$$A = \begin{bmatrix} a_0 & a_{-1} & a_{-2} & \cdots & \cdots & a_{-n+1} \\ a_1 & a_0 & a_{-1} & \ddots & & \vdots \\ a_2 & a_1 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & a_{-1} & a_{-2} \\ \vdots & & \ddots & a_1 & a_0 & a_{-1} \\ a_{n-1} & \cdots & \cdots & a_2 & a_1 & a_0 \end{bmatrix} \tag{1}$$

$$A = \begin{bmatrix} a_0 & a_{n-1} & a_{n-2} & \cdots & \cdots & a_1 \\ a_1 & a_0 & a_{n-1} & a_{n-2} & & \vdots \\ a_2 & a_1 & a_0 & a_{n-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & a_1 & a_{n-2} \\ \vdots & & \ddots & a_1 & a_0 & a_{n-1} \\ a_{n-1} & a_{n-2} & a_{n-3} & \cdots & \cdots & a_0 \end{bmatrix} \tag{2}$$

$$A = \begin{bmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & a_3 & b_3 & \ddots & \\ & & \ddots & \ddots & c_{n-1} \\ & & & a_n & b_n \end{bmatrix} \tag{3}$$

$$A = \begin{bmatrix} b_1 & c_1 & & & a_1 \\ a_2 & b_2 & c_2 & & \\ & a_3 & b_3 & \ddots & \\ & & \ddots & \ddots & c_{n-1} \\ c_n & & & a_n & b_n \end{bmatrix} \tag{4}$$

$$A = \begin{bmatrix} c_1 & d_1 & e_1 & & & & & \\ b_2 & c_2 & d_2 & e_2 & & & & \\ a_3 & b_3 & c_3 & d_3 & e_3 & & & \\ & a_4 & b_4 & c_4 & d_4 & e_4 & & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & & a_{n-2} & b_{n-2} & c_{n-2} & d_{n-2} & e_{n-2} \\ & & & & a_{n-1} & b_{n-1} & c_{n-1} & d_{n-1} \\ & & & & & a_n & b_n & c_n \end{bmatrix} \tag{5}$$

三对角

$$c_i^* = \begin{cases} \frac{c_i}{b_i} & \text{if } i = 1, \\ \frac{c_i}{b_i - a_i c_i^*} & \text{if } i = 2, 3, \ldots, n-1. \end{cases} \tag{6}$$

$$d_i^* = \begin{cases} \frac{d_i}{b_i} & \text{if } i = 1, \\ \frac{d_i - a_i d_{i-1}^*}{b_i - a_i c_{i-1}^*} & \text{if } i = 2, 3, \ldots, n. \end{cases} \tag{7}$$

$$\begin{cases} x_n = d_n^*, \\ x_i = d_i^* - c_i x_{i+1} \text{ for } i = n-1, n-2, \ldots, 1. \end{cases} \tag{8}$$

五对角

$$b_i^* = \begin{cases} b_i & \text{if } i = 2, \\ b_i - a_i d_{i-2}^* & \text{if } i = 3, 4, \ldots, n. \end{cases} \tag{9}$$

$$c_i^* = \begin{cases} c_i & \text{if } i = 1, \\ c_i - \frac{b_i^* d_{i-1}}{c_{i-1}} & \text{if } i = 2, \\ c_i - a_i e_{i-2}^* - b_i^* d_{i-1}^* & \text{if } i = 3, 4, \ldots, n. \end{cases} \tag{10}$$

$$d_i^* = \begin{cases} \frac{d_i}{c_i} & \text{if } i = 1, \\ \frac{d_i - b_i^* d_{i-1}^*}{c_i^*} & \text{if } i = 2, 3, \ldots, n-1. \end{cases} \tag{11}$$

$$\tag{12}$$

$$e_i^* = \frac{e_i}{c_i^*} \text{ if } i = 1, 2, \ldots, n-2. \tag{13}$$

$$y_i^* = \begin{cases} \frac{y_i}{c_i} & \text{if } i = 1, \\ \frac{y_i - b_i^* y_{i-1}}{c_i^*} & \text{if } i = 2 \\ \frac{y_i - a_i y_{i-2}^* - b_i^* y_{i-1}^*}{c_i^*} & \text{if } i = 3, 4, \ldots, n. \end{cases} \tag{14}$$

$$
\begin{cases}
x_n = y_n^* \\
x_{n-1} = y_{n-1}^* - d_{n-1}^* x_n \\
x_i = y_i^* - d_i^* x_{i+1} - e_i^* x_{i+2} \text{ for } i = n-2, n-3, \ldots, 1.
\end{cases}
\tag{15}
$$

循环三对角

$$
L = \begin{bmatrix}
d_1 & & & & & \\
a_2 & d_2 & & & & \\
 & \ddots & \ddots & & & \\
 & & a_{n-2} & d_{n-2} & & \\
 & & & a_{n-1} & d_{n-1} & \\
\alpha_1 & \alpha_2 & \cdots & \alpha_{n-2} & \alpha_{n-1} & d_n
\end{bmatrix},
\tag{16}
$$

$$
U = \begin{bmatrix}
1 & u_1 & & & & \beta_1 \\
 & 1 & u_2 & & & \beta_2 \\
 & & \ddots & \ddots & & \vdots \\
 & & & 1 & u_{n-2} & \beta_{n-2} \\
 & & & & 1 & \beta_{n-1} \\
 & & & & & 1
\end{bmatrix}
\tag{17}
$$

$$
d_1 = b_1, \quad u_1 = c_1/d_1, \quad \alpha_1 = c_n, \quad \beta_1 = \alpha_1/d_1,
\tag{18}
$$

$$
\begin{cases}
d_i & = b_i - a_i u_{i-1} \\
u_i & = c_i/d_i, \quad i = 2, 3, \ldots, n-2 \\
\alpha_i & = -\alpha_{i-1} u_{i-1}
\end{cases}
\tag{19}
$$

$$
d_{n-1} = b_{n-1} - \alpha_{n-2} u_{n-2}, \tag{20}
$$

$$
\alpha_{n-1} = a_n - \alpha_{n-2} u_{n-2}, \tag{21}
$$

$$
\beta_{n-1} = (c_{n-1} - a_{n-1}\beta_{n-2})/d_{n-1}, \tag{22}
$$

$$
d_n = b_n - \sum_{i=1}^{n-1} \alpha_i \beta_i. \tag{23}
$$

$$
\begin{cases}
y_1 & = f_1/d_1, \\
y_i & = (f_i - a_i y_{i-1})/d_i, \quad i = 2, 3, \ldots, n-1, \\
y_n & = (f_n - \sum_{i=1}^{n-1} \alpha_i y_i)/d_n.
\end{cases}
\tag{24}
$$

$$\begin{cases} x_n &= y_n, \\ x_{n-1} &= y_{n-1} - \beta_{n-1} x_n, \\ x_i &= y_i - u_i x_{i+1} - \beta_i x_n, \quad i = n-2, n-3, \ldots, 2, 1. \end{cases} \tag{25}$$