrading correlations among the categorical representations. Furthermore, a weighted class-specific group constraint on representation coefficients is taken into account in the DGCCR model. The group constraint fully employs the similarities of the categorical representations and can make the class-specific group of the training samples closer to the query sample have more contribution to representing and classifying the query sample. To demonstrate the effectiveness of the proposed DGCCR, experiments are conducted on the face recognition, scene classification and facial expression recognition data sets, in comparison with the competing RBC methods. The experimental results show that the proposed DGCCR performs very well.

2. RELATED WORK

In this section, some related RBC methods are briefly reviewed. For convenient use of the common denotations below, let $X \in \mathbb{R}^{m\times N}$ denotes the training set of N training samples from C classes and each column of X stands for one training sample. $X_i \in \mathbb{R}^{m\times n_i}$ is a collection of the ith class with n_i training samples and $N = \sum_{i=1}^{C} n_i$. Given a query sample y, the RBC methods classify it into the class with the minimum reconstruction residual among all classes generally.

2.1. The related SRC models

SRC is to seek a sparse representation coefficient vector α to reconstruct and then classify the query sample [1]. α in the benchmark SRC is solved as follows:

$$\arg\min_{\alpha}\Big\{ \parallel y - X\alpha \parallel_2^2 + \lambda \parallel \alpha \parallel_1 \Big\}, \tag{1}$$

where $\alpha \in \mathbb{R}^n$ is the vector of the representation coefficients of all the training samples and λ is a small positive parameter.

As argued in [9], SRC could choose a single sample from a group of correlated training samples instead of the group of all training samples from the true class of the query sample to represent and classify the query sample. To overcome this limitation of SRC, the group sparsity classification (GSC) method [9] was designed by the l_2-norm over the group of class-specific coefficients as

$$\arg\min_{\alpha}\Big\{ \parallel y - X\alpha \parallel_2^2 + \lambda \sum_{i=1}^{C} \parallel \alpha_i \parallel_2 \Big\}, \tag{2}$$

where $\alpha_i \in \mathbb{R}^{n_i}$ is the representation coefficient vector of the ith class and $\alpha = [\alpha_1; \alpha_2; ...; \alpha_C]$. GSC can enforces group sparsity of very few classes from all the classes for favorable classification.

Considering the computational complexity of most SRC models with l_1-norm regularization of representation coefficients, the discriminative sparse representation-based classification (DSRC) method [3] designs l_2-norm regularization of the categorical representations. The DSRC model is defined as

$$\arg\min_{\alpha}\Big\{ \parallel y - X\alpha \parallel_2^2 + \lambda \sum_{i=1}^{C}\sum_{j=1}^{C} \parallel X_i\alpha_i + X_j\alpha_j \parallel_2^2 \Big\}. \tag{3}$$

The second term in Eq. (3) can discriminatively degrade correlations among the categorical representations and enforce the sparsity. Moreover, DSRC is very simple and efficient for good classification.

2.2. The related CRC models

CRC [2] argues the collaboration of all the training samples for classification. It adopts l_2-norm regularization of the representation coefficient vector α and makes the entire training data to collaboratively represent a query sample. The collaboration representation coefficient vector α in the benchmark CRC is solved as

$$\arg\min_{\alpha}\Big\{ \parallel y - X\alpha \parallel_2^2 + \lambda \parallel \alpha \parallel_2^2 \Big\}. \tag{4}$$

To strengthen the ability of pattern discrimination of CRC, the collaborative-competitive representation-based classifier (CCRC) [14] adds a regularization term into CRC, which can competitively achieve the class-specific representation among all the classes. The CCRC model is formulated as

$$\arg\min_{\alpha}\Big\{ \parallel y - X\alpha \parallel_2^2 + \lambda_1 \sum_{i=1}^{C} \parallel y - X_i\alpha_i \parallel_2^2 + \lambda_2 \parallel \alpha \parallel_2^2 \Big\}, \tag{5}$$

where $\parallel y - X_i\alpha_i \parallel_2^2$ is the categorical reconstruction residual that can yield the ability of competitions among all the classes.

3. THE PROPOSED METHOD

In this section, the proposed DGCCR method is presented in details, and then its advantage and rationale are analyzed.

3.1. The Motivation

Generally speaking, CRC represents a query sample y with all the training samples collaboratively in the first representation phase, and then classifies y with the categorical reconstruction residuals in the classification phase. However, these two phases are separate, so as to degrade the classification performance of CRC. To solve this issue, CCRC adds the categorical reconstruction residuals into the collaborative representation and achieves the class-specific competitive representations among all the classes. Although CCRC holds the good classification performance, the crucial discrimination among different classes for classification is not considered in it. In DSRC, the discrimination term of different categorical representations is developed to degrade the correlations among all the classes for favorable classification. Using the properties of both CCRC and DSRC, we propose DGCCR model that integrates the competitions in CCRC and discriminative decorrelations in DSRC among all the classes. Moreover, we assume that one and very few similar classes could have the more competitive power in the representation phase. That is to say, the true class of the query sample in the representation phase could has stronger competition. In view of this, we design the weighted class-specific group constraint in the DGCCR model. This group constraint can not only enforce the true class of the query sample in the representation phase, but also consider the similarity between the query sample and each class.

3.2. The DGCCR Method

In CRC, a query sample y is linearly approximated by the entire training samples X within C classes and is represented as

$$y \approx X_1\alpha_1+X_2\alpha_2+\ldots+X_i\alpha_i+\ldots+X_C\alpha_C \ , \qquad (6)$$

where $X_i\alpha_i$ denotes the representation from the ith class. Then, the label $l(y)$ of y is assigned to the class that has the minimum reconstruction residual among all the classes

$$l(y) = \arg\min_{i} \| y-X_i\alpha_i \|_2^2 \ , \qquad (7)$$

where $i = 1, 2, \ldots, C$. The minimization of $\| y-X_i\alpha_i \|_2^2$ is equal to minimize $\| y \|_2^2 + \| X_i\alpha_i \|_2^2 -2y^T(X_i\alpha_i)$, which means that the value of $y^T(X_i\alpha_i)$ is maximized. In other words, Eq. (7) is to seek the class i that minimizes $X_i\alpha_i$ and makes the correlation between y and $X_i\alpha_i$ strong. $\| y-X_i\alpha_i \|_2^2$ implies the similar reason why CCRC works well. Nevertheless, the discrimination among multiple classes that is very important in representation and classification phases is ignored. Moreover, the separation of the representation and classification phases in CRC is detrimental to classification. Thus, we add the competition term $\sum_{i=1}^{C} \| y-X_i\alpha_i \|_2^2$ in the proposed DGCCR model.

To enhance the discrimination among the different classes, we consider the decorrelations among multiple classes and added the corresponding term $\sum_{i=1}^{C}\sum_{j\neq i}^{C} \| X_i\alpha_i+X_j\alpha_j \|_2^2$, which is similar to the discrimination term in DSRC. To improve the classification performance of the proposed model, we design the group constraint of the class-specific representation coefficients $w_i \| \alpha_i \|_2^2$ in the proposed model. w_i is a weight scalar which preserves similarities between the query sample and the categorical representation from class i to constrain group coefficients α_j. Inspired by LRC [4], the weight w_i is calculated as

$$w_i = \frac{r_i}{\sum_{j=1}^{C} r_j} \ , \qquad (8)$$

where $r_i = \| y - X_i\alpha_i \|_2^2$ and α_i is obtained by

$$\arg\min_{\alpha_i} \| y-X_i\alpha_i \|_2^2 \ . \qquad (9)$$

Moreover, this group constraint can enhance the pattern discrimination and competition among different classes. Thus, the whole model of the proposed DGCCR is formulated as

$$\arg\min_{\alpha}\{ \| y-X\alpha \|_2^2+\sum_{i=1}^{C}(\gamma \| y-X_i\alpha_i \|_2^2+ \beta\sum_{j\neq i}^{C} \| X_i\alpha_i+X_j\alpha_j \|_2^2+\lambda w_i \| \alpha_i \|_2^2)\} \ , \qquad (10)$$

where γ, β, λ are positive parameters for balancing the influence on each term.

For a given query sample y from the class i, the proposed DGCCR can make the reconstruction residual between y and $X_i\alpha_i$ very smaller, can strength the correlation between y and the class i, and can well degrade the correlations among different classes, in order that the classification performance can be fully improved.

3.3. The Solution of DGCCR

Since the objective function in Eq. (10) is convex and differentiable, the proposed DGCCR can be efficiently resolved by derivative.

Firstly, we can easily get $(d \ \| \ y - X\alpha \ \|_2^2 \ /d\alpha) = 2X^T(X\alpha - y)$. Then, we expand the objective function $G(\alpha) = \beta\sum_{i=1}^{C}\sum_{j\neq i}^{C} \| X_i\alpha_i+X_j\alpha_j \|_2^2$ and compute the derivative of $G(\alpha)$ with aspect to α. Since α cannot be directly included from $G(\alpha)$, we seek all partial derivations $\partial G(\alpha)/\partial\alpha_k$ $(k=1,2,\ldots,C)$ to get $dG(\alpha)/d\alpha$. We rewrite $G(\alpha)$ as

$$G(\alpha) = \beta(\sum_{i\neq k}^{C}\sum_{j\neq i,k}^{C}\|X_i\alpha_i+X_j\alpha_j\|_2^2+2\sum_{i\neq k}^{C}\|X_i\alpha_i+X_k\alpha_k \|_2^2), \qquad (11)$$

and then easily get $\partial G(\alpha)/\partial\alpha_k = 4\beta X_k^T\sum_{i\neq k}^{C}(X_i\alpha_i+X_k\alpha_k) = 4\beta X_k^TX\alpha + 4\beta(C-2)X_k\alpha_k$. Using $\partial G(\alpha)/\partial\alpha_k$, we get

$$\frac{dG(\alpha)}{d\alpha} = 4\beta X^TX\alpha + 4\beta(C-2)M\alpha \ , \qquad (12)$$

where C is the number of classes and $M = diag(X_1^TX_1, X_2^TX_2, \ldots, X_C^TX_C) \in \mathbb{R}^{N\times N}$ is a block diagonal matrix. Similarly, we obtain

$$\frac{d}{d\alpha}(\gamma\sum_{i=1}^{C} \| y-X_i\alpha_i \|_2^2) = 2\gamma(M\alpha-X^Ty), \qquad (13)$$

and

$$\frac{d}{d\alpha}(\lambda \sum_{i=1}^{C} w_i \parallel \alpha_i \parallel_2^2) = 2\lambda W \alpha, \tag{14}$$

where $W = diag(W_1, W_2, \ldots, W_C) \in \mathbb{R}^{N \times N}$ is also a block diagonal matrix and $W_i = diag(w_i, w_i, \ldots, w_i) \in \mathbb{R}^{n_i \times n_i}$ is the weight matrix for the class i. Through the computation above, the solution of objective function in Eq. (10) is obtained:

$$\alpha = (1+\gamma)((1+2\beta)X^T X + (2\beta(C-2)+\gamma)M + \lambda W)^{-1} X^T y. \tag{15}$$

Then the label $l(y)$ of y is determined by the decision function presented in Eq. (7). According to the detailed description of DGCCR, we summarize the crucial steps in Algorithm 1.

Algorithm 1 Discriminative Group Collaborative Competitive Representation Algorithm (DGCCR)

Input: A training sample set $X \in \mathbb{R}^{m \times N}$, a test sample $y \in \mathbb{R}^m$, a weight matrix $W \in \mathbb{R}^{N \times N}$ and positive constants λ, β, γ.

Output: Coefficient vector α and the class label $l(y)$.

1: Normalize each column of X.
2: Calculate α by Eq. (15).
3: Obtain the reconstruction residual of each class:
$$r_i = \parallel y - X_i \alpha_i \parallel_2^2,\ i = 1, 2, \ldots, C.$$
4: Predict the label $l(y) = \arg\min_i (r_i)$.

3.4. The Analysis of DGCCR

In this subsection, we will analyze the rationale and advantage of the proposed DGCCR method. DGCCR makes each class as a group to collaboratively, competitively and discriminatively represent a query sample by integrating the competition term, the discrimination term and the group constraint term.

As mentioned in Section 3.2, the competition term $\parallel y - X_i\alpha_i \parallel_2^2$ in Eq. (10) can be rewritten as $\parallel y \parallel_2^2 + \parallel X_i\alpha_i \parallel_2^2 - 2y^T(X_i\alpha_i)$ where the term $y^T(X_i\alpha_i)$ indicates the similarity and correlation between the query sample y and the representation from class i and the terms $\parallel y \parallel_2^2$ and $\parallel X_i\alpha_i \parallel_2^2$ is a regularized norms of y and $X_i\alpha_i$. The smaller the value of $\parallel y - X_i\alpha_i \parallel_2^2$ is, the closer the distance between y and $X_i\alpha_i$ is. This means the possibility y belongs to class i is greater. Thus, to minimize $\parallel y - X_i\alpha_i \parallel_2^2$ can enhance the true class of y to competitively achieve more similar representation for the discriminative classification. Moreover, since this competition term is the same as the decision function, the proposed DGCCR appropriately integrates the representation and classification phases in some degree.

The discrimination term $\parallel X_i\alpha_i + X_j\alpha_j \parallel_2^2$ in Eq. (10) can be written as $\parallel X_i\alpha_i \parallel_2^2 + \parallel X_j\alpha_j \parallel_2^2 + 2(X_i\alpha_i)^T(X_j\alpha_j)$ where $(X_i\alpha_i)^T(X_j\alpha_j)$ describes the correlation between the representation from class i and the representation from class j. Clearly, to minimize $\parallel X_i\alpha_i + X_j\alpha_j \parallel_2^2$ is to mainly degrade the correlation among the classes. The low correlations among classes is beneficial for classification. To further strengthen the power of the pattern discrimination, we design the class-specific group regularization of the representation coefficients. In this group constraint, we consider the similarities of the query sample and the categorical representation. The similarities are reflected by the weight W in Eq. (14). A smaller similarity weight w_i between the query sample and the categorical representation of the class i implies that the class i has great contribution to representing the query sample with larger representation coefficients α_i by minimizing the term $w_i \parallel \alpha_i \parallel_2^2$. Thus, to minimize $w_i \parallel \alpha_i \parallel_2^2$ can further improve the ability of the competition and the discrimination among the classes.

To visually present the superiority of the proposed DGCCR, we give a comparative example of the representation residuals achieved by DGCCR, CCRC and DSRC for a given query sample, because our DGCCR is based on CCRC and DSRC. The experimental example is given on GT data with 50 classes shown in Section 4.1. We choose the first five samples per class as the training samples and the rest are the testing samples. The given query sample is from class 25. The comparative and experimental example is shown in in Fig. 1. Note that the representation residual of the correct class of the query sample is plotted with a red bar and the green bar indicates the class that the query sample is wrongly classified into. Obviously, we can see the proposed DGCCR method rightly classifies the query sample, but both CCRC and DSRC wrongly classify the query sample to class 50. Thus, it can be see from Fig. 1 that the proposed DGCCR combining the categorical competitive representation and the discrimination among the classes can obtain the promising classification performance.

4. EXPERIMENTS

In this section, several experiments on the Georgia Tech (GT) face data set [15], Fifteen Scene Category data set [16] and CMU Multi-PIE facial expression data set [17] are exploited to verify the classification performance of the proposed DGCCR method. On the GT and Fifteen Scene Category data sets, the training samples are chosen randomly per class and we repeat the experiments ten times to achieve the average classification results. On the CMU Multi-PIE facial expression data set, the ResNet-50 [18] is used to extract CNN features. For each sample, we obtain a feature vector with a size of 2048×1 from the average pooling layer of ResNet. There are three parameters γ, β, λ in DGCCR. We choose the values of them from 0.001, 0.01, 0.1, 1, 10 and 100, which is the same setting of other RBC methods for their corresponding parameters.

4.1. Face Recognition

The Georgia Tech (GT) data set contains 750 frontal or tilted face images from 50 subjects. Each face image has the size 150×150 pixels and we resize it to 32×32 pixels. Each subject has 15 images taken in various positions, expressions and illuminations. The numbers of training samples per class are chosen from 7 to 11 with a step 1. The experiments on GT are conducted by comparing the proposed DGCCR with SR-

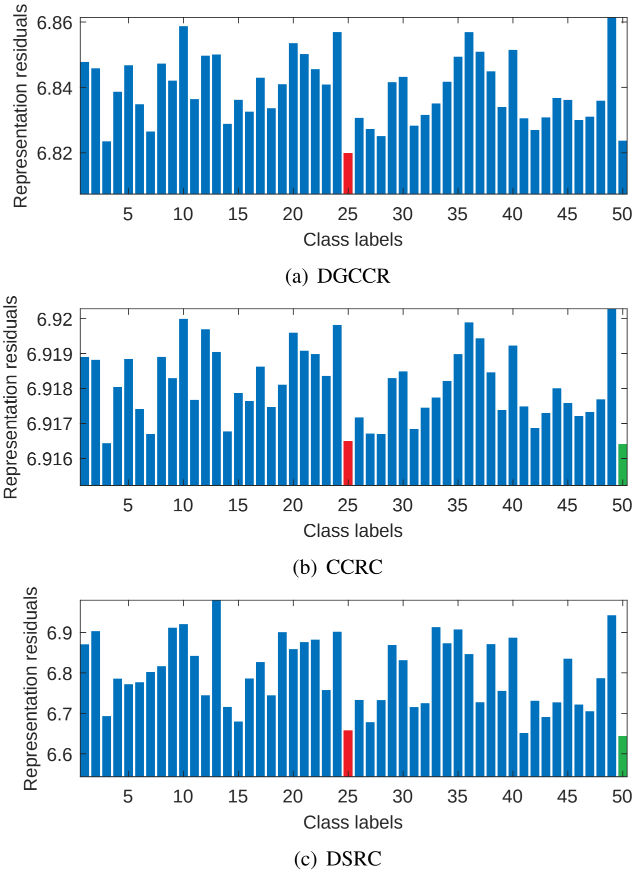

Fig. 1. The class-specific residuals of one given test sample from class 25 on GT using DGCCR, CCRC and DSRC, respectively.

C [1], CRC [2], GSC [9], NSC [12], DSRC [3], CCRC [14], ProCRC [7], EProCRC [8], CROC [11].

The comparative experiments on GT are shown in Table 1. It can be seen that DGCCR achieves the best classification performance among all the competing methods with different numbers of the training samples. This means the combination of the competitive representation from each class and the discrimination among classes in the proposed DGCCR can well improve the classification performance. Note that since CROC combines both NSC and CRC, CROC often degenerates to NSC with the same classification performance when CRC has lower classification accuracies than NSC. Moreover, the fact that DGCCR significantly performs better than CCRC means the effective utilization of the discrimination among different classes is very crucial for classification.

4.2. Scene category recognition

The Fifteen Scene Category data set is collected from 15 natural scenes, each of which contains 200-400 images. The size of each image is 250×300 pixels. For each image, we extract a spatial pyramid feature with 1000 dimensionalities using the methods in [16]. We randomly select 10, 18, 26, 34, 42 images from each class as the training samples. The same competing methods used in subsection 4.1 are also adopted to verify the classification performance of the proposed DGCCR.

The experimental comparisons on the Fifteen Scene Category data set in terms of the classification accuracies are shown in Table 2. It can be seen from Table 2 that the proposed DGCCR performs best among all the competing methods. Different from EProCRC which has low classification accuracies on GT but high accuracies on Fifteen Scene Category, the performance of DGCCR always achieves the good classification performance. The experimental results in Table 2 implies the proposed DGCCR has effective and robust classification performance by fully considering both competition and discrimination among the classes.

Table 1. The classification accuracies (%) of the various competing methods on the GT face data set

Method	7	8	9	10	11
SRC	70.17	72.57	73.45	72.20	73.17
CRC	69.21	71.29	73.89	74.20	73.50
GSC	71.58	73.10	75.38	77.00	79.25
NSC	76.17	78.52	81.39	81.60	83.92
DSRC	77.83	78.67	82.00	82.80	85.42
CCRC	77.17	78.43	81.55	82.20	84.17
ProCRC	71.88	72.85	74.50	75.20	76.92
EProCRC	64.29	65.96	67.72	66.67	67.92
CROC	76.17	78.52	81.39	81.60	83.92
DGCCR	**78.08**	**78.95**	**82.39**	**82.93**	**85.50**

Table 2. The classification accuracies (%) of the various competing methods on the Fifteen Scene Category data set

Method	10	18	26	34	42
SRC	82.96	89.92	93.52	95.53	96.92
CRC	87.02	93.95	95.92	97.16	97.55
GSC	83.95	91.22	95.01	97.45	97.73
NSC	85.04	93.70	96.46	97.91	98.36
DSRC	87.92	94.46	96.03	96.88	96.93
CCRC	87.81	94.41	96.67	97.83	98.33
ProCRC	82.71	90.90	94.92	97.07	97.80
EProCRC	85.21	93.49	96.53	**97.96**	98.41
CROC	87.64	94.03	96.48	97.80	98.26
DGCCR	**88.20**	**94.52**	**96.71**	**97.96**	**98.49**

4.3. Expression recognition

The CMU Multi-PIE facial expression data set is formed by a subset of the CMU Multi-PIE data set [17]. There are 6123 images for training and 1531 images for testing. Six expressions including scream, smile, squint, surprise, neutral and disgust are contained in the CMU Multi-PIE facial expression data set. We train the ResNet-50 model with 6123 training images and then get the CNN features of both training images and testing images for classification. In addition to the competing methods used in the above experiments, the ResNet-50 with softmax classifier is also used to demonstrate the classification performance of the proposed DGCCR method.

Table 3. The classification accuracies (%) of the various competing methods on the CMU Multi-PIE facial expression data set

Method	CMU Multi-PIE Expression Data Set
SRC	86.68
CRC	87.52
GSC	87.63
NSC	62.77
DSRC	71.06
CCRC	87.85
ProCRC	87.85
EProCRC	87.72
CROC	87.52
ResNet-50	87.39
DGCCR	**88.31**

It is clear from Table 3 that the proposed DGCCR outperforms all the other methods. This means the proposed DGCCR has more power of pattern discrimination on expression recognition. Most interestingly, using the deep learning based features, most competing representation-based methods has better classification performance than ResNet-50, which implies that some RBC methods could be used rather than the softmax classifier if we want obtain better classification performance after extracting deep feature.

5. CONCLUSION

In this paper, we propose a novel representation-based classification method, called the discriminative group collaborative competitive representation-based classification method (DGCCR) for visual classification. The proposed DGCCR integrates the competitive representation of each class that is beneficial for representation and the discriminative decorrelations among the classes that is crucial for classification. Meanwhile, the class-specific group constraint on representation coefficients is designed in the proposed model. The group constraint considers the similarities between each query sample and the categorical representations, and further strengthens the ability of discrimination and competition among the classes. Experiments on three different types of visual data sets demonstrate the superiority of the proposed DGCCR method, in comparisons with the other state-of-the-art representation-based classification methods.

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