**A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems**

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# ABSTRACT

To accurately predict the energy consumption of radiant slab (RS) systems with large thermal inertia, we explored a novel hybrid approach, which is enabled by the combination of resistor-capacitor (RC) network and growing Gaussian mixture regression (GGMR) model. The proposed method will predict RS system loads with updating Gaussians in online setting, where RC network will feed GGMR module with real-time predicted RS system loads as one of GGMR inputs. Three modeling approaches have been compared with a case study for predicting the hourly RS system load of a Living Laboratory office space at Purdue University for 50 days, from January 15th to March 7th, 2022: 1) an RC network model; 2) a GGMR method; and 3) the proposed hybrid approach. The first two weeks of data were used for training, while the remaining data was used as a testing data set in all three modeling methodologies. The RC model has a normalized root mean square error (NRMSE) of 13.56%, a coefficient of variation of root means square error (CVRMSE) of 15.59%, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53%. GGMR has NRMSE of 15.89%, CVRMSE of 17.76%, MAE of 6.40 kW, and MAPE of 27.68%. The hybrid approach has NRMSE of 8.77% (35% lower from RC, 45% lower from GGMR), CVRMSE of 9.95% (36% lower from RC, 44% lower from GGMR), MAE of 3.62 kW (37 lower from RC, 43% lower from GGMR), and MAPE of 19.31% (82% lower from RC, 30% lower from GGMR).

# 1. INTRODUCTION

Recently, hydronic radiant slab systems have demonstrated many advantages for conditioned space thermal management, such as improved thermal comfort and reduced energy cost. And the above superior performance relies on the advanced control strategies, such as Model Predictive Control (MPC), since conventional control strategies usually end up with over-heated conditioned space due to the large thermal inertia of radiant slab systems. To maximize control potential of MPC accurate energy load predictions are required.

Conventional prediction strategies include reduced order method (e.g. thermal resistor-capacitor (RC) network) models. Creating and calibrating such models require substantial model development efforts. Apart from the physics based methods, data-driven methods have been proved as simpler but efficient models that can be readily trained using in-site measurements. For example, Wang et al.(Wang et al., 2018) researched the hourly energy consumption prediction using Gaussian Mixer Model Regression. And Karami et al.(Karami & Wang, 2018) used Unscented Kalman filter to construct a novel adaptive Gaussian mixture model (AGMM). Bouchachia et al.(Bouchachia & Vanaret, 2011) utilized incremental learning to learn Growing Gaussian Mixture Regression (GGMR).

In the present paper, we explored the thermal load prediction performance from a hybrid approach, where we combined the prediction from an RC network with GGMR algorithm to enhance both prediction performance.

In the sequel, the performance metrics, methodology and model development are detailed in Sec. 2. Section 3 presented the real-world hybrid approach prediction performance for an existing office located at Purdue University before a conclusion in Sec. 4.

# 2. METHODLOGY

This section will elaborate on the methodology developed to improve the prediction performance, which began with prediction performance criteria metrics, then the RC network model development are described before moving on to the GGMR approach, and finally to the Hybrid Modeling approach combining the RC and GGMR model.

## 2.1 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | (1) |
|  |  | (2) |
|  |  | (3) |
|  |  | (4) |
|  | = | (5) |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

## 2.2 RC Network Model

Heat balance equations on each temperature or state variable are used to create a gray-box RC network model(Braun & Chaturvedi, 2002; Joe & Karava, 2017). A general heat balance equation has been listed below. represent the node temperature, the specific heat capacity, the resistance between two nodes, the heat flux input to the node. And neighboring temperature node is denoted as .

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surrounded by the estimated resistors and capacitors. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | (8) |
|  |  | (9) |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

Particle swarm optimization (PSO) from python package (pyswarms (James V. Miranda, 2018)) was used to solve the above optimization problem.

Based on the state-space formulation from Equation (7), three data-driven RC network models have been constructed as illustrated in Figure **1,** in which represents temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. And the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements. Figure 1 includes 4-states Model 1, 6-states Model 2 and 5-states Model 3. shows those distinct electrical analogs for the radiant slab systems RC networks, In Model 1, the detailed thermal structure of radiant flow has been neglected. And Model 2 has higher order than Model 3 to incorporate the temperature state of thermal insulation beneath pipes. Figure 2 shows the predicted and measured results during testing period. Model 2 has a substantially lower CVRMSE, as detailed in Table 2, and has been chosen as the optimum model for the RC network technique.

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table 1:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

**Table 1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

Description automatically generated

**Figure 1** Structure of RC network. Left: Model 1 with 4 states; Middle: Model 2 with 6 states; Middle: Model 3 with 5 states.

Timeline

Description automatically generated with medium confidence

**Figure 2** Testing results for Model 1, Model 2 and Model 3

**Table 2** Comparison of proposed RC models (5-mins interval)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5.76 | 87.88 |
| Model 2 | **16.15** | **21.31** | **0.84** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1.28 | 35.89 |

## 2.3 GGMR

Gaussian mixture regression (GMR)(Fabisch, 2021) is a regression approach that models probability distributions rather than functions. It consists of training phase, (learning a Gaussian Mixture Model (GMM), see Equation (6) through iterative expectation maximization (EM) algorithm), and predicting phase using Equation (7).

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

where K is the number of Gaussians, is the prior or weight coefficient for each gaussian, , is the gaussian distribution notation with mean and covariance .

As for the prediction phase, GMR can be used to predict distributions of variables y by computing the conditional distribution . The conditional distribution of each individual Gaussian , where , is defined as

|  |  |  |
| --- | --- | --- |
|  |  | (17) |
|  |  | (18) |

And the posterior for each gaussian is:

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

Thus we can obtain the conditional distribution

|  |  |  |
| --- | --- | --- |
|  |  | (20) |

In the current study, we are interested in the expectation of y among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | (21) |

According to Bouchachia et al.(Bouchachia & Vanaret, 2011), GGMR can be used to either update, generate, split or merge Gaussians to accommodate new data in an online setting. We utilized the updating gaussians part in the present paper. More details can be seen in Bouchachia et al(Bouchachia & Vanaret, 2011). The best match Gaussian will be updated with the following formulas:

|  |  |  |
| --- | --- | --- |
|  |  | (22) |
|  |  | (23) |
|  |  | (24) |
|  |  | (25) |
|  |  | (26) |
|  |  | (27) |

in which is the match probability calculated with new input and best match Gaussian , is the expected posterior, is the sum of the expected posterior for best match Gaussian, is the weights of best match Gaussian, is the on-going learning rate for j-th Gaussian, is the converging learning rate.

## 2.4 Hybrid Approach

The schema shown below illustrates the underlying structure of the hybrid approach. Enabled by the real time predicted system load from RC network model and incremental learning framework from (Bouchachia & Vanaret, 2011), those trained gaussian components from Expectation Maximization (EM) will be updated accordingly. Specifically, the RC network module will get the target time step index from GGMR. Then RC module will predict the energy load with a certain period of warming up, where the warming up period is statistically selected (Figure 4). As referred from Wang et al.(Wang et al., 2018), we utilized correlation coefficients between RS system load and all other variables to select the best input variables for the hybrid approach, as shown in Figure 5. In the present study, we also investigated the impact of predicted water flow rate through pipes as shown in Table 3. And we finally selected as the hybrid approach inputs.

Diagram

Description automatically generated

**Figure 3** Underlying Communication for Hybrid Approach

Chart

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Figure Determination of warming up steps for hybrid approach

Table

Description automatically generated

**Figure 5** Correlation coefficients heatmap for hybrid approach input out variables.

**Table 3** Prediction performance comparison for different G/GMR inputs

|  |  |
| --- | --- |
| **Inputs** | **CVRMSE (%)** |
|  | 25.34% |
|  | 11.22 % |
|  | 25.02% |
|  | 9.95% |

# 3. CASE STUDY

This section gives a case study for the creation of the Hybrid Approach. It begins with a description of the data collection procedure, then moved to the hyperparameters selections, and finally presents with the comparison of each modeling approach performance.

## 3.1 Data Description

The minutely data consists of two types, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted as , Façade cavity space temperature denoted as , slab concrete temperature denoted as , flowing water temperature within slab pipe denoted as , solar radiation retrieved from a weather station denoted as , air handling unit consumed heating power . The estimated input values are calculated in accordance with ASHRAE 90.1 (*ANSI/ASHRAE/IES 90.1-2016, Energy Standard for Buildings Except Low Rise Residential Buildings.*, n.d.), such as internal heating radiation denoted as , lighting radiation .

## 3.2 Determination of hybrid approach

Chart, line chart

Description automatically generated

**Figure 6** The effect of number of Gaussian components

## 3.3 Performance Comparison across RC, GGMR and Hybrid

**Table 5** Hourly prediction performance comparison of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| RC | 13.56 | 15.59 | 5.76 | 108.53 |
| GGMR | 15.89 | 17.67 | 6.40 | 27.68 |
| GGMR+RC | **8.77** | **9.95** | **3.62** | **19.31** |

# 4. Conclusion

In the present paper, we explored the thermal load prediction performance from a hybrid approach, where we combined the prediction from an RC network with GGMR algorithm to enhance both prediction performance.

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

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