OPTIMIZATION MODEL FORMULATION

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INDEXED SETS

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i = plant i (i = 1…5)

j = warehouse j (j = 1…4)

k = retail center k (k = 1…8)

t = year number t (t = 1…10)

DATA

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d_k = demand of retail center k (k = 1…8) at year-1

C_{ij} = shipping cost from plant i to warehouse j at year 1

P_i = capacity of plant i (won't change by time)

S_{ik} = Shipping cost from warehouse j to retail center k at year 1

A_i = the cost of alloy for plant i

WC_i = cost of Widget

f_i = fixed construction cost of plant i

e_i = operating cost of plant i

RC_i = reopening cost for plant i

D_i = shutdown cost of plant i

W_j = capacity of warehouse j (1000 flugels per month)

T = Total demand at year-1

DECISION VARIABLES

{Demand}

X_{ijt} = Flugels produced at plant i and sent to warehouse j in year t

  N_{1-ijt} = Flugels produced at plant i and sent to warehouse j in year t (resources)

  N_{2-ijt} = Flugels produced at plant i and sent to warehouse j in year t (resources)

I_{jt} = Flugels stored in warehouse j in year t

Y_{jkt} = Flugels shipped from warehouse j to retail k in year t

{Production Lines at Plants}
$B_{it} = 1\ if$ plant i line is constructed at beginning of year t (i = 1…5, t = 1..10); 0 if plant not constructed
$O_{it} = 1\ if$ plant i line is operating/maintained at year t; 0 if the plant shutdown (i = 1…5)
$R_{it} = 1\ if$ plant i line is reopened at beginning of year t; 0 if the plant stays shutdown/operating (i = 1…5)
$S_{it} = 1\ if$ plant i line is shutdown at end of year t; 0 if plant keeps operating (i = 1…5)
$o(minus)_{it} = 1\ if$ plant i line is open at end of year t, but shut down at end of year t-1; 0 otherwise (i = 1…10)

{Cost Structure for Widget}
$\lambda1_i$ = proportion of flugels produced in plant i in year t less than 0
$\lambda2_i$ = proportion of flugels produced in plant i in year t between 0 and 3000 flugels
$\lambda3_i$ = proportion of flugels produced in plant i in year t between 3000 and plant i capacity flugels

OBJECTIVE

Minimize Total Cost of Meeting Demand over the next 10 years.

Cost Equation: Alloy + Widget + Shipping Cost from Plant to Warehouse + Shipping Cost from Warehouse to Retail Center + Plant Costs

{Alloy}

$$\sum_j 4.7 \cdot 0.02 \cdot X_{ij1} + \sum_j 1.03 * 4.7 \cdot 0.02 \cdot X_{ij2} + ... + \sum_j 1.03^9 * 4.7 \cdot 0.02 \cdot X_{ij10}$$

{Widget}

$$\sum (0(\lambda1_i) + 450000(\lambda2_i) + (P_i * 120) * (\lambda3_i))$$

{Shipping Cost from Plant to Warehouse}

$$\sum_i C_{ij} X_{ij1} + \sum_i 1.03 C_{ij} X_{ij2} + \sum_i 1.03^2 C_{ij} X_{ij3} + ... + \sum_i 1.03^9 C_{ij} X_{ij10}$$

{Shipping Cost from Warehouse to Retail}

$$\sum_j S_{ik} Y_{jk1} + \sum_j 1.03 S_{ik} * Y_{jk2} + \sum_j 1.03^2 S_{ik} * Y_{jk3} + \ldots + \sum_j 1.03^9 S_{ik} * Y_{jk10}$$

{Plant Costs}

$$(\sum_i f_i B_{i1} + \sum_i e_i O_{i1} + \sum_i RC_i R_{i1} + \sum_i D_i S_{i1}) + 1.03(\sum_i f_i B_{i2} + \sum_i e_i O_{i2} + \sum_i RC_i R_{i2} + \sum_i D_i S_{i2}$$

$$) + \ldots + 1.03^9 (\sum_i f_i B_{i10} + \sum_i e_i O_{i10} + \sum_i RC_i R_{i10} + \sum_i D_i S_{i10})$$

CONSTRAINTS

{Resource Constraint - Pounds of Alloy}

$$\sum_j 4.7 X_{ijt} \le 60000 \ \ for\ all\ plant\ i\ in\ year\ t$$

{Cost Structures - Widget Subassemblies}

t_i : 0, 3000, 12000 $\qquad$ $f(t_i)$: 0, 450000, 1530000

f(xvars_1):	150 (xvars_1)	0 <= xvars_1 <= 3000
	450000 + 120 (xvars_1)	3000 <= xvars_1 <= P_i

$$\sum_j x_{ijt} = 0(\lambda 1_i) + 3000(\lambda 2_i) + P_i * (\lambda 3_i)$$

$\lambda 1_i + \lambda 2_i + \lambda 3_i = 1$

Y1 + Y2 = 1

$\lambda 1_i$ <= Y1

$\lambda 2_i$ <= Y1 + Y2

$\lambda 3_i$ <= Y2

$\lambda 1_i$, $\lambda 2_i$, $\lambda 3_i$ >= 0

Y1, Y2, Y3 = binary

{Plant Construction Only Occurs Once}

$$\sum_t B_{it} \le 1$$

{Plant Production Capacity}

$$\sum_j X_{ijt} \le P_i * O_{it}$$

{Inventory Definition}
$\sum Y_{jkt} - \sum_i X_{ijt} = I_{jt}$
{No Ending Inventory}
$I_{jt} = 0 \quad where\ t = 10$

{Warehouse Capacity}
$\sum_i X_{ijt} + I_{jt-1} \leq 12000$
$\sum Y_{jkt} \leq 12000$

{Flow Equilibrium from Plant to Warehouse and Warehouse to Retailer}
$\sum_i X_{ijt} + I_{jt-1} = \sum (Y_{jkt}) + I_{jt}$

{Meet Demand of Retail Centers}
$\sum Y_{jkt} = d_k + (.20 * d_k * (t - 1)) \quad where\ k = 1, 2, 4, 5, 8$
$\sum Y_{jkt} = d_k + (.25 * d_k * (t - 1)) \quad where\ k = 3, 6, 7$

{Plant Production Lines}
If $R_{it} = 1$, then $O_{it} = 1$ {if line is reopened, the plant incurs operating costs}
If $B_{it} = 1$, then $R_{it} = 1$ {if line is constructed, the plant incurs reopening costs}
If $B_{it} = 1$, then $O_{it} = 1$ {if line is constructed, the plant incurs operating costs}
If $S_{it} = 1$, then $X_{ijt+1} = 0$ {if there is not production in plant i, plant i is close in the year t - 1}
If $o(minus)_{it} = 1$, then $B_{it} = 1$ {if line i is open at year t and shut down at year t-1, then plant i is constructed at year t}
If $O_{i1} = 1$, then $B_{i1} = 1$ {if line i is operating at year 1, then plant i is constructed at year 1}

{Non-negativity}
$X_{ijt}, Y_{jkt}, I_{jt}, N_{1-ijt}, N_{2-ijt} >= 0$

{Binary}
$B_{it}, O_{it}, R_{it}, S_{it}$