

Project: Multigrid code

Problem

1. Use two Jacobi iterations as the smoother to implement a 2-level multigrid method solving

$$-u_{xx} = 1, x \in [0, 1], \quad u(0) = u(1) = 0, \quad (1)$$

with central difference FD spatial discretization and grid size $\Delta x = 1/N$ and $N = 50, 100, 200$.

Suppose the matrix-vector form for this problem is $Au = b$. Set the stopping criteria for the multigrid as the residual $\|Au - b\|_2 / \|b\|_2 < 10^{-8}$ where $\|\cdot\|_2$ is the l_2 -norm.

- Write out the algorithm details.
- Report the number of iterations for convergence with different N . Explain your observations.

2. Modify your multigrid code to solve

$$u_x = 1, x \in [0, 1], \quad u(0) = 0, u(1) = 1, \quad (2)$$

with 1st order upwind finite difference discretization with grid size $\Delta x = 1/N$ for $N = 50, 100, 200$.

Suppose the matrix-vector form for this problem is $Au = b$. Set the stopping criteria for the multigrid as the residual $\|Au - b\|_2 / \|b\|_2 < 10^{-8}$ where $\|\cdot\|_2$ is the l_2 -norm. Set the maximum number of iterations as 200.

Report the number of iterations for convergence with different N . Explain your observations. You may see multigrid method does not work while this time.

3. Replace the smoother for part 2 with one Gauss-Siedel iteration and explain your observations.