**A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems**

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# ABSTRACT

Accurately predicting the performance of radiant slab systems can be challenging due to the large thermal capacitance of the radiant slab and room temperature stratification. Current methods for predicting heating and cooling energy consumption of hydronic radiant slabs include detail first-principle-based (e.g, finite difference) and reduced-order (e.g, thermal resistor-capacitor (RC) network) models. Creating and calibrating detailed first-principle models, as well as RC network models for predicting the performance of radiant slabs requires substantial modeling efforts. To develop improved control, monitoring, and diagnostic methods, there is a need for simpler models that can be readily trained using in-situ measurements.

In this study, we explored a novel hybrid modeling method integrating a simple RC network model with an evolving learning-based algorithm growing Gaussian mixture regression (GGMR) modeling approach to predict the heating and cooling rates of a radiant slab system for a Living Laboratory office space. The RC network model provides heating or cooling load of the radiant slab system to the GGMR model as one of the inputs. The three modeling approaches have been compared with a case study for predicting the hourly RS system load of a Living Laboratory office space from January 15th to March 7th, 2022: 1) an RC network model; 2) a GGMR method; and 3) the proposed hybrid approach. The first two weeks of data were used for training, while the remaining data was used as a testing data set in all three modeling methods. The RC model has a normalized root mean square error (NRMSE) of 13.56%, a coefficient of variation of root means square error (CVRMSE) of 15.59%, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53%. The GGMR model has NRMSE of 15.89%, CVRMSE of 17.76%, MAE of 6.40 kW, and MAPE of 27.68%. The hybrid approach has NRMSE of 8.77% (35% lower from RC, 45% lower from GGMR), CVRMSE of 9.95% (36% lower from RC, 44% lower from GGMR), MAE of 3.62 kW (37% lower from RC, 43% lower from GGMR), and MAPE of 19.31% (82% lower from RC, 30% lower from GGMR).

# 1. INTRODUCTION

Recently, hydronic radiant slab systems (HRSS) have demonstrated advantages for conditioned space thermal management, such as improved thermal comfort and reduced energy cost. Apart from the above benefits, the large thermal storage of HRSS brings negative influences. One problem is that it delays cooling output when there are increase of internal heat and adjustment of supply water(Liu et al., 2011). Overcooling or overheating are common problems for RS control. Due to large thermal time constant, conventional control using room temperature feedback even can lead to higher primary energy use than the conventional air system(Sourbron et al., 2009). In addition, the operation of Hydronic Radiant Slab Systems are usually subject to multiple thermal disturbances simultaneously, such as solar radiation, internal heat, and air systems. As indicated in (Joe & Karava, 2019), Model Predictive Control (MPC) is required to reduce overheating in the heating season. And accurately predicting RS system load is crucial for MPC. Generally, there are two categories of building energy models, e.g. physics-based model from the theory of heat transfer and thermodynamics, and data-driven model as summarized in ASHRAE(Handbook, 1997) and Dong et al.(Dong et al., 2016).

## 1.1 Physics-Based Model

In terms of physics-based model, they consist of first-principle based methods, such as Finite Difference Method, and reduced-order model (such as thermal resistor-capacitor (RC) network model). For the FDM, it usually characterized as computational expensive thus not suitable for integrating with traditional simulation programs, such as TRNSYS and EnergyPlus. And according to Rodríguez Jara et al. (Rodríguez Jara et al., 2016), the Response Factor Method or the Transfer Function Method are the most used methods in building simulation programs. As for RC network model, it can be treated as a simplified model compared with FDM, or a set of linear ordinary differential equations (ODEs). RC model nodes are generally in the form of 2R1C, 3R2C, or lumped RC parameter model with corresponding self-adjusting methods(Rodríguez Jara et al., 2016). As stated in (O’Dwyer et al., 2016), there theoretically is a guaranteed thermal passivity solution for RC network when resistance and capacitance values are provided to be positive. When it comes to the control design for RC network model, simpler RC models are preferred. As stated (A. Li et al., 2017), it is worthwhile to quantify RC network parameters for optimal system control. There are many research efforts aiming for the tradeoff of model accuracy and model complexity(Ahn & Song, 2010; Goyal et al., 2011). Koschenz came up with core temperature layer for RS (Koschenz & Dorer, 1999; Liu et al., 2011). And Liu et al. proposed a method utilizing systematical geometric structure parameters to define heat resistance and heat capacity of assumed core layer, which were difficult to determine.

The RC model application is usually limited, although researchers have the freedom to construct different orders of model to meet their own needs. In addition, the finer RC network of HRSS usually has more restriction for the onsite configuration, such as a start-type RC model proposed by Li et al.(A. Li et al., 2017) has aspect ratios limitation. As stated in(Rodríguez Jara et al., 2016), the accuracy of lumped parameter methods, one instance of RC model, depends largely on the value of their characteristic parameters. Although Rodríguez Jara et al. proposed self-adjusting methods for simplified RC model, the simplification method is dependent on the reasonable estimation of element properties (e.g. thermal diffusivity), element thickness and special excitation for training experimental setup. When considering the actual operation of HRSS, the RC model accuracy deteriorated when the slab was subject to rapid thermal disturbances(Rhee & Kim, 2015). Firstly, RC network model should be established based on the linear energy balance equation. Besides, according to Dong et al., the construction of RC model are usually dependent on lots of onsite input data, which are not necessarily available. As stated in Koschenz et al., the ventilation system, which is required, has a large influence on the energy related operation of RS system. Jaewan et al. also indicates that the conditioned RS room were frequently overheated during the daytime due to solar radiation and uncertainty of internal heat gain during the occupied hours.

## 1.2 Data-Driven Model

As for data-driven model in the present study, we focus on black box data driven model. Common data-driven methods, such as partial least square method (PLS) and principal component analysis (PCA) are assuming the target model is inherently single normal distribution (Karami & Wang, 2018), or normally used to describe non-Gaussian and linear relationships(D. Li & Song, 2020), which is not the case for the complex dynamic system as HRSS. As summarized in Dong et al.(Dong et al., 2016), other conventional data-driven methods, such as artificial neural networks (ANN) and support vector machine (SVM) cannot conduct uncertainty analysis spontaneously, while uncertainty analysis is crucial for HRSS load prediction. Alternatively, Gaussian family models, including gaussian process regression (GPR) and gaussian mixture model (GMM), has been used as a data-driven method for building system load prediction. The major advantages of gaussian family methods are their non-linear, non-Gaussian, inherent uncertainty formulation component and multimode characteristics. As indicated in Guenther et al.(Guenther & Sawodny, 2019), GPR has been used to capture the complex and highly subjective relations between room temperature and personal perception. The GMM is generally recognized to model the multimode characteristics and handle process uncertainties(D. Li & Song, 2020). As for GMR, Li et al.(D. Li & Song, 2020) claimed that GMR carries the potential of dealing with nonlinear and non-Gaussian industry problems. The number of unique building operational patterns can be identified with different Gaussians in GMR, as indicated in Srivastav et al.(Srivastav et al., 2013) for baseline building energy modeling. In addition, Wang et al.(Wang et al., 2018) has used GMMR for adaptive learning to predict hourly energy uses in building. However, it is more challenging to address time-varying processes using GMR, which does not have an online adaptive mechanism. There are numerous efforts put on the design of adaptive mechanism for GMR, which can be found in the field of incremental learning, or growing GMR (GGMR)(Bouchachia & Vanaret, 2011; Cederborg et al., 2010; Karami & Wang, 2018; D. Li & Song, 2020). The overall benefits from GGMR can be summarized as follows(D. Li & Song, 2020): using incremental learning to avoid maintaining all historical data; maintain the compactness of models; improve model updating efficiency. For example, Majid et al.(Karami & Wang, 2018) used Unscented Kalman filter (UKF) to update those existing gaussian.

## 1.3 Research Gap and Objective

As summarized in (O’Dwyer et al., 2016), buildings thermal responses are intrinsically thermodynamically complex, especially subject to many disturbances (such as solar radiation, various miscellaneous electrical load and air systems load). However, RC is limited to due to its dependencies on unusual sensor inputs and requires substantial efforts to develop and calibrate for onsite buildings. Also, uncertainty analysis is crucial for building energy prediction since it is inherently thermodynamic complex, which is not covered in RC model. As for data driven method, such as adaptive gaussian mixture regression, there are few studies using GGMR for HRSS load prediction.

To close the above research gap, we propose a hybrid method in the present study, which use a simple RC model out as one of inputs for GGMR model. And the proposed hybrid method combines the advantage of the GGMR (data-driven method) and overcomes the limitation in RC model (physics-based method).

In the sequel, the methodology and performance metrics are detailed in Sec. 2. Section 3 presented model development and one real-world hybrid method case study for an existing office located at Purdue University before a conclusion in Sec. 4.

# 2. METHODLOGY

This section discussed the methodology developed to improve the prediction performance, which began with the RC network model development are described before moving on to the GGMR approach, and then moved to the Hybrid Modeling approach combining the RC and GGMR. In the last subsection, model prediction performance criteria metrics are described.



## 2.1 RC Network Method



Heat balance equations on each temperature or state variable are used to create a gray-box RC network model(Braun & Chaturvedi, 2002; Joe & Karava, 2017). A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surrounded by the estimated resistors and capacitors. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | () |















## 2.2 GGMR Method

Gaussian mixture regression (GMR)(Sung, n.d.) is a regression approach that models probability distributions rather than functions. Assume the data follow the join density

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where , . The above Gaussian mixture probability function shown in Equation (5) can be portioned as

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where

|  |  |  |
| --- | --- | --- |
|  |  | (7) |
|  |  | (8) | |

From Equation (6), the marginal density of X is

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

The conditional probability density function of is

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

with the mixing weight

|  |  |  |
| --- | --- | --- |
|  |  | (11) |















In the current study, we are interested in the expectation of y among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | () |

To control GMR model complexity and improve its model accuracy, GGMR has been proposed by Bouchachia et al.(Bouchachia & Vanaret, 2011) with growing and shrinking mechanisms. According to Bouchachia et al., their GGMR can be used to either update, generate, split or merge Gaussians to accommodate new data in an online setting. We utilized the updating gaussians part in the present paper. More details can be seen in Bouchachia et al(Bouchachia & Vanaret, 2011). The best match Gaussian will be updated with the following formulas:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |

in which is the match probability calculated with new input and best match Gaussian , is the expected posterior, is the sum of the expected posterior for best match Gaussian, is the weights of best match Gaussian, is the on-going learning rate for j-th Gaussian, is the converging learning rate.

## 2.3 Hybrid Method

The basic idea of the Hybrid Model is to improve the forecasting accuracy for both RC model and GGMR model. In fact, it is evident that we should provide a close estimation as one of GGMR inputs to enhance its prediction power. In the present study, we have designed the Hybrid Model schema as shown in **Figure 1**, which illustrates the underlying structure of the hybrid approach. Enabled by the real time predicted system load from RC network model and incremental learning framework from the GGMR model, those trained gaussian components from Expectation Maximization (EM) will be updated accordingly as the update rules shown in Equation 11~ 16. Specifically, the RC network module will get the target time step index from GGMR and return the predicted RS system load back to GGMR module.

Diagram

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**Figure 1** Underlying Communication for Hybrid Approach



## 2.4 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | (19) |
|  |  | (20) |
|  |  | (21) |
|  |  | (22) |
|  | = | (23) |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section presents a case study for all the three proposed methods, including RC network, GGMR and hybrid approach. It begins with a description of the data collection procedure, then moved to the moved to the model development and selections, and finally presents with the comparison of each modeling approach performance.

## 3.1 Test bed

The dataset contained in-situ measurements for a living laboratory office space from January 15th to March 7th, 2022, with a 5-minute sampling rate. And we used the first two weeks data for training and the rest of data used for testing. In addition, the data consists of two types, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted as , Façade cavity space temperature denoted as , slab concrete temperature denoted as , flowing water temperature within slab pipe denoted as , solar radiation retrieved from a weather station denoted as (This public data is available on *www.ambientweather.net*), air handling unit consumed heating power . The estimated input values are calculated with predefined schedule in accordance with ASHRAE 90.1 (ANSI/ASHRAE/IES 90.1-2010, 2010), such as internal heating radiation denoted as , lighting radiation .

## 3.2 RC Network Model Development

Considering model accuracy-complexity trade-off, the overall design logic for RC network construction are listed as followings:

1. Increase model accuracy. The physical description of an RC model should capture the key and most thermal behaviors of targeted space to maintain model robustness for different operating conditions.
2. Decrease model complexity. Reduce the number of input variables or training data to avoid over-complex model.

The major thermal components of the living laboratory office space (Joe & Karava, 2017) include external walls, roof/ceiling, internal wall, south-facing double façade system, conditioned air from air handling unit (AHU) system, and hydronic radiant floor system (as shown in Figure 2).

In the present study, we experimented different RC network design by considering model robustness and different levels of complexity or model orders. We designed three models, 4-states Model 1, 6-states Model 2 and 5-states Model 3, for RC networks as shown in Figure 3, in which represents temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. As for the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements, respectively.

All those three models consist of two parts: room part and concrete slab part. We chose the same room part network structure to well capture its thermal properties: two-node envelope, one-node internal wall, one node cavity for double façade system, and room air node to capture the provided disturbance heating or cooling from AHU system. It is worth noting that we used envelope node to represent external wall and roof/ceiling to maintain model complexity. As for radiant floor system, we have tried different model orders to capture its thermal behaviors. In Model 1, the detailed thermal structure of radiant floor has been neglected. We treated the entire slab as one node. Compared with Model 1, Model 3 has additional source node, which represents the water flowing through slab pipes. And Model 2 has one more sink node than Model 3 to represent the thermal insulation below concrete as shown in Figure 2.

Figure 4 shows the predicted and measured results during testing period (10892 sampling points for around 37 days). Compared with Model 2 and Model 3, the Model 1 has much higher error which can be attributed to the oversimplified floor representation. As for Model 2, it has lower CVRMSE than Model 3 which can be explained by the additional sink node. More detailed performance comparison can be view in Table 2. Model 2 has been chosen as the optimum model for the RC network method as it has better performance to capture the peaking load than Model 3.

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | (24) |
|  |  | (25) |
|  |  | (26) |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table 1:

|  |  |  |
| --- | --- | --- |
|  |  | (27) |
|  |  | (28) |

As stated in Equation 4, the RC network model training is essentially an optimization problem. In the present paper, p

A picture containing chart

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Figure 2 Floor slab section view

**Table 1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

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**Figure 3** Structure of RC network. Left: Model 1 with 4 states; Middle: Model 2 with 6 states; Middle: Model 3 with 5 states.

Timeline

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**Figure 4** Testing results for Model 1, Model 2 and Model 3

**Table 2** Comparison of proposed RC models (5-mins interval)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5.76 | 87.88 |
| Model 2 | **16.15** | **21.31** | **0.84** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1.28 | 35.89 |

## 3.2 GGMR Model Development

In this subsection, we mainly focus on the determination of input variables for GGMR model. As referred to Wang et al. (Wang et al., 2018), correlation coefficients has been used as a indicator of the strength and direction for of the linear relationship between inputs and model outputs. And the correlation coefficient is ranging from -1 to +1, where -1 represents the prefect negative linear correlation and +1 represents the prefect positive linear correlation. By referring the Table 3, we have experimented different inputs combinations for GGMR model as shown in Table 4. It is worth noting that strong linear correlation does not necessarily mean better GGMR prediction performance. has been selected as the GGMR Model inputs.

**Table 3** Correlation coefficients between Radiant Slab systems load and input variables

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| -0.06 | -0.08 | -0.16 | -0.89 | 0.35 | -0.16 | 1 |

**Table 4** Prediction performance comparison for different GGMR inputs

|  |  |
| --- | --- |
| **Inputs** | **CVRMSE (%)** |
|  | 107.42 % |
|  | 25.81 % |
|  | 26.93 % |

## 3.3 Hybrid Model Development

As mentioned in Sec. 2.4, the development of hybrid approach is mainly the determination of warming up steps for RC module, number of Gaussians and learning rate used in GGMR module. In the present study, the warming up period is statistically selected as shown in the left plot of Figure 5. And the ideal warming up steps for RC prediction has been selected as 15. Moreover, the best number of Gaussians and learning rate have been chosen as 15 and 8e-3 respectively as indicated from the middle and right plots of Figure 5. In addition, we investigated the impact of predicted water flow rate through pipes as shown in Table 3. And we finally selected as the Hybrid Model inputs.

Chart, line chart

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5hyperparameters for Hybrid Approach. Left: Warming up steps for RC model; Middle: Number of Gaussians for GGMR model; Right: Learning rate for GGMR Model.

**5**Hybrid Model

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

## 3.3 Performance Comparison for Proposed Models

As illustrated in Table 5, all the three proposed models met the requirement of ASHRAE Guideline 14(Landsberg et al., n.d.). This table indicates that Hybrid Model is the best model for predicting Radiant Slab system energy consumption. To furtherly analyze the prediction performance for those models, typical days has been selected and plotted in Figure 6. All of three models have a reasonably good prediction performance, although they cannot well capture the peak load (which occurs usually at 6:00 PM when the space is not occupied). Furthermore, GGMR Model tends to have to overshoot or oscillate a lot around the measured data, while RC Model lean for a undershoot prediction and smoothed the ups and downs. In addition, it is evident to observe that the Hybrid Model integrates the information from both RC model and GGMR model to make the best prediction for RS system load.

As shown in Table 5, the RC model has a normalized root mean square error (NRMSE) of 13.56%, a coefficient of variation of root means square error (CVRMSE) of 15.59%, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53%. The GGMR model has NRMSE of 15.89%, CVRMSE of 17.76%, MAE of 6.40 kW, and MAPE of 27.68%. The hybrid approach has NRMSE of 8.77% (35% lower from RC, 45% lower from GGMR), CVRMSE of 9.95% (36% lower from RC, 44% lower from GGMR), MAE of 3.62 kW (37 lower from RC, 43% lower from GGMR), and MAPE of 19.31% (82% lower from RC, 30% lower from GGMR).

**Table 5** Performance comparison for hourly prediction of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| RC | 13.56 | 15.59 | 5.76 | 108.53 |
| GGMR | 15.89 | 17.67 | 6.40 | 27.68 |
| Hybrid | **8.77** | **9.95** | **3.62** | **19.31** |

Chart, histogram

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**Figure 6** Radiant slab load between RC model, GGMR model, Hybrid model and measured data.

# 4. Conclusion

In this paper, a novel Hybrid Model been proposed to predict the energy use of a hydronic radiant slab system, which integrate advantages of both RC model and GGMR model. The proposed method was tested with the real word radiant slab operation data, located at Purdue University. From the case study, the Hybrid Model has demonstrated the best prediction performance among all the three models, RC, GGMR and Hybrid Model. And the proposed Hybrid model has an hourly prediction CVRMSE as 9.95% (36% lower from RC, 44% lower from GGMR), which surely met the criteria for ASHRAE Guideline 14 (Landsberg et al., n.d.). Specifically, it has been proved that the RC model prediction can be used as input for a GGMR model to furtherly reduce the predictions for both RC model and GGMR model.

During the model development process for the input variables selection of GGMR model, we found that the stronger linear correlation does not necessarily mean better prediction performance. This observation indicates that there might be further potential to explore different input variables for both GGMR model and Hybrid model.

It is worth noting that the case study is limited to only one onsite dataset source. In the future, we need to do more case studies for various data source.

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

# REFERENCES

Ahn, B.-C., & Song, J.-Y. (2010). Control characteristics and heating performance analysis of automatic thermostatic valves for radiant slab heating system in residential apartments. *Energy*, *35*(4), 1615–1624. https://doi.org/10.1016/j.energy.2009.11.007

ANSI/ASHRAE/IES 90.1-2010. (2010). *Energy Standard for Buildings Except Low-Rise Residential Buildings*. American Society of Heating, Refrigerating and Air-Conditioning Engineers.

Bouchachia, H., & Vanaret, C. (2011). *Incremental Learning Based on Growing Gaussian Mixture Models*. *2*. https://doi.org/10.1109/ICMLA.2011.79

Braun, J. E., & Chaturvedi, N. (2002). An Inverse Gray-Box Model for Transient Building Load Prediction. *HVAC&R Research*, *8*(1), 73–99. https://doi.org/10.1080/10789669.2002.10391290

Cederborg, T., Li, M., Baranes, A., & Oudeyer, P.-Y. (2010). Incremental local online Gaussian Mixture Regression for imitation learning of multiple tasks. *2010 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 267–274. https://doi.org/10.1109/IROS.2010.5652040

Dong, B., Li, Z., Rahman, S. M. M., & Vega, R. (2016). A hybrid model approach for forecasting future residential electricity consumption. *Energy and Buildings*, *117*, 341–351. https://doi.org/10.1016/j.enbuild.2015.09.033

Goyal, S., Liao, C., & Barooah, P. (2011). Identification of multi-zone building thermal interaction model from data. *2011 50th IEEE Conference on Decision and Control and European Control Conference*, 181–186. https://doi.org/10.1109/CDC.2011.6161387

Guenther, J., & Sawodny, O. (2019). Feature selection and Gaussian Process regression for personalized thermal comfort prediction. *Building and Environment*, *148*, 448–458.

Handbook, A. (1997). Fundamentals SI edition. *American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, GA*.

James V. Miranda, L. (2018). PySwarms: A research toolkit for Particle Swarm Optimization in Python. *The Journal of Open Source Software*, *3*(21), 433. https://doi.org/10.21105/joss.00433

Joe, J., & Karava, P. (2017). Agent-based system identification for control-oriented building models. *Journal of Building Performance Simulation*, *10*(2), 183–204. https://doi.org/10.1080/19401493.2016.1212272

Joe, J., & Karava, P. (2019). A model predictive control strategy to optimize the performance of radiant floor heating and cooling systems in office buildings. *Applied Energy*, *245*, 65–77. https://doi.org/10.1016/j.apenergy.2019.03.209

Karami, M., & Wang, L. (2018). Fault detection and diagnosis for nonlinear systems: A new adaptive Gaussian mixture modeling approach. *Energy and Buildings*, *166*, 477–488. https://doi.org/10.1016/j.enbuild.2018.02.032

Koschenz, M., & Dorer, V. (1999). Interaction of an air system with concrete core conditioning. *Energy and Buildings*, *30*(2), 139–145. https://doi.org/10.1016/S0378-7788(98)00081-4

Landsberg, D. R., Shonder, J. A., Barker, K. A., Hall, C. R. L., & Reindl, D. T. (n.d.). *Or trans©miAsSsiHonRAinE e(iwthwewr.parsihnrtaoer.odrigg)it.aFl oforrpmerissonnoatl upseermointtlye.dAwdidthitoiountaAl SreHpRrAoEd’uscptiroionr, dwirsittrtiebnutpioenrm, ission. ASHRAE Guideline Project Committee 1 4 Cognizant TC: TC 7.6, Building Energy Performance SPLS Liaison: Waller S. Clements*. 150.

Li, A., Sun, Y., & Xu, X. (2017). Development of a simplified resistance and capacitance (RC)-network model for pipe-embedded concrete radiant floors. *Energy and Buildings*, *150*, 353–375. https://doi.org/10.1016/j.enbuild.2017.06.011

Li, D., & Song, Z. (2020). A Novel Incremental Gaussian Mixture Regression and Its Application for Time-varying Multimodal Process Quality Prediction. *2020 IEEE 9th Data Driven Control and Learning Systems Conference (DDCLS)*, 645–650. https://doi.org/10.1109/DDCLS49620.2020.9275082

Liu, K., Tian, Z., Zhang, C., Ding, Y., & Wang, W. (2011). Establishment and validation of modified star-type RC-network model for concrete core cooling slab. *Energy and Buildings*, *43*(9), 2378–2384. https://doi.org/10.1016/j.enbuild.2011.05.029

O’Dwyer, E., De Tommasi, L., Kouramas, K., Cychowski, M., & Lightbody, G. (2016). Modelling and disturbance estimation for model predictive control in building heating systems. *Energy and Buildings*, *130*, 532–545. https://doi.org/10.1016/j.enbuild.2016.08.077

Rhee, K.-N., & Kim, K. W. (2015). A 50 year review of basic and applied research in radiant heating and cooling systems for the built environment. *Building and Environment*, *91*, 166–190. https://doi.org/10.1016/j.buildenv.2015.03.040

Rodríguez Jara, E. Á., Sánchez de la Flor, F. J., Álvarez Domínguez, S., Molina Félix, J. L., & Salmerón Lissén, J. M. (2016). A new analytical approach for simplified thermal modelling of buildings: Self-Adjusting RC-network model. *Energy and Buildings*, *130*, 85–97. https://doi.org/10.1016/j.enbuild.2016.08.039

Sourbron, M., De Herdt, R., Van Reet, T., Van Passel, W., Baelmans, M., & Helsen, L. (2009). Efficiently produced heat and cold is squandered by inappropriate control strategies: A case study. *Energy and Buildings*, *41*(10), 1091–1098. https://doi.org/10.1016/j.enbuild.2009.05.015

Srivastav, A., Tewari, A., & Dong, B. (2013). Baseline building energy modeling and localized uncertainty quantification using Gaussian mixture models. *Energy and Buildings*, *65*, 438–447. https://doi.org/10.1016/j.enbuild.2013.05.037

Sung, H. G. (n.d.). *Gaussian Mixture Regression and Classiﬁcation*. 117.

Wang, L., Kubichek, R., & Zhou, X. (2018). Adaptive learning based data-driven models for predicting hourly building energy use. *Energy and Buildings*, *159*, 454–461. https://doi.org/10.1016/j.enbuild.2017.10.054