**A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems**

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# ABSTRACT

We have compared three modeling approaches for the radiant system: 1) an RC network model, 2) a GGMR approach, and 3) a combination of the RC and GGMR models. The study performed minutely heating and cooling rates prediction for using 50 days, from January 15th to March 7th, 2022, for a Living Laboratory office space at Purdue University. Both three modeling approaches used the first two weeks data for training and the rest of data as testing data set. For the performance comparison, the RC model has achieved normalized root mean square error (NRMSE) as 3%, coefficient of variation of root mean square error (CVRMSE) as 144.18, mean absolute error (MAE) as 855.30 watts, and mean absolute percentage error (MAPE) as 26%.

# 1. INTRODUCTION

# 2. METHODLOGY

This section will elaborate on the methodology developed to improve the prediction performance, which is starting from the RC network model (Braun & Chaturvedi, 2002; Joe & Karava, 2017), the GGMR approach, and the Hybrid Modeling approach from RC model and GGMR. Finally, all the performance criteria metrics are described in the last subsection.

## 2.1 RC Network Model

A gray-box RC network model is formed from heat balance equations on each temperature or state variable. A general heat balance equation has been listed below. represent the node temperature, the specific heat capacity, the resistance between two nodes, the heat flux input to the node. And neighboring temperature node is denoted as .

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surround by the estimated resistors and capacitors. The input vector includes all the driving conditions, such as the hot water or chilled water temperature and derivation along the sampling time within tubes, outdoor air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | (3) |
|  |  | (4) |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

In general, the above gray-box RC model optimization problem is not linear nor convex in terms of the estimate parameter and output variable trajectory. Particle swarm optimization (PSO) from python package (pyswarms (James V. Miranda, 2018)) was used to solve the above optimization problem.

## 2.2 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | (6) |
|  |  | (7) |
|  |  | (8) |
|  |  | (9) |
|  | = | (10) |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section presents the case-study for the development of the Hybrid Model of RC network model and GGMR approach. It starts with the description of data collection process, then presents with the development and performance of each modeling approach.

## 3.1 Data Description

The minutely data consists of two types, onsite sensor data and estimated data. In terms of the onsite sensor data, they include the followings: outdoor air temperature denoted as , Façade cavity space temperature denoted as , slab concrete temperature denoted as , flowing water temperature within slab pipe denoted as , solar radiation retrieved from a weather station denoted as , air handling unit consumed heating power . As for the estimated input data, they are estimated according to ASHRAE 90.1 (*ANSI/ASHRAE/IES 90.1-2016, Energy Standard for Buildings Except Low Rise Residential Buildings.*, n.d.), such as internal heating radiation denoted as , lighting radiation .

## 3.1 RC Network Model Development

Two data-driven RC network models have been developed based on the state-space formulation from Equation (**1**) as shown in Figure **1** including 4-states model and 6-states model. Different from 4-states model, 6-states model explicitly set the thermal insulation layer node below slab pipes as additional state node. Figure **1** depicts two different electrical analogs for radiant slab systems RC network, in which denote temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. And the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements. Figure 2 and Figure 3 show the predicted and measured results for testing period. As shown in comparison Table 2, Model 2 has much lower MAPE and has been selected as the best model for RC network approach.

The Model 2 can be represented with a state-space model with the following definition for state, input, and output variables:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | (12) |
|  |  | (13) |

Correspondingly, thermal resistances, (), and thermal capacity, (), are estimated from the following equations, whose estimated values have been shown in Table 1:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | (15) |

**Table 1** Estimated Values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

Description automatically generated

**Figure** **1** Structure of RC Network. Left: Model 1 with 4 states; Right: Model 2 with 6 states.

A picture containing chart

Description automatically generated

**Figure 2** Testing Results for Model 1 and Model 2

Chart, line chart, histogram

Description automatically generated

**Figure 3** Predicted Absolute Percentage Error for Model 1 and Model l

**Table 2** Comparison of proposed model

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE** | **CVRMSE** | **MAE** | **MAPE** |
| Model 1 | 2.66 | 4093.97 | 5471.16 | 1.05 |
| Model 2 | **0.03** | **144.18** | **835.30** | **0.26** |

**NOMENCLATURE**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | L | thickness | *m* |
|  | capacitors | J/K |  | conductivity | *w/m/K* |
|  | Specific heat | J/Kg/K | Q | heating flux | *W* |
|  | heat transfer coefficient |  |  | density | *Kg/m3* |
| *R* | resistors | K/W |  |  |  |
| *T* | temperature | K |  |  |  |
| *t* | time | second |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *env* | envelope |  |
| *AHU* | Air handling unit |  | *intwall* | Internal wall |  |
| *cav* | cavity |  | *int* | Internal heating |  |

# REFERENCES

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