**A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems**

First AUTHOR1\*, Second AUTHOR2

1Organization, Department or Equivalent,

City, State, Country

Contact Information (Phone, Fax, E-mail)

2Organization, Department or Equivalent,

City, State, Country

Contact Information (Phone, Fax, E-mail)

\* Corresponding Author

# ABSTRACT

For the radiant system, we compared three modeling approaches: 1) an RC network model; 2) a GGMR method; and 3) a hybrid of the RC and GGMR models. The study predicted heating and cooling rates for a Living Laboratory office space at Purdue University for 50 days, from January 15th to March 7th, 2022. The first two weeks of data were used for training, while the remaining data was used as a testing data set in all three modeling methodologies. In terms of performance, the RC model has a normalized root mean square error (NRMSE) of 16.15%, a coefficient of variation of root mean square error (CVRMSE) of 21.31%, a mean absolute error (MAE) of 835.30 watts, and a mean absolute percentage error (MAPE) of 26.10%.

# 1. INTRODUCTION

# 2. METHODLOGY

This section will elaborate on the methodology developed to improve the prediction performance, which began with the RC network model (Braun & Chaturvedi, 2002; Joe & Karava, 2017), then moved on to the GGMR approach, and finally to the Hybrid Modeling approach combining the RC and GGMR model. In the last subsection, all the performance criteria metrics are described.

## 2.1 RC Network Model

Heat balance equations on each temperature or state variable are used to create a gray-box RC network model. A general heat balance equation has been listed below. represent the node temperature, the specific heat capacity, the resistance between two nodes, the heat flux input to the node. And neighboring temperature node is denoted as .

|  |  |  |
| --- | --- | --- |
|  |  | () |

A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surrounded by the estimated resistors and capacitors. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | () |

In terms of the estimate parameter and output variable trajectory, the above gray-box RC model optimization issue is neither linear nor convex. Particle swarm optimization (PSO) from python package (pyswarms (James V. Miranda, 2018)) was used to solve the above optimization problem.

Based on the state-space formulation from Equation (2), two data-driven RC network models have been constructed as illustrated in Figure **1**, including 4-states Model 1, 6-states Model 2 and 5-states Model 3. In Model 1, the detailed structure of radiant flow has been neglected. And Model 2 has higher order than Model 3 to incorporate the temperature state of thermal insulation beneath pipes. FigureError! Reference source not found.shows those distinct electrical analogs for the radiant slab systems RC networks, in which represents temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. And the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements. Figure 2 shows the predicted and measured results during testing period. Model 2 has a substantially lower MAPE, as demonstrated in Table 2, and has been chosen as the optimum model for the RC network technique

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |
|  |  | (7) |
|  |  | (8) |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table 1:

|  |  |  |
| --- | --- | --- |
|  |  | (9) |
|  |  | (10) |

**Table 1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

Description automatically generated

**Figure 1** Structure of RC network. Left: Model 1 with 4 states; Middle: Model 2 with 6 states; Middle: Model 3 with 5 states.

Timeline

Description automatically generated with medium confidence

**Figure 2** Testing results for Model 1, Model 2 and Model 3

**Table 2** Comparison of proposed RC models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (W)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5755.32 | 87.88 |
| Model 2 | **16.15** | **21.31** | **835.30** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1278.94 | 35.89 |

## 2.2 GGMR

Gaussian mixture regression (GMR) is a regression approach that models probability distributions rather than functions. It consists of training phase, (learning a Gaussian Mixture Model (GMM), see Equation (6) through iterative expectation maximization (EM) algorithm), and predicting phase using Equation (7).

|  |  |  |
| --- | --- | --- |
|  |  | () |

where K is the number of Gaussians, is the priors or weight coefficient for each gaussian, , is the gaussian distribution notation with mean and covariance .

As for the prediction phase, GMR can be used to predict distributions of variables y by computing the conditional distribution . The conditional distribution of each individual Gaussian , where , is defined as

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

And the posterior for each gaussian is:

|  |  |  |
| --- | --- | --- |
|  |  | () |

Thus we can obtain the conditional distribution

|  |  |  |
| --- | --- | --- |
|  |  | () |

In the current study, we are interested in the expectation of y among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | () |

According to Bouchachia et al., an advanced version of GMR, Growing Gaussian Mixture Model, enabled by incremental learning has been proposed. The overall algorithm can be summarized as three phases:

A. Probability of Match. In this phase, we will find the best match Gaussian using posteriors after filtering with Mahalanobis distance and closeness threshold .

B. Accommodating New Data. Depending on if existing the best match Gaussian, the best match Gaussian will be updated with new data or less contributing Gaussian will be replaced.

C. Refinement of the Model. Depending on the calculated volumes and Kullback-Leiber divergence (*kld*) between two Gaussians, Gaussians might be split or merged.

The Mahalanobis distance between an input and a Gaussian :

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | (25) |
|  |  | (26) |
|  |  | (27) |
|  |  | (28) |
|  |  | (29) |

Table Prediction performance comparison for different G/GMR inputs

|  |  |
| --- | --- |
| **Input** | **CVRMSE (%)** |
|  | GMR: 24.5~31.08%; GGMR 23.93~29.5% |
|  | GMR:17.23~21.51% ; GGMR 13.72 ~ 14.56 % |

## 2.3 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  | = | () |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section gives a case study for the creation of the Hybrid Model of the RC network model and the GGMR technique. It begins with a description of the data collection procedure and then presents with the development and performance of each modeling approach.

## 3.1 Data Description

The minutely data consists of two types, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted as , Façade cavity space temperature denoted as , slab concrete temperature denoted as , flowing water temperature within slab pipe denoted as , solar radiation retrieved from a weather station denoted as , air handling unit consumed heating power . The estimated input values are calculated in accordance with ASHRAE 90.1 (*ANSI/ASHRAE/IES 90.1-2016, Energy Standard for Buildings Except Low Rise Residential Buildings.*, n.d.), such as internal heating radiation denoted as , lighting radiation .

## 3.2 Performance Comparison

**Table 4** Comparison of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (W)** | **MAPE (%)** |
| RC | 16.15 | 21.37 | 835.30 | 26.10 |
| GMR | 15.69 | 20.33 | 501.68 | 10.66 |
| GGMR | 6.00 | 7.56 | 361.60 | 7.68 |

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

# REFERENCES

*ANSI/ASHRAE/IES 90.1-2016, Energy Standard for Buildings Except Low Rise Residential Buildings.* (n.d.). American Society of Heating, Refrigerating and Air-Conditioning Engineers.

Braun, J. E., & Chaturvedi, N. (2002). An Inverse Gray-Box Model for Transient Building Load Prediction. *HVAC&R Research*, *8*(1), 73–99. https://doi.org/10.1080/10789669.2002.10391290

James V. Miranda, L. (2018). PySwarms: A research toolkit for Particle Swarm Optimization in Python. *The Journal of Open Source Software*, *3*(21), 433. https://doi.org/10.21105/joss.00433

Joe, J., & Karava, P. (2017). Agent-based system identification for control-oriented building models. *Journal of Building Performance Simulation*, *10*(2), 183–204. https://doi.org/10.1080/19401493.2016.1212272