A Novel Hybrid Modeling Method for Predicting Energy Use of Hydronic Radiant Slab Systems

First AUTHOR1\*, Second AUTHOR2

1Organization, Department or Equivalent,

City, State, Country

Contact Information (Phone, Fax, E-mail)

2Organization, Department or Equivalent,

City, State, Country

Contact Information (Phone, Fax, E-mail)

\* Corresponding Author

# ABSTRACT

Accurately predicting the performance of radiant slab systems can be challenging due to the large thermal capacitance of the radiant slab and room temperature stratification. Current methods for predicting heating and cooling energy consumption of hydronic radiant slabs include detail first-principle-based (e.g, finite difference) and reduced-order (e.g, thermal resistor-capacitor (RC) network) models. Creating and calibrating detailed first-principle models, as well as detailed RC network models for predicting the performance of radiant slabs require substantial efforts. To develop improved control, monitoring, and diagnostic methods, there is a need for simpler models that can be readily trained using in-situ measurements.

In this study, we explored a novel hybrid modeling method integrating a simple RC network model with an evolving learning-based algorithm growing Gaussian mixture regression (GGMR) modeling approach to predict the heating and cooling rates of a radiant slab system for a Living Laboratory office space. The RC network model provides heating or cooling load of the radiant slab system to the GGMR model as one of the inputs in real time. The three modeling approaches: 1) an RC network model; 2) a GGMR model; and 3) the proposed hybrid modeling between RC and GGMR. The three modeling methods have been compared for predicting the energy use of a radiant slab system of a Living Laboratory office space using measurement data from January 15th to March 7th, 2022. The first two weeks of data were used for training, while the remaining data was used for testing in all three modeling methods. The hybrid approach had an NRMSE of 8.77 percent (4.79 percent less than RC and 11.98 percent less than GGMR), a CVRMSE of 9.95 percent (5.64 percent less than RC and 12.6 percent less than GGMR), an MAE of 3.62 kW (2.14 kW less than RC and 3.99 kW less than GGMR), and a MAPE of 19.31 percent (89.22 percent lower from RC, 8.43 percent lower from GGMR). The hybrid modeling approach outperformed both RC model and GGMR model.

# 1. INTRODUCTION

Recently, hydronic radiant slab systems (HRSS) demonstrated significant benefits for thermal management of conditioned spaces, including increased thermal comfort and energy savings. Apart from these benefits, the large thermal storage capacity of HRSS has a few disadvantages. One disadvantage of the large thermal time constant is that it causes cooling output to be delayed when supply water flow rates and temperature were adjusted (Liu et al. 2011). Additionally, conventional control based on room temperature feedback may consume more primary energy than a conventional air system (Sourbron et al. 2009). Moreover, HRSS frequently experience concurrent thermal disturbances caused by solar radiation, internal heat, and air systems (Koschenz and Dorer 1999). As a result, conventional HRSS control frequently encounters overcooling or overheating issues. To address these issues, HRSS requires Model Predictive Control (MPC) with accurate load prediction, as stated in (Joe and Karava 2019). In general, load prediction for buildings fall into three categories: first principle based models, thermal resistor-capacitor (RC) network models, and data-driven models, as summarized in ASHRAE's (Handbook 2001) and (Dong et al. 2016). The following sections will review those models in detail, followed by the present research objective.

## 1.1 First principle-based models

The first principle-based models refers to the models using computational fluid dynamics (CFD) (Zhang et al. 2013) or building energy simulation software such as EnergyPlus(Crawley et al. 2001), and ESP-r (Clarke 2001). The computational cost of CFD makes them incompatible with large-scale simulation programs(Neumann, Gamisch, and Gschwander 2021; Rodríguez Jara et al. 2016). Most current building energy software requires a detailed physical and operational description of building, as well as the well-stirred zone air assumption, in order to design buildings and their heating ventilation and air condition (HVAC) system.

## 1.2 Thermal RC network model

The inverse grey-box RC model, which strikes a balance between physical based models and data-driven model(Braun and Chaturvedi 2002). A RC network model is considered of as a collection of linear ordinary differential equations (ODEs). RC models are typically in the form of 2R1C, 3R2C, or lumped RC parameter models with associated self-adjusting methods (Rodríguez Jara et al. 2016). According to (O’Dwyer et al. 2016), when the resistance and capacitance values are positive, there is theoretically a guaranteed thermal passivity solution for RC models. As for the training of RC model, there is considerable research devoted to optimizing the trade-off between model accuracy and complexity (Ahn and Song 2010; Goyal, Liao, and Barooah 2011; Koschenz and Dorer 1999; Liu et al. 2011).

Nevertheless, there are some limitations in terms of the RC model application. The accuracy of lumped parameter methods is highly dependent on the values of their characteristic parameters(Rodríguez Jara et al. 2016). Moreover, the accuracy of the RC model degrades when the slab is subjected to rapid thermal disturbances (Neumann, Gamisch, and Gschwander 2021; Rhee and Kim 2015).

## 1.3 Data-Driven Model

Many data-driven/machine learning algorithms have been evaluated for building energy modeling such as partial least squares (PLS), Principal component analysis (PCA), Gaussian process regression (GPR) and Gaussian mixture models (GMM). GPR had been used to capture the complex and highly subjective relationships between room temperature and subjective thermal perception(Guenther and Sawodny 2019). The GMM is widely recognized for its ability to model multimode characteristics and deal with process uncertainty (Billard et al. 2008; Li and Song 2020).

Considerable efforts have been made in the field of incremental learning GMR, or growing GMR (GGMR), to develop a mechanism for GMR adaptation (Bouchachia and Vanaret 2011; Cederborg et al. 2010; Karami and Wang 2018; Li and Song 2020; Wang, Kubichek, and Zhou 2018).

In this study, we propose a hybrid approach, in which the outputs from a simpler RC model used as one of the inputs to a GGMR model. Additionally, the proposed hybrid model can benefit from the GGMR model while overcoming the RC model’s limitations. The methodology and performance metrics are detailed in Sec. 2. Section 3 presented model development and case study for an existing office at Purdue University before a conclusion in Sec. 4.

# 2. METHODLOGY

This section discussed the methodology used to improve prediction performance, beginning with the development of RC network models and progressing to the GGMR approach, and finally to the hybrid modeling approach, which combines the RC and GGMR approaches. The final subsection describes the model prediction performance criteria metrics.

## 2.1 RC Network Model

Heat balance equations on each temperature or state variable are used to create a RC network model (Braun and Chaturvedi 2002; Joe and Karava 2017). A general state-space model for estimating radiant slab systems load is of the form

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where estimated resistances, capacities and heat flux coefficient form matrices *A, B* and vector *c* and *d*. And *x, u, y* represents vector of state variables, vector of inputs and output variable respectively. For HRSS, the output variable is the cooling and heating load. The state vector contains all the temperature nodes. The input vector contains all the driving conditions, such as the heated or chilled water temperature and its derivation along the sampling time within tubes, exterior air temperature, solar radiation, lighting, and occupancy schedule.

The discrete version of the above state-space model can be written in terms of a recursive formula as

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  | (3) |

where the subscript *d* indicates these variables are the discretized forms of *A, B, c, d* in equation (1). A typical objective function for RC network model is to minimize the root-mean-square error (RMSE) for the training duration, denoted as the following, where *N* stands for the number of samples.

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

## 2.2 GGMR Method

Gaussian mixture regression (GMR)(Sung 2004) is a regression approach that models probability distributions rather than functions. Assume the data follow the joint density

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

where K is the number of Gaussian mixtures, is the weight coefficient mean , covariance . The above Gaussian mixture probability function shown in equation (5) can be portioned as

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () | |

From equation (6), the marginal density of X is

|  |  |  |
| --- | --- | --- |
|  |  | () |

The conditional probability density function of is

|  |  |  |
| --- | --- | --- |
|  |  | () |

with the mixing weight

|  |  |  |
| --- | --- | --- |
|  |  | () |

In the current study, we are interested in the expectation of *y* among all gaussian components:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

To accommodate new data in an online setting, control model complexity and allow to modeling time-varying processes, GGMR has been proposed by (Bouchachia and Vanaret 2011) with growing and shrinking mechanisms. We utilized its updating gaussians algorithm in the present paper. More details can be seen in (Bouchachia and Vanaret 2011). The best match Gaussian will be updated with the following formulas:

|  |  |  |
| --- | --- | --- |
|  |  | (13) |
|  |  | (14) |
|  |  | (15) |
|  |  | (16) |
|  |  | (17) |
|  |  | (18) |

in which is the match probability calculated with new input and best match Gaussian , is the expected posterior, is the sum of the expected posterior for best match Gaussian, is the weights of best match Gaussian, is the on-going learning rate for j-th Gaussian, is the converging learning rate.

## 2.3 Hybrid Approach

In the present study, we have designed the Hybrid Model schema as shown in Figure **1**, which illustrates the underlying structure of the hybrid approach. Enabled by the real time predicted system load from RC model and incremental learning framework from the GGMR model, those trained gaussian components from Expectation Maximization (EM) will be updated accordingly as the update rules shown in equation (13)~ (18). Specifically, the RC network module will get the target time step index from GGMR and return the predicted RS system load back to GGMR module.

Diagram

Description automatically generated

**Figure** **1** Underlying Communication for Hybrid Approach

## 2.4 Model Performance Evaluation Criteria

Four indices, normalized root mean square error (NRMSE), coefficient of variation of root mean square error (CVRMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |
|  |  | () |
|  | = | () |

where n the number of observations, is the standard deviation of predictions, is the average of measured values.

# 3. CASE STUDY

This section presents a case study for all the three proposed methods, including RC, GGMR and hybrid approach.

## 3.1 Test bed

The dataset included in-situ measurements for a living laboratory office space from January 15th to March 7th, 2022, with a 5-minute sampling rate. The first two weeks data were used for training and the rest of data used for testing. The dataset can be divided into of two categories, onsite sensor data and estimated data. Onsite sensor data includes the followings: outdoor air temperature denoted by , Façade cavity space temperature denoted by , slab concrete temperature denoted by , flowing water temperature within slab pipe denoted by , solar radiation retrieved from a weather station denoted by , air handling unit consumed heating power . The estimated input values are determined using a predefined schedule in accordance with (ANSI/ASHRAE/IES 90.1-2010 2010, 1), such as internal heating radiation denoted by , and lighting radiation .

## 3.2 RC Network Model Development

The current subsection describes the design logic for the RC model, followed by a description of the target room's physical structures and, finally, our consideration of various RC model designs and their associated performance. Ultimately, the chosen design will be detailed.

Considering model accuracy-complexity trade-off, the following is the overall design logic for RC network construction:

1. Improve the model’s accuracy. The RC model should capture the key and most thermal behaviors of targeted space to maintain model robustness under a variety of operating conditions.
2. Reduce the complexity of model. Reduce the number of input variables or training data to avoid creating an excessively complex model.

The major thermal components of the living laboratory office space include external walls, roof/ceiling, internal wall, south-facing double façade system, conditioned air from air handling unit (AHU) system, and hydronic radiant floor system.

In the present study, we experimented three RC network designs by considering model robustness and various levels of complexity or model orders. As illustrated in Figure **2**, we developed three models for RC networks, four-states Model 1, six-states Model 2 and five-states Model 3, in which represent temperature, capacitances, resistances, heat flux due to radiation and corresponding coefficients. As for the subscripts, , represent outdoor air, façade cavity, slab concrete, hot water or chilled water within tubes, insulation below tubes, envelope, room air, internal wall, solar radiation, internal heat, lighting, air handling unit, thermal heat flux load requirements, respectively.

Each of three models is composed of two components: room and concrete slab. We chose the same RC network model for room to effectively capture its thermal properties: a two-node envelope, one-node internal wall, one node cavity for double façade system, and room air node to capture the provided disturbance heating or cooling from AHU system. It is worth noting that we used the envelope node to represent the external wall and roof/ceiling to keep the model simple. In the case of the concrete slab, we experimented with various model orders to capture its thermal behaviors. The detailed thermal structure of radiant floor was omitted from Model 1. And we considered the entire slab to be a single node. In comparison to Model 1, Model 3 included an additional source node to represent the flow of water through slab pipes. Furthermore, Model 2 had one additional sink node than Model 3 to represent the heat transfer between source node and another space.

Figure **3** depicts the predicted and actual results obtained during the testing period (10892 sampling points for around 37 days). Model 1 has a significantly higher errors than Models 2 and 3, which can be attributed to the oversimplified concrete slab representation. Model 2 has a lower CVRMSE than Model 3, which is consistent with the addition of a sink node. Table **2** contains a more detailed comparison of performance. Model 2 was chosen as the optimal model for the RC network method since it performs better than Model 3 at capturing peaking loads.

The Model 2 can be represented by a state-space model with the following state, input, and output variables definitions:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |

Thermal resistances, () and thermal capacity ( are evaluated using the following equations, the results of which are displayed in Table **1**:

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

As stated in equation (4), the RC network model training is essentially an optimization problem. In the present paper, particle swarm optimization (PSO) from python package (James V. Miranda 2018) was used to solve the above optimization problem.

**Table** **1** Estimated values for Rs (K/W) and Cs (J/K)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| 3.6E-3 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Diagram, schematic

Description automatically generated

Diagram, schematic

Description automatically generatedDiagram

Description automatically generated

**Figure** **2** Structure of RC network. Up: Model 1 with four states; Left: Model 2 with six states; Right: Model 3 with five states.

Timeline

Description automatically generated with medium confidence

**Figure** **3** Testing results for Model 1, Model 2 and Model 3

**Table** **2** Comparison of proposed RC models (5-mins interval)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| Model 1 | 156.96 | 117.52 | 5.76 | 87.88 |
| Model 2 | **16.15** | **21.31** | **0.84** | **26.10** |
| Model 3 | 27.60 | 31.37 | 1.28 | 35.89 |

## 3.2 GGMR Model Development

This subsection primarily discusses how to determine the input variables for the GGMR model. Correlation coefficients R were used to determine the strength and direction of the linear relationship between inputs and model outputs. We experimented with various input combinations for the GGMR model, as its subset presented in Table **3** and Table **4**. It is worth noting that larger correlation coefficients do not necessarily mean better prediction. For instance, the correlation coefficient of was not more trivial than while the inputs including did not provide additional prediction power as shown in case 1 and 2 of Table **4**. Moreover, it was found additional prediction performance can almost be gained for almost free if we provide flow rate information as additional input during the process of model development. In comparison to case 1, case 3 had additional 3.26% lower of CVRMSE after adding from another GGMR prediction. In the end, case 3 inputs,   
 have been selected for GGMR model.

**Table** **3** Correlation coefficients between Radiant Slab systems load and input variables

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| -0.06 | -0.08 | -0.16 | -0.89 | 0.35 | -0.16 | 1 |

**Table** **4** Prediction performance comparison for different GGMR inputs

|  |  |  |
| --- | --- | --- |
| **Case #** | **Inputs** | **CVRMSE (%)** |
| 1 |  | 25.81 |
| 2 |  | 26.93 |
| 3 |  | 22.55 |

## 3.3 Hybrid Model Development

As mentioned in Sec. 2.3, the development of the hybrid approach is primarily concerned with determining the number of warming up steps for the RC module, the number of Gaussians used in the GGMR module, and the learning rate used in the GGMR module. The warming up period is statistically chosen in this study, as illustrated in the up plot of Figure **4** And 15 has been chosen as the optimal number of warming-up steps for RC prediction. Additionally, as indicated by the left and right plots of Figure **4** the optimal number of Gaussians and learning rate have been chosen as 15 and 8e-3, respectively. Additionally, different input combinations had also been experimented for hybrid model as presented in Table **5** . Compared with case 1, case 2 had additional 1.27% lower of CVRMSE, which was consistent as shown in Table **4**. And we finally selected as the Hybrid Model inputs.

Chart

Description automatically generated

Chart, line chart

Description automatically generatedChart, line chart

Description automatically generated

**Figure** **4** Determination of hyperparameters for Hybrid Approach. Up: Warming up steps for RC model; Left: Number of Gaussians for GGMR model; Right: Learning rate for GGMR Model.

**Table** **5** Prediction performance comparison for different hybrid model inputs

|  |  |  |
| --- | --- | --- |
| **Case #** | **Inputs** | **CVRMSE** |
| 1 |  | 11.22 % |
| 2 |  | 9.95 % |

## 3.3 Performance Comparison for Proposed Models

As shown from the statistical results in Table **5**, all three proposed models complied with ASHRAE Guideline 14 (ASHRAE 2014). Moreover, this table demonstrates that the hybrid model is the most accurate model for predicting the energy consumption of radiant slab systems, as it incorporates information from both RC and GGMR models. Specifically, the hybrid approach has an NRMSE of 8.77 percent (4.79 percent less than RC and 11.98 percent less than GGMR), a CVRMSE of 9.95 percent (5.64 percent less than RC and 12.6 percent less than GGMR), an MAE of 3.62 kW (2.14 kW less than RC and 3.99 kW less than GGMR), and a MAPE of 19.31 percent (89.22 percent lower from RC, 8.43 percent lower from GGMR).

**Table** **5** Performance comparison for hourly prediction of proposed models

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Models** | **NRMSE (%)** | **CVRMSE (%)** | **MAE (kW)** | **MAPE (%)** |
| RC | 13.56 | 15.59 | 5.76 | 108.53 |
| GGMR | 20.75 | 22.55 | 7.61 | 27.74 |
| Hybrid | **8.77** | **9.95** | **3.62** | **19.31** |

# 4. Conclusion

In this paper, a novel hybrid modeling approach has been proposed to predict the energy consumption of a hydronic radiant slab system that incorporates the advantages of both the RC and GGMR models. The proposed method was validated using data from actual radiant slab operations at Purdue University. According to the case study, the hybrid model outperformed the RC, GGMR in terms of prediction performance. And the proposed hybrid model has a CVRMSE of 9.95 percent for hourly prediction (5.64 percent less than RC, 12.6 percent less than GGMR), which clearly meets the criteria for ASHRAE Guideline 14. Specifically, it has been demonstrated that the RC model prediction can be used as input for a GGMR model to further reduce prediction errors of RC and GGMR models.

In addition, it's worth noting that the case study makes use of a single onsite dataset source. In the future, we need to conduct additional case studies using a variety of data sources.

# NOMENCLATURE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | area |  | *R* | resistors | K/W |
|  | capacitors | J/K |  | density | *kg/m3* |
|  | Specific heat | J/Kg/K | *T* | temperature | K |
|  | heat transfer coefficient |  | *t* | time | second |
| L | thickness | *m* |  |  |  |
|  | conductivity | *w/m/K* |  |  |  |
| Q | heating flux | *W* |  |  |  |
| **Subscript** |  |  |  |  |  |
| *adj* | adjacent |  | *intwall* | internal wall |  |
| *AHU* | air handling unit |  | *int* | internal heating |  |
| *cav* | cavity |  | *rad* | radiant heating flux |  |
| *env* | envelope |  |  |  |  |

# REFERENCES

Ahn, Byung-Cheon, and Jae-Yeob Song. 2010. “Control Characteristics and Heating Performance Analysis of Automatic Thermostatic Valves for Radiant Slab Heating System in Residential Apartments.” *Energy* 35(4): 1615–24.

“Ambient Weather Network.” 2022. *Ambient Weather Network*. https://ambientweather.net/ (April 11, 2022).

ANSI/ASHRAE/IES 90.1-2010. 2010. *Energy Standard for Buildings Except Low-Rise Residential Buildings*. American Society of Heating, Refrigerating and Air-Conditioning Engineers.

ASHRAE. 2014. “ASHRAE Guideline 14: Measurement of Energy, Demand and Water Savings.” : 150.

Billard, Aude, Sylvain Calinon, Rüdiger Dillmann, and Stefan Schaal. 2008. “Robot Programming by Demonstration.” In *Springer Handbook of Robotics*, eds. Bruno Siciliano and Oussama Khatib. Berlin, Heidelberg: Springer, 1371–94. https://doi.org/10.1007/978-3-540-30301-5\_60 (April 12, 2022).

Bouchachia, Hamid, and Charlie Vanaret. 2011. “Incremental Learning Based on Growing Gaussian Mixture Models.”

Braun, James E., and Nitin Chaturvedi. 2002. “An Inverse Gray-Box Model for Transient Building Load Prediction.” *HVAC&R Research* 8(1): 73–99.

Cederborg, Thomas, Ming Li, Adrien Baranes, and Pierre-Yves Oudeyer. 2010. “Incremental Local Online Gaussian Mixture Regression for Imitation Learning of Multiple Tasks.” In *2010 IEEE/RSJ International Conference on Intelligent Robots and Systems*, , 267–74.

Clarke, Joseph. 2001. *Energy Simulation in Building Design*. 2nd ed. London: Routledge.

Crawley, Drury B. et al. 2001. “EnergyPlus: Creating a New-Generation Building Energy Simulation Program.” *Energy and Buildings* 33(4): 319–31.

Dong, Bing, Zhaoxuan Li, S. M. Mahbobur Rahman, and Rolando Vega. 2016. “A Hybrid Model Approach for Forecasting Future Residential Electricity Consumption.” *Energy and Buildings* 117: 341–51.

Goyal, Siddharth, Chenda Liao, and Prabir Barooah. 2011. “Identification of Multi-Zone Building Thermal Interaction Model from Data.” In *2011 50th IEEE Conference on Decision and Control and European Control Conference*, , 181–86.

Guenther, Janine, and Oliver Sawodny. 2019. “Feature Selection and Gaussian Process Regression for Personalized Thermal Comfort Prediction.” *Building and Environment* 148: 448–58.

Handbook, ASHRAE. 2001. “Fundamentals SI Edition.” *American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, GA*.

James V. Miranda, Lester. 2018. “PySwarms: A Research Toolkit for Particle Swarm Optimization in Python.” *The Journal of Open Source Software* 3(21): 433.

Joe, Jaewan, and Panagiota Karava. 2017. “Agent-Based System Identification for Control-Oriented Building Models.” *Journal of Building Performance Simulation* 10(2): 183–204.

———. 2019. “A Model Predictive Control Strategy to Optimize the Performance of Radiant Floor Heating and Cooling Systems in Office Buildings.” *Applied Energy* 245: 65–77.

Karami, Majid, and Liping Wang. 2018. “Fault Detection and Diagnosis for Nonlinear Systems: A New Adaptive Gaussian Mixture Modeling Approach.” *Energy and Buildings* 166: 477–88.

Koschenz, Markus, and Viktor Dorer. 1999. “Interaction of an Air System with Concrete Core Conditioning.” *Energy and Buildings* 30(2): 139–45.

Li, Deyang, and Zhihuan Song. 2020. “A Novel Incremental Gaussian Mixture Regression and Its Application for Time-Varying Multimodal Process Quality Prediction.” In *2020 IEEE 9th Data Driven Control and Learning Systems Conference (DDCLS)*, , 645–50.

Liu, Kuixing et al. 2011. “Establishment and Validation of Modified Star-Type RC-Network Model for Concrete Core Cooling Slab.” *Energy and Buildings* 43(9): 2378–84.

Neumann, Hannah, Sebastian Gamisch, and Stefan Gschwander. 2021. “Comparison of RC-Model and FEM-Model for a PCM-Plate Storage Including Free Convection.” *Applied Thermal Engineering* 196: 117232.

O’Dwyer, Edward et al. 2016. “Modelling and Disturbance Estimation for Model Predictive Control in Building Heating Systems.” *Energy and Buildings* 130: 532–45.

Rhee, Kyu-Nam, and Kwang Woo Kim. 2015. “A 50 Year Review of Basic and Applied Research in Radiant Heating and Cooling Systems for the Built Environment.” *Building and Environment* 91: 166–90.

Rodríguez Jara, Enrique Á. et al. 2016. “A New Analytical Approach for Simplified Thermal Modelling of Buildings: Self-Adjusting RC-Network Model.” *Energy and Buildings* 130: 85–97.

Sourbron, M. et al. 2009. “Efficiently Produced Heat and Cold Is Squandered by Inappropriate Control Strategies: A Case Study.” *Energy and Buildings* 41(10): 1091–98.

Sung, Hsi Guang. 2004. “Gaussian Mixture Regression and Classification.” Ph.D. Rice University. https://www.proquest.com/docview/305155652/abstract/8C63788CCF824897PQ/1 (April 12, 2022).

Wang, Liping, Robert Kubichek, and Xiaohui Zhou. 2018. “Adaptive Learning Based Data-Driven Models for Predicting Hourly Building Energy Use.” *Energy and Buildings* 159: 454–61.

Zhang, Rui, Khee Poh Lam, Shi-chune Yao, and Yongjie Zhang. 2013. “Coupled EnergyPlus and Computational Fluid Dynamics Simulation for Natural Ventilation.” *Building and Environment* 68: 100–113.