Artificial Intelligence
UNSW Sydney
Topic exercises: Search - **Solutions**

Question 1: Romanian map

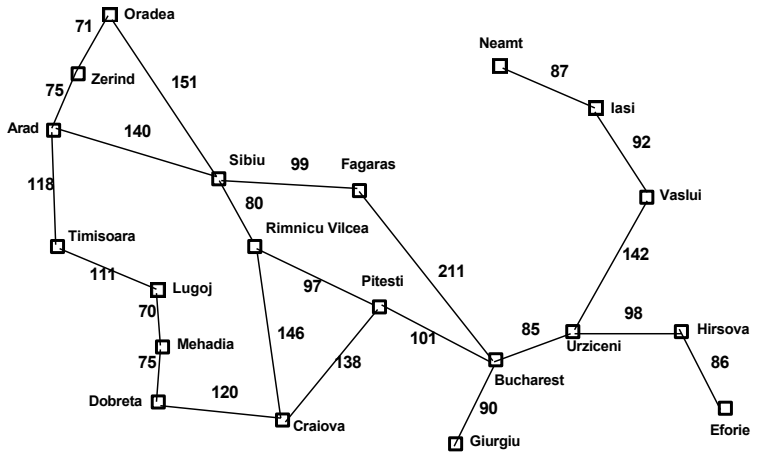
This exercise uses the route-finding example with the Romanian map from Russell & Norvig (Artificial Intelligence: A Modern Approach). See Fig. 1.

For the route from Arad to Bucharest, in what order are nodes in the state space expanded for each of the following algorithms when searching for the shortest path between Arad and Bucharest? Where there is a choice of nodes, take the first one alphabetically. Make sure you understand the key properties of the different algorithms, as listed below.

1. Depth-first search.
2. Breadth-first search.
3. Uniform-cost search.
4. Greedy best-first search.
5. A* search.

Answer:

1. (Depth-first) Arad, Sibiu, Fagaras, Bucharest (note that the solution is found on the first branch only because of the rule for ordering the successors alphabetically; this is not usually the case!).



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Figure 1: Romanian map from Russell & Norvig (Artificial Intelligence: A Modern Approach).

2. (Breadth-first) Arad, Sibiu, Timisoara, Zerind, Fagaras, Bucharest (assuming that the search stops once the goal state is generated and that when expanding a node, previously expanded nodes are checked to ensure that nodes with states already explored are not added to the frontier, e.g. Arad which is generated via the paths Arad → Sibiu → Arad and Arad → Sibiu → Oradea → Zerind → Arad).

3. (Uniform-cost) Arad (0), Zerind (75), Timisoara (118), Sibiu (140), Oradea (146), Rimnicu Vilcea (220), Lugoj (229), Fagaras (239), Mehadia (299), Pitesti (317), Craiova (366), Drobeta (374), Bucharest (418) (assuming a check that nodes with states previously generated are not added to the frontier, except when they have a lower path cost than a node with that state already on the frontier, in which case the node with higher path cost is removed, so ignore Oradea (291) reached via Sibiu, and Sibiu (297) reached via Zerind and Oradea).

4. (Greedy) Arad (366), Sibiu (253), Fagaras (178), Bucharest (0).

5. (A*) Arad (366), Sibiu (393), Rimnicu Vilcea (413), Pitesti (415), Fagaras (417), Bucharest (418) (unexpanded states on the frontier are Timisoara (447), Zerind (449), Craiova (526), Oradea (671)) – For example, f(Rimnicu Vilcea) = g(Rimnicu Vilcea) + h(Rimnicu Vilcea) =

$140 + 80 + 193 = 413$ (remember to use the total path cost from Arad to Rimnicu Vilcea here).

Question 2: Relationships between search strategies

Prove each of the following statements, or give a counterexample:

1. Breadth First Search is a special case of Uniform Cost Search.
2. Breadth First Search, Depth First Search and Uniform Cost Search are special cases of best-first search.
3. Uniform Cost Search is a special case of A*Search.

Answer:

1. Uniform Cost Search reduces to Breadth First Search when all edges have the same cost.
2. Best-first search reduces to Breadth-First Search when f(n)=number of edges from start node to n, to UCS when f(n)=g(n). It can be reduced to DFS by, for example, setting f(n)=-(number of nodes from start state to n) (thus forcing deep nodes on the current branch to be searched before shallow nodes on other branches). Another way to produce DFS is to set f(n)=-g(n) (but this might produce a particularly bad choice of nodes within the DFS framework – For example, try tracing the order in which nodes are expanded when travelling from Urziceni to Craiova).
3. A* Search reduces to UCS when the heuristic function is zero everywhere, i.e. h(n)=0 for all n. This heuristic is clearly admissible since it always (grossly!) underestimates the distance remaining to reach the goal.

Question 3: Modified Romanian map

Suppose in Question 1, the heuristic value for Fagaras is 176 rather than 178, and the value for Pitesti is 100 rather than 98. What difference does this make?

Answer: Fagaras (415) is expanded before Pitesti (417).

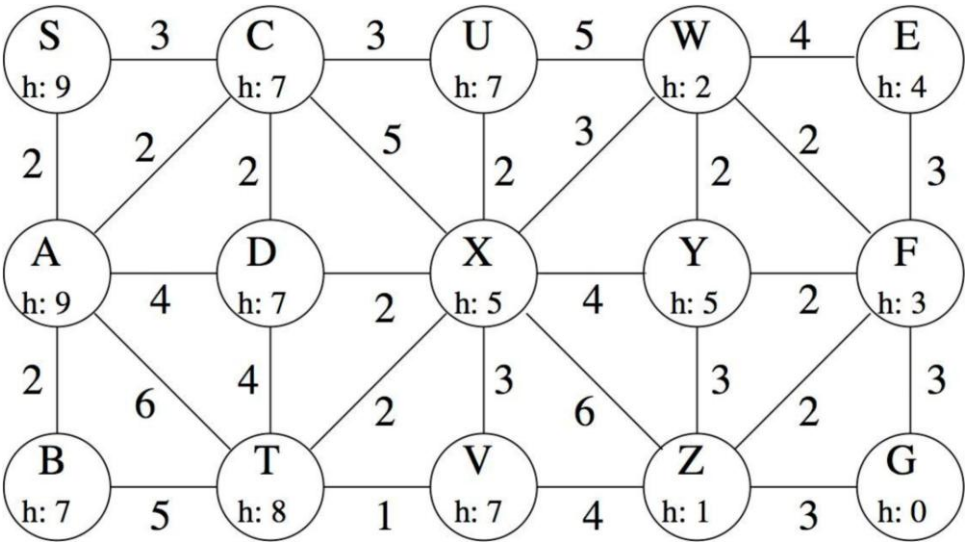


Figure 2: Path search.

Question 4: Path search

Consider the task of finding a path from start state S to goal state G, given the distances and heuristic values in Fig. 2.

For each of the following strategies, list the order in which the states are expanded. Whenever there is a choice of states, you should select the one that comes first in alphabetical order. In each case, you should skip any states that have previously been expanded, and you should continue the search until the goal node is expanded.

1. Depth-first search.
2. Breadth-first search.
3. Uniform-cost search.
4. Greedy best-first search.
5. A* search.

Answer:

1. Breadth-first search: S, A, C, B, D, T, U, X, V, W, Y, Z, E, F, G.

2. Depth-first search: S, A, B, T, D, C, U, W, E, F, G.

3. Uniform-cost search [Hint: first compute g() for each state in the graph]: S(0), A(2), C(3), B(4), D(5), U(6), X(7), T(8), V(9), W(10), Y(11), F(12), Z(13), E(14), G(15).

4. Greedy search: using the heuristic shown S, C, X, Z, G.

5. A* search: using the heuristic shown S(9), C(10), A(11), B(11), D(12), X(12), W(12), U(13), Z(14), F(15), G(15). Note that g(X) becomes 8 after C is expanded, but drops to 7 after D is expanded.