**A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems**

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# ABSTRACT

Accurately predicting the performance of radiant slab systems can be challenging due to the large thermal capacitance of the radiant slab and room temperature stratification. Current methods for predicting heating and cooling energy consumption of hydronic radiant slabs include detail first-principle-based (e.g, finite difference) and reduced-order (e.g, thermal resistor-capacitor (RC) network) models. Creating and calibrating detailed first-principle models, as well as RC network models for predicting the performance of radiant slabs requires substantial modeling efforts. To develop improved control, monitoring, and diagnostic methods, there is a need for simpler models that can be readily trained using in-situ measurements.

In this study, we explored a novel hybrid modeling method integrating a simple RC network model with an evolving learning-based algorithm growing Gaussian mixture regression (GGMR) modeling approach to predict the heating and cooling rates of a radiant slab system for a Living Laboratory office space. The RC network model provides heating or cooling load of the radiant slab system to the GGMR model as one of the inputs. The three modeling approaches have been compared with a case study for predicting the hourly RS system load of a Living Laboratory office space from January 15th to March 7th, 2022: 1) an RC network model; 2) a GGMR method; and 3) the proposed hybrid approach. The first two weeks of data were used for training, while the remaining data was used as a testing data set in all three modeling methods. The RC model has a normalized root mean square error (NRMSE) of 13.56%, a coefficient of variation of root means square error (CVRMSE) of 15.59%, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53%. The GGMR model has NRMSE of 15.89%, CVRMSE of 17.76%, MAE of 6.40 kW, and MAPE of 27.68%. The hybrid approach has NRMSE of 8.77% (35% lower from RC, 45% lower from GGMR), CVRMSE of 9.95% (36% lower from RC, 44% lower from GGMR), MAE of 3.62 kW (37 lower from RC, 43% lower from GGMR), and MAPE of 19.31% (82% lower from RC, 30% lower from GGMR).

# 1. INTRODUCTION

Recently, hydronic radiant slab systems have demonstrated many advantages for conditioned space thermal management, such as improved thermal comfort and reduced energy cost.

Conventional prediction strategies include reduced order method (e.g. thermal resistor-capacitor (RC) network) models. Creating and calibrating such models require substantial model development efforts. Apart from the physics based methods, data-driven methods have been proved as simpler but efficient models that can be readily trained using in-site measurements. For example, Wang et al.(Wang et al., 2018) researched the hourly energy consumption prediction using Gaussian Mixer Model Regression. And Karami et al.(Karami & Wang, 2018) used Unscented Kalman filter to construct a novel adaptive Gaussian mixture model (AGMM). Bouchachia et al.(Bouchachia & Vanaret, 2011) utilized incremental learning to learn Growing Gaussian Mixture Regression (GGMR).

In the present paper, we explored the thermal load prediction performance from a hybrid approach, where we combined the prediction from an RC network with GGMR algorithm to enhance both prediction performance.

In the sequel, the performance metrics, methodology and model development are detailed in Sec. 2. Section 3 presented the real-world hybrid approach prediction performance for an existing office located at Purdue University before a conclusion in Sec. 4.

# 2. METHODLOGY

This section discussed the methodology developed to improve the prediction performance, which began with the RC network model development are described before moving on to the GGMR approach, and then moved to the Hybrid Modeling approach combining the RC and GGMR. In the last subsection, model prediction performance criteria metrics are described.



## 2.1 RC Network Model



Heat balance equations on each temperature or state variable are used to create a gray-box RC network model(Braun & Chaturvedi, 2002; Joe & Karava, 2017). A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surrounded by the estimated resistors and capacitors. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | () |















## 2.2 GGMR Model

Gaussian mixture regression (GMR)(Sung, n.d.) is a regression approach that models probability distributions rather than functions. Assume the data follow the join density

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where , . The above Gaussian mixture probability function shown in Equation (5) can be portioned as

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where

|  |  |  |
| --- | --- | --- |
|  |  | (7) |
|  |  | (8) | |

From Equation (6), the marginal density of X is

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

The conditional probability density function of is

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

with the mixing weight

|  |  |  |
| --- | --- | --- |
|  |  | (11) |















In the current study, we are interested in the expectation of y among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | () |

According to Bouchachia et al.(Bouchachia & Vanaret, 2011), GGMR can be used to either update, generate, split or merge Gaussians to accommodate new data in an online setting. We utilized the updating gaussians part in the present paper. More details can be seen in Bouchachia et al(Bouchachia & Vanaret, 2011). The best match Gaussian will be updated with the following formulas:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |

in which is the match probability calculated with new input and best match Gaussian , is the expected posterior, is the sum of the expected posterior for best match Gaussian, is the weights of best match Gaussian, is the on-going learning rate for j-th Gaussian, is the converging learning rate.

## 2.3 Hybrid Model

The basic idea of the Hybrid Model is to improve the forecasting accuracy for both RC model and GGMR model. In fact, it is evident that we should provide a close estimation as one of GGMR inputs to enhance its prediction power. In the present study, we have designed the Hybrid Model schema as shown in **Figure 1**, which illustrates the underlying structure of the hybrid approach. Enabled by the real time predicted system load from RC network model and incremental learning framework from the GGMR model, those trained gaussian components from Expectation Maximization (EM) will be updated accordingly as the update rules shown in Equation 11~ 16. Specifically, the RC network module will get the target time step index from GGMR and return the predicted RS system load back to GGMR module.

Diagram

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**Figure 1** Underlying Communication for Hybrid Approach



## 2.4 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | (19) |
|  |  | (20) |
|  |  | (21) |
|  |  | (22) |
|  | = | (23) |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section presents a case study for all the three proposed methods, including RC network, GGMR and hybrid approach. It begins with a description of the data collection procedure, then moved to the moved to the model development and selections, and finally presents with the comparison of each modeling approach performance.

## 3.1 Data Description

The dataset contained in-situ measurements for a living laboratory office space from January 15th to March 7th, 2022, with a 5-minute sampling rate. In addition, the data consists of two types, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted as , Façade cavity space temperature denoted as , slab concrete temperature denoted as , flowing water temperature within slab pipe denoted as , solar radiation retrieved from a weather station denoted as (This public data is available from *www.ambientweather.net*), air handling unit consumed heating power . The estimated input values are calculated with predefined schedule in accordance with ASHRAE 90.1 (ANSI/ASHRAE/IES 90.1-2010, 2010), such as internal heating radiation denoted as , lighting radiation .

## 3.2 RC Network Model Development

Considering model accuracy-complexity trade-off, the overall design logic for RC network construction are listed as followings:

1. Increase model accuracy. The physical description of an RC model should capture the key and most thermal behaviors of targeted space to maintain model robustness for different operating conditions.
2. Decrease model complexity. Reduce the number of input variables or training data to avoid over-complex model.

The major thermal components of the living laboratory office space (Joe & Karava, 2017) include external walls, roof/ceiling, internal wall, south-facing double façade system, conditioned air from air handling unit (AHU) system, and hydronic radiant floor system (as shown in Figure 2).

In the present study, we experimented different RC network design by considering model robustness and different levels of complexity or model orders. We designed three models, 4-states Model 1, 6-states Model 2 and 5-states Model 3, for RC networks as shown in Figure 3, in which represents temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. As for the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements, respectively.

All those three models consist of two parts: room part and concrete slab part. We chose the same room part network structure to well capture its thermal properties: two-node envelope, one-node internal wall, one node cavity for double façade system, and room air node to capture the provided disturbance heating or cooling from AHU system. It is worth noting that we used envelope node to represent external wall and roof/ceiling to maintain model complexity. As for radiant floor system, we have tried different model orders to capture its thermal behaviors. In Model 1, the detailed thermal structure of radiant floor has been neglected. We treated the entire slab as one node. Compared with Model 1, Model 3 has additional source node, which represents the water flowing through slab pipes. And Model 2 has one more sink node than Model 3 to represent the thermal insulation below concrete as shown in Figure 2.

Figure 4 shows the predicted and measured results during testing period (10892 sampling points for around 37 days). Compared with Model 2 and Model 3, the Model 1 has much higher error which can be attributed to the oversimplified floor representation. As for Model 2, it has lower CVRMSE than Model 3 which can be explained by the additional sink node. More detailed performance comparison can be view in Table 2. Model 2 has been chosen as the optimum model for the RC network method as it has better performance to capture the peaking load than Model 3.

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | (24) |
|  |  | (25) |
|  |  | (26) |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table 1:

|  |  |  |
| --- | --- | --- |
|  |  | (27) |
|  |  | (28) |

As stated in Equation 4, the RC network model training is essentially an optimization problem. In the present paper, p

A picture containing chart

Description automatically generated

Figure 2 Floor slab section view

**Table 1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

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**Figure 3** Structure of RC network. Left: Model 1 with 4 states; Middle: Model 2 with 6 states; Middle: Model 3 with 5 states.

Timeline

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**Figure 4** Testing results for Model 1, Model 2 and Model 3

**Table 2** Comparison of proposed RC models (5-mins interval)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5.76 | 87.88 |
| Model 2 | **16.15** | **21.31** | **0.84** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1.28 | 35.89 |

## 3.2 GGMR Model Development

In this subsection, we mainly focus on the determination of input variables for GGMR model. As referred to Wang et al. (Wang et al., 2018), correlation coefficients has been used as a indicator of the strength and direction for of the linear relationship between inputs and model outputs. And the correlation coefficient is ranging from -1 to +1, where -1 represents the prefect negative linear correlation and +1 represents the prefect positive linear correlation. By referring the Table 3, we have experimented different inputs combinations for GGMR model as shown in Table 4. It is worth noting that strong linear correlation does not necessarily mean better GGMR prediction performance. has been selected as the GGMR Model inputs.

**Table 3** Correlation coefficients between Radiant Slab systems load and input variables

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| -0.06 | -0.08 | -0.16 | -0.89 | 0.35 | -0.16 | 1 |

**Table 4** Prediction performance comparison for different GGMR inputs

|  |  |
| --- | --- |
| **Inputs** | **CVRMSE (%)** |
|  | 107.42 % |
|  | 25.81 % |
|  | 26.93 % |

## 3.3 Hybrid Model Development

As mentioned in Sec. 2.4, the development of hybrid approach is mainly the determination of warming up steps for RC module, number of Gaussians and learning rate used in GGMR module. In the present study, the warming up period is statistically selected as shown in the left plot of Figure 5. And the ideal warming up steps for RC prediction has been selected as 15. Moreover, the best number of Gaussians and learning rate have been chosen as 15 and 8e-3 respectively as indicated from the middle and right plots of Figure 5. In addition, we investigated the impact of predicted water flow rate through pipes as shown in Table 3. And we finally selected as the Hybrid Model inputs.

Chart, line chart

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Description automatically generated

5hyperparameters for Hybrid Approach. Left: Warming up steps for RC model; Middle: Number of Gaussians for GGMR model; Right: Learning rate for GGMR Model.

**5**Hybrid Model

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

## 3.3 Performance Comparison for Proposed Models

As illustrated in Table 5, all the three proposed models met the requirement of ASHRAE Guideline 14(Landsberg et al., n.d.). This table indicates that Hybrid Model is the best model for predicting Radiant Slab system energy consumption. To furtherly analyze the prediction performance for those models, typical days has been selected and plotted in Figure 6. All of three models have a reasonably good prediction performance, although they cannot well capture the peak load (which occurs usually at 6:00 PM when the space is not occupied). Furthermore, GGMR Model tends to have to overshoot or oscillate a lot around the measured data, while RC Model lean for a undershoot prediction and smoothed the ups and downs. In addition, it is evident to observe that the Hybrid Model integrates the information from both RC model and GGMR model to make the best prediction for RS system load.

As shown in Table 5, the RC model has a normalized root mean square error (NRMSE) of 13.56%, a coefficient of variation of root means square error (CVRMSE) of 15.59%, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53%. The GGMR model has NRMSE of 15.89%, CVRMSE of 17.76%, MAE of 6.40 kW, and MAPE of 27.68%. The hybrid approach has NRMSE of 8.77% (35% lower from RC, 45% lower from GGMR), CVRMSE of 9.95% (36% lower from RC, 44% lower from GGMR), MAE of 3.62 kW (37 lower from RC, 43% lower from GGMR), and MAPE of 19.31% (82% lower from RC, 30% lower from GGMR).

**Table 5** Performance comparison for hourly prediction of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| RC | 13.56 | 15.59 | 5.76 | 108.53 |
| GGMR | 15.89 | 17.67 | 6.40 | 27.68 |
| Hybrid | **8.77** | **9.95** | **3.62** | **19.31** |

Chart, histogram

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**Figure 6** Radiant slab load between RC model, GGMR model, Hybrid model and measured data.

# 4. Conclusion

In this paper, a novel Hybrid Model been proposed to predict the energy use of a hydronic radiant slab system, which integrate advantages of both RC model and GGMR model. The proposed method was tested with the real word radiant slab operation data, located at Purdue University. From the case study, the Hybrid Model has demonstrated the best prediction performance among all the three models, RC, GGMR and Hybrid Model. And the proposed Hybrid model has an hourly prediction CVRMSE as 9.95% (36% lower from RC, 44% lower from GGMR), which surely met the criteria for ASHRAE Guideline 14 (Landsberg et al., n.d.). Specifically, it has been proved that the RC model prediction can be used as input for a GGMR model to furtherly reduce the predictions for both RC model and GGMR model.

During the model development process for the input variables selection of GGMR model, we found that the stronger linear correlation does not necessarily mean better prediction performance. This observation indicates that there might be further potential to explore different input variables for both GGMR model and Hybrid model.

It is worth noting that the case study is limited to only one onsite dataset source. In the future, we need to do more case studies for various data source.

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

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