A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems

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# ABSTRACT

Accurately predicting the performance of radiant slab systems can be challenging due to the large thermal capacitance of the radiant slab and room temperature stratification. Current methods for predicting heating and cooling energy consumption of hydronic radiant slabs include detail first-principle-based (e.g, finite difference) and reduced-order (e.g, thermal resistor-capacitor (RC) network) models. Creating and calibrating detailed first-principle models, as well as detailed RC network models for predicting the performance of radiant slabs requires substantial modeling efforts. To develop improved control, monitoring, and diagnostic methods, there is a need for simpler models that can be readily trained using in-situ measurements.

In this study, we explored a novel hybrid modeling method integrating a simple RC network model with an evolving learning-based algorithm growing Gaussian mixture regression (GGMR) modeling approach to predict the heating and cooling rates of a radiant slab system for a Living Laboratory office space. The RC network model provides heating or cooling load of the radiant slab system to the GGMR model as one of the inputs in real time. The three modeling approaches: 1) an RC network model; 2) a GGMR method; and 3) the proposed hybrid approach have been compared with a case study for predicting the energy use of a radiant slab system of a Living Laboratory office space from January 15th to March 7th, 2022. The first two weeks of data were used for training, while the remaining data was used as a testing data set in all three modeling methods. The RC model had a normalized root mean square error of 13.56 percent, a coefficient of variation of root mean square error of 15.59 percent, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53 percent. The NRMSE of the GGMR model was 20.75 percent, the CVRMSE was 22.56 percent, the MAE was 7.61 kW, and the MAPE was 27.74 percent. The hybrid approach had an NRMSE of 8.77 percent (4.79 percent less than RC and 11.98 percent less than GGMR), a CVRMSE of 9.95 percent (5.64 percent less than RC and 12.6 percent less than GGMR), an MAE of 3.62 kW (2.14 kW less than RC and 3.99 kW less than GGMR), and a MAPE of 19.31 percent (89.22 percent lower from RC, 8.43 percent lower from GGMR). The hybrid modeling approach outperformed the RC model and GGMR model.

# 1. INTRODUCTION

Recently, hydronic radiant slab systems (HRSS) demonstrated significant benefits for thermal management of conditioned spaces, including increased thermal comfort and energy savings. Apart from these benefits, the large thermal storage capacity of HRSS has a few disadvantages. One disadvantage of the large thermal time constant is that it causes cooling output to be delayed when supply water flow rates and temperature were adjusted (Liu et al., 2011). Additionally, conventional control based on room temperature feedback may consume more primary energy than a conventional air system (Sourbron et al., 2009). Moreover, Hydronic Radiant Slab Systems frequently experience concurrent thermal disturbances caused by solar radiation, internal heat, and air systems (Koschenz & Dorer, 1999). As a result, conventional HRSS control frequently encounters overcooling or overheating issues. To address these issues, HRSS requires Model Predictive Control (MPC) with accurate load prediction, as stated in (Joe & Karava, 2019). In general, energy models for buildings fall into three categories: first principle based models, reduced-order models, and purely data-driven models, as summarized in ASHRAE's (Handbook, 2001) and Dong et al. (Dong et al., 2016). The following sections will review those models in detail, followed by a discussion of the current research gap and objective.

## 1.1 First principle-based models

The first principle-based models are calculated from Navier-Stokes set of equations and turbulence models, such as Computational Fluid Dynamics (CFD) (Zhang et al., 2013). However, CFD will become unstable for conjugating heat transfer model, due to the different response times of thermal energy from solid and fluid. Additionally, the computational cost of these methods makes them incompatible with large-scale simulation programs(Neumann et al., 2021; Rodríguez Jara et al., 2016), such as EnergyPlus(Crawley et al., 2001). As a result, building elements with large thermal storage capacity are challenging to be correctly modeled with CFD. Rather than that, most current building energy model programs, like EnergyPlus, employ forward nodal modeling (Handbook, 2001; Neumann et al., 2021; Zhang et al., 2013), which requires a detailed physical and operational description of building, as well as the well-stirred zone air assumption, in order to design buildings and their heating ventilation and air condition (HVAC) system. Although EnergyPlus provides a large number of house templates represented by high-order thermal resistor-capacitor (RC) networks, developing and calibrating an RC model for onsite building requires substantial efforts(Dong et al., 2016).

## 1.2 Thermal RC network model

The inverse grey-box RC model, which strikes a balance between purely physical based models and purely empirical data-driven model. (Braun & Chaturvedi, 2002). Typically, an RC model is used to accurately represent the target space using historical data set. An RC network model is considered of as a collection of linear ordinary differential equations (ODEs). RC models are typically in the form of 2R1C, 3R2C, or lumped RC parameter models with associated self-adjusting methods (Rodríguez Jara et al., 2016). According to ,when the resistance and capacitance values are positive, there is theoretically a guaranteed thermal passivity solution for RC models. As previously stated by Li et al. (A. Li et al., 2017), quantifying RC network parameters is advantageous for optimizing system control. As for the training of RC model, there is considerable research devoted to optimizing the trade-off between model accuracy and complexity (Ahn & Song, 2010; Goyal et al., 2011; Koschenz & Dorer, 1999). For instance, (Liu et al., 2011) proposed a method for defining the heat resistance and heat capacity of an assumed core layer through the use of systematical geometric structure parameters.

There are some limitations in terms of the RC model application. Generally, the finer RC network of HRSS usually has more restriction for the onsite configuration, such as a start-type RC model proposed by (A. Li et al., 2017) has aspect ratios limitation. According to (Rodríguez Jara et al., 2016), the accuracy of lumped parameter methods, one type of RC model, is highly dependent on the values of their characteristic parameters. Although Rodrguez Jara et al.(Rodríguez Jara et al., 2016) proposed self-adjusting methods for simplification of the RC model, the method is dependent on reasonable estimation of element properties (e.g. thermal diffusivity), element thickness, and special excitation for the training experimental setup. In practice, the accuracy of the RC model degrades when the slab is subjected to rapid thermal disturbances (Neumann et al., 2021; Rhee & Kim, 2015).

## 1.3 Data-Driven Model

There are a lot of data-driven model candidates for building energy modeling. Some research indicates that PLS and PCA are typically used to describe non-Gaussian and linear relationships (D. Li & Song, 2020), which is not the case for complex dynamic systems such as HRSS. Alternatively, Gaussian family models, such as gaussian process regression (GPR) and gaussian mixture models (GMM), have been used to develop data-driven system load prediction. The primary advantages of gaussian family methods are their nonlinearity, inherent uncertainty formulation component, and multimode properties. As Guenther et al. (Guenther & Sawodny, 2019) demonstrated, GPR had been used to capture the complex and highly subjective relationships between room temperature and subjective thermal perception. The GMM is widely recognized for its ability to model multimode characteristics and deal with process uncertainty (Billard et al., 2008; D. Li & Song, 2020). Li et al. (D. Li & Song, 2020) asserted that GMR is appropriate for resolving nonlinear and non-Gaussian industry problems. As Srivastav et al. (Srivastav et al., 2013) demonstrated for baseline building energy modeling, the number of distinct building operational patterns can be identified using different Gaussians in GMR. Additionally, (Wang et al., 2018) used GMR to forecast hourly energy consumption in buildings. On the other hand, the lack of an online adaptive mechanism makes GMR more difficult to address time-varying processes.

Considerable efforts have been made in the field of incremental learning GMR, or growing GMR (GGMR), to develop a mechanism for GMR adaptation (Bouchachia & Vanaret, 2011; Cederborg et al., 2010; Karami & Wang, 2018; D. Li & Song, 2020; Wang et al., 2018). Generally, GGMR outperforms GMR from the following perspectives (D. Li & Song, 2020): avoiding the maintenance of all historical data through incremental learning; maintaining model compactness; and increasing model updating efficiency.

## 1.4 Research Gap and Objective

As summarized in (O'Dwyer et al., 2016), buildings' thermal responses are intrinsically complex and particularly susceptible to numerous disturbances (such as solar radiation, various miscellaneous electrical load and air systems load). However, RC model is usually restricted to many onsite buildings due to its reliance on lots of sensor inputs and the significant effort required to develop and calibrate. Additionally, uncertainty analysis is critical for predicting building energy consumption, which is not captured by the RC model. From the posterior distribution, one can derive the forecast uncertainty (Zavala et al., 2009). Bayesian estimation is a component of the GGMR method's uncertainty formulation. However, few studies have been conducted to investigate its application to HRSS load prediction.

To address the above research gap, we propose a hybrid approach, in which we use the outputs from a RC model as one of the inputs to a GGMR model. Additionally, the proposed Hybrid Model can inherit the benefits from the GGMR model and overcome the limitations of the RC model.

The methodology and performance metrics are detailed in Sec. 2. Section 3 presented model development and case study for an existing office at Purdue University before a conclusion in Sec. 4.

# 2. METHODLOGY

This section discussed the methodology used to improve prediction performance, beginning with the development of RC network models and progressing to the GGMR approach, and finally to the Hybrid Modeling approach, which combines the RC and GGMR approaches. The final subsection describes the model prediction performance criteria metrics.



## 2.1 RC Network Model



Heat balance equations on each temperature or state variable are used to create a RC network model (Braun & Chaturvedi, 2002; Joe & Karava, 2017).

|  |  |  |
| --- | --- | --- |
|  |  | () |

For a radiant slab system model, the output variable is the cooling and heating load. The state vector contains all the temperature nodes, which are surrounded by the estimated resistors and capacitors. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

A typical objective function for RC network model is to minimize the root-mean-square error for the training duration, denoted as

|  |  |  |
| --- | --- | --- |
|  |  | (4) |















## 2.2 GGMR Method

Gaussian mixture regression (GMR)(Sung, n.d.) is a regression approach that models probability distributions rather than functions. Assume the data follow the joint density

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where , . The above Gaussian mixture probability function shown in equation (5) can be portioned as

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where

|  |  |  |
| --- | --- | --- |
|  |  | (7) |
|  |  | (8) | |

From equation (6), the marginal density of X is

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

The conditional probability density function of is

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

with the mixing weight

|  |  |  |
| --- | --- | --- |
|  |  | (11) |















In the current study, we are interested in the expectation of y among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | () |

To accommodate new data in an online setting, control model complexity and allow to modeling time-varying processes, GGMR has been proposed by (Bouchachia & Vanaret, 2011) with growing and shrinking mechanisms. We utilized its updating gaussians algorithm in the present paper. More details can be seen in (Bouchachia & Vanaret, 2011). The best match Gaussian will be updated with the following formulas:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | (18) |

in which is the match probability calculated with new input and best match Gaussian , is the expected posterior, is the sum of the expected posterior for best match Gaussian, is the weights of best match Gaussian, is the on-going learning rate for j-th Gaussian, is the converging learning rate.

## 2.3 Hybrid Approach

In the present study, we have designed the Hybrid Model schema as shown in Figure **1**, which illustrates the underlying structure of the hybrid approach. Enabled by the real time predicted system load from RC network model and incremental learning framework from the GGMR model, those trained gaussian components from Expectation Maximization (EM) will be updated accordingly as the update rules shown in equation (13)~ (18). Specifically, the RC network module will get the target time step index from GGMR and return the predicted RS system load back to GGMR module.

Diagram

Description automatically generated

**Figure** **1** Underlying Communication for Hybrid Approach



## 2.4 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | (19) |
|  |  | (20) |
|  |  | (21) |
|  |  | (22) |
|  | = | (23) |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section presents a case study for all the three proposed methods, including RC network, GGMR and hybrid approach. It begins with a description of the test bed, then moved to the moved to the model development and selections, and concludes with a comparison of the performance of each modeling approach.

## 3.1 Test bed

The dataset included in-situ measurements for a living laboratory office space from January 15th to March 7th, 2022, with a 5-minute sampling rate. And further, we used the first two weeks data for training and the rest of data used for testing. The dataset can be divided into of two categories, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted by , Façade cavity space temperature denoted by , slab concrete temperature denoted by , flowing water temperature within slab pipe denoted by , solar radiation retrieved from a weather station denoted by , air handling unit consumed heating power . The estimated input values are determined using a predefined schedule in accordance with ASHRAE 90.1(ANSI/ASHRAE/IES 90.1-2010, 2010, p. 1)(ANSI/ASHRAE/IES 90.1-2010, 2010), such as internal heating radiation denoted by , and lighting radiation .

## 3.2 RC Network Model Development

The current subsection describes the design logic for the RC model, followed by a description of the target room's physical structures and, finally, our consideration of various RC model designs and their associated performance. Ultimately, the chosen design will be detailed.

Considering model accuracy-complexity trade-off, the following is the overall design logic for RC network construction:

1. Improve the model’s accuracy. The RC model should capture the key and most thermal behaviors of targeted space to maintain model robustness under a variety of operating conditions.
2. Reduce the complexity of model. Reduce the number of input variables or training data to avoid creating an excessively complex model.

The major thermal components of the living laboratory office space (Joe & Karava, 2017) include external walls, roof/ceiling, internal wall, south-facing double façade system, conditioned air from air handling unit (AHU) system, and hydronic radiant floor system.

In the present study, we experimented three RC network designs by considering model robustness and various levels of complexity or model orders. As illustrated in Figure **2**, we developed three models for RC networks, four-states Model 1, six-states Model 2 and five-states Model 3, in which represent temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. As for the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements, respectively.

Each of three models is composed of two components: room and concrete slab. We chose the same RC network model for room to effectively capture its thermal properties: a two-node envelope, one-node internal wall, one node cavity for double façade system, and room air node to capture the provided disturbance heating or cooling from AHU system. It is worth noting that we used the envelope node to represent the external wall and roof/ceiling to keep the model simple. In the case of the concrete slab, we experimented with various model orders to capture its thermal behaviors. The detailed thermal structure of radiant floor was omitted from Model 1. And we considered the entire slab to be a single node. In comparison to Model 1, Model 3 included an additional source node to represent the flow of water through slab pipes. Furthermore, Model 2 had one additional sink node than Model 3 to represent the heat transfer between source node and another space.

Figure **3** depicts the predicted and actual results obtained during the testing period (10892 sampling points for around 37 days). Model 1 has a significantly higher errors than Models 2 and 3, which can be attributed to the oversimplified concrete slab representation. Model 2 has a lower CVRMSE than Model 3, which is consistent with the addition of a sink node. Table **2** contains a more detailed comparison of performance. Model 2 was chosen as the optimal model for the RC network method since it performs better than Model 3 at capturing peaking loads.

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | (24) |
|  |  | (25) |
|  |  | (26) |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table **1**:

|  |  |  |
| --- | --- | --- |
|  |  | (27) |
|  |  | (28) |

As stated in equation (4), the RC network model training is essentially an optimization problem. In the present paper, p(James V. Miranda, 2018)

**Table** **1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

Description automatically generated

**Figure** **2** Structure of RC network. Left: Model 1 with four states; Middle: Model 2 with six states; Middle: Model 3 with five states.

Timeline

Description automatically generated with medium confidence

**Figure** **3** Testing results for Model 1, Model 2 and Model 3

**Table** **2** Comparison of proposed RC models (5-mins interval)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5.76 | 87.88 |
| Model 2 | **16.15** | **21.31** | **0.84** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1.28 | 35.89 |

## 3.2 GGMR Model Development

This subsection primarily discusses how to determine the input variables for the GGMR model. According to Wang et al. (Wang et al., 2018), correlation coefficients R were used to determine the strength and direction of the linear relationship between inputs and model outputs. And the correlation coefficient is between -1 and +1, with -1 indicating perfect negative linear correlation and +1 indicating perfect positive linear correlation. We experimented with various input combinations for the GGMR model, as its subset presented in Table **3** and Table **4**. It is worth noting that larger correlation coefficients do not necessarily mean better prediction. For instance, the correlation coefficient of was not more trivial than while the inputs including did not provide additional prediction power as shown in case 1 and 2 of Table **4**. Moreover, it was found additional prediction performance can almost be gained for free if we provide flow rate information as additional input during the process of model development. In comparison to case 1, case 3 had additional 3.26% lower of CVRMSE after adding from another GGMR prediction. In the end, case 3 inputs,   
 have been selected for GGMR Model.

**Table** **3** Correlation coefficients between Radiant Slab systems load and input variables

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| -0.06 | -0.08 | -0.16 | -0.89 | 0.35 | -0.16 | 1 |

**Table** **4** Prediction performance comparison for different GGMR inputs

|  |  |  |
| --- | --- | --- |
| **Case #** | **Inputs** | **CVRMSE (%)** |
| 1 |  | 25.81 |
| 2 |  | 26.93 |
| 3 |  | 22.55 |

## 3.3 Hybrid Model Development

As mentioned in Sec. 2.3, the development of the hybrid approach is primarily concerned with determining the number of warming up steps for the RC module, the number of Gaussians used in the GGMR module, and the learning rate used in the GGMR module. The warming up period is statistically chosen in this study, as illustrated in the left plot of Figure **4** And 15 has been chosen as the optimal number of warming-up steps for RC prediction. Additionally, as indicated by the middle and right plots of Figure **4** the optimal number of Gaussians and learning rate have been chosen as 15 and 8e-3, respectively. Additionally, different input combinations had also been experimented for Hybrid Model as presented in Table **5** Compared with case 1, case 2 had additional 1.27% lower of CVRMSE, which was consistent as shown in Table **4**. And we finally selected as the Hybrid Model inputs.

Chart, line chart

Description automatically generatedChart, line chart

Description automatically generated

**4**hyperparameters for Hybrid Approach. Left: Warming up steps for RC model; Middle: Number of Gaussians for GGMR model; Right: Learning rate for GGMR Model.

**5**Hybrid Model

|  |  |  |
| --- | --- | --- |
| **Case #** |  |  |
| 1 |  | 11.22 % |
| 2 |  | 9.95 % |

## 3.3 Performance Comparison for Proposed Models

As shown in Table **5**, all three proposed models complied with ASHRAE Guideline 14.(Landsberg et al., n.d.). This table indicates that the Hybrid Model is the most accurate model for predicting the energy consumption of Radiant Slab systems. To conduct a more detailed analysis of those models' prediction performance, typical days were selected and plotted in Figure **5**. All three models performed reasonably well in terms of prediction, though they fall short of accurately capturing peak load (which occurs usually at 6:00 PM when the people are leaving). Additionally, the GGMR Model was prone to overshoot or oscillate significantly around the measured data, whereas the RC Model was lean in terms of undershoot prediction and smooths the ups and downs. Furthermore, it is obvious that the Hybrid Model incorporates information from both the RC and GGMR models to provide the most accurate prediction of RS system load. Specifically, the RC model has a normalized root mean square error of 13.56 percent, a coefficient of variation of root mean square error of 15.59 percent, a mean absolute error (MAE) of 5.76 kilowatts (kW), and a mean absolute percentage error (MAPE) of 108.53 percent. The NRMSE of the GGMR model is 20.75 percent, the CVRMSE is 22.56 percent, the MAE is 7.61 kW, and the MAPE is 27.74 percent. The hybrid approach has an NRMSE of 8.77 percent (4.79 percent less than RC and 11.98 percent less than GGMR), a CVRMSE of 9.95 percent (5.64 percent less than RC and 12.6 percent less than GGMR), an MAE of 3.62 kW (2.14 kW less than RC and 3.99 kW less than GGMR), and a MAPE of 19.31 percent (89.22 percent lower from RC, 8.43 percent lower from GGMR).

**Table** **5** Performance comparison for hourly prediction of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| RC | 13.56 | 15.59 | 5.76 | 108.53 |
| GGMR | 20.75 | 22.55 | 7.61 | 27.74 |
| Hybrid | **8.77** | **9.95** | **3.62** | **19.31** |

Chart, histogram

Description automatically generated

**Figure** **5** Radiant slab load between RC model, GGMR model, Hybrid model and measured data.

# 4. Conclusion

In this paper, a novel hybrid modeling approach has been proposed to predict the energy consumption of a hydronic radiant slab system that incorporates the advantages of both the RC and GGMR models. The proposed method was validated using data from actual radiant slab operations at Purdue University. According to the case study, the Hybrid Model outperformed the RC, GGMR, and the hybrid model in terms of prediction performance. And the proposed Hybrid model has a CVRMSE of 9.95 percent for hourly prediction (5.64 percent less than RC, 12.6 percent less than GGMR), which clearly meets the criteria for ASHRAE Guideline 14. (Landsberg et al., n.d.). Specifically, it has been demonstrated that the RC model prediction can be used as input for a GGMR model to further reduce both the RC and GGMR model predictions.

During the model development process for the GGMR model's input variable selection, we discovered that a higher linear correlation does not always imply a higher prediction performance. This observation implies that there may be additional opportunity to investigate alternative input variables for both the GGMR and Hybrid models.

In addition, it's worth noting that the case study makes use of a single onsite dataset source. In the future, we need to conduct additional case studies using a variety of data sources.

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

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