

GPU-parallel Gibbs sampling of a hierarchical model of hybrid vigor in RNA-seq experiments

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Outline

- 1 Biological background
 - Hybrid vigor
- 2 The model
- 3 The Gibbs sampler
 - Gibbs steps
 - Estimated heterosis probabilities
 - GPU parallelism
- 4 The software

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High-parent heterosis: child's trait surpasses both parents

Parent 1



Parent 2



Child



Low-parent heterosis: child's trait is weaker than in each parent

Parent 1



Parent 2



Child



Mid-parent heterosis: child's trait is different than average of parents

Parent 1



Parent 2



Child



High-parent heterosis in gene expression

	Parent 1			Child		Parent 2	
Gene 1	100	225	0	70	279	300	106
Gene 2	0	1	1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
...
Gene 34897	10	13	6	819	761	902	912

Low-parent heterosis in gene expression

	Parent 1			Child		Parent 2	
Gene 1	100	225	0	70	279	300	106
Gene 2	0	1	1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
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Mid-parent heterosis in gene expression

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The model

$$\mu(n, \phi_g, \alpha_g, \delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent 2} \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$c_n \stackrel{\text{ind}}{\sim} \text{N}(0, \sigma_c^2)$$

$$\sigma_c \sim \text{U}(0, \sigma_{c0})$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \text{N}(0, \eta_g^2)$$

$$\eta_g^2 \stackrel{\text{ind}}{\sim} \text{Inv-Gamma} \left(\text{shape} = \frac{d}{2}, \text{rate} = \frac{d \cdot \tau^2}{2} \right)$$

$$d \sim \text{U}(0, d_0)$$

$$\tau^2 \sim \text{Gamma}(\text{shape} = a_\tau, \text{rate} = b_\tau)$$

The model

$$\mu(n, \phi_g, \alpha_g, \delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent 2} \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} N(\theta_\phi, \sigma_\phi^2)$$

$$\theta_\phi \sim N(0, \gamma_\phi^2)$$

$$\sigma_\phi \sim U(0, \sigma_{\phi 0})$$

$$\alpha_g \stackrel{\text{ind}}{\sim} (1 - I(\alpha_g)) \cdot \pi_\alpha + I(\alpha_g) \cdot (1 - \pi_\alpha) \cdot N(\alpha_g \mid \theta_\alpha, \sigma_\alpha^2)$$

$$\theta_\alpha \sim N(0, \gamma_\alpha^2)$$

$$\sigma_\alpha \sim U(0, \sigma_{\alpha 0})$$

$$\pi_\alpha \sim \text{Beta}(a_\alpha, b_\alpha)$$

$$\delta_g \stackrel{\text{ind}}{\sim} (1 - I(\delta_g)) \cdot \pi_\delta + I(\delta_g) \cdot (1 - \pi_\delta) \cdot N(\delta_g \mid \theta_\delta, \sigma_\delta^2)$$

$$\theta_\delta \sim N(0, \gamma_\delta^2)$$

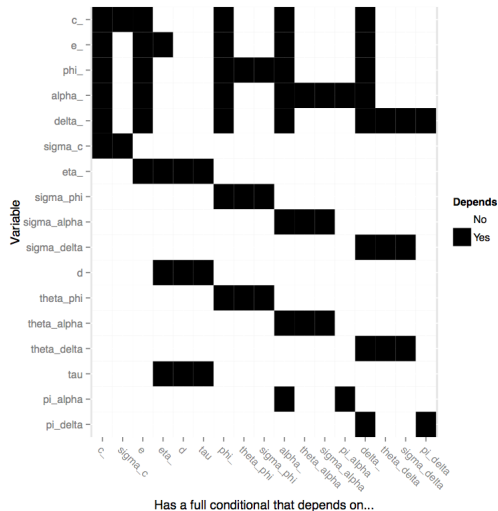
$$\sigma_\delta \sim U(0, \sigma_{\delta 0})$$

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Partition parameters by conditional independence.



Use these partitions as Gibbs steps.

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- From the appropriate full conditional distributions, sample the following:

- 1 c_1, \dots, c_N
- 2 $\tau, \pi_\alpha, \pi_\delta$
- 3 $d, \theta_\phi, \theta_\alpha, \theta_\delta$
- 4 $\sigma_c, \sigma_\phi, \sigma_\alpha, \sigma_\delta, \eta_1^2, \dots, \eta_G^2$
- 5 $\varepsilon_{1,1}, \varepsilon_{1,2}, \dots, \varepsilon_{1,N}, \varepsilon_{2,N}, \dots, \varepsilon_{G,N}$
- 6 ϕ_1, \dots, ϕ_G
- 7 $\alpha_1, \dots, \alpha_G$
- 8 $\delta_1, \dots, \delta_G$

- and then repeat.

Estimated heterosis probabilities

$$\mu(n, \phi_g, \alpha_g, \delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent 2} \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Consider one chain with M iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^M I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

$$P(\text{low-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^M I(\delta_g^{(i)} < -|\alpha_g^{(i)}|)$$

$$P(\text{mid-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^M I(\delta_g^{(i)} \neq 0)$$

Tons of opportunity for GPU parallelism across genes!

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:

- ϕ_g 's
- α_g 's
- δ_g 's
- $\varepsilon_{g,n}$'s
- η_g 's

Example: ϕ_g 's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\theta_\phi, \sigma_\phi^2)$$

$$\theta_\phi \sim \text{N}(0, \gamma_\phi^2)$$

$$\sigma_\phi \sim \text{U}(0, \sigma_{\phi 0})$$

- Using parallel random walk Metropolis steps, sample the ϕ_g 's from their full conditional distributions,

$$p(\phi_g \mid \cdots) \propto \exp \left(\sum_{n=1}^N [y_{g,n} \cdot \mu(n, \phi_g, \alpha_g, \delta_g) - \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))] - \frac{(\phi_g - \theta_\phi)^2}{2\sigma_\phi^2} \right)$$

Tons of opportunity for GPU parallelism across genes!

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Use **parallel reductions** to calculate sufficient statistics for:
 - c_n 's
 - τ, d
 - $\theta_\phi, \theta_\alpha, \theta_\delta$
 - $\sigma_\phi, \sigma_\alpha, \sigma_\delta, \sigma_c$
 - π_α, π_δ

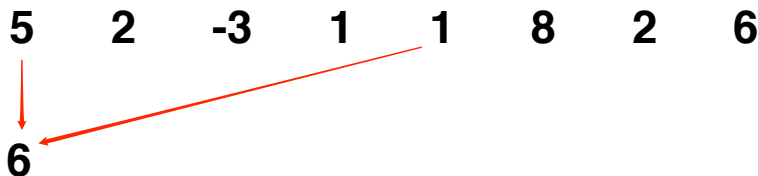
Pairwise summation: an example reduction

- Let's take the pairwise sum of the vector,

$$(5, 2, -3, 1, 1, 8, 2, 6)$$

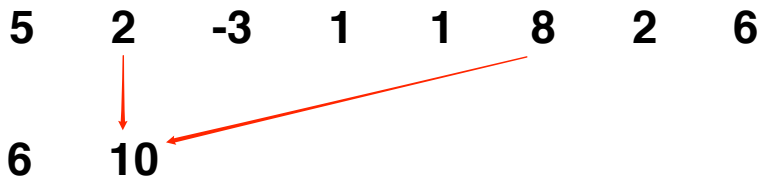
using 1 block of 4 threads.

Pairwise summation: an example reduction



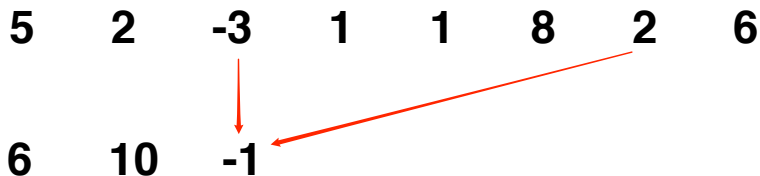
Thread 0

Pairwise summation: an example reduction



Thread 1

Pairwise summation: an example reduction



Thread 2

Pairwise summation: an example reduction

5	2	-3	1	1	8	2	6
6	10	-1	7				

Thread 3

Pairwise summation: an example reduction

5 2 -3 1 1 8 2 6

6 10 -1 7

Synchronize threads

Pairwise summation: an example reduction

5 2 -3 1 1 8 2 6

6 10 -1 7



5



Thread 0

Pairwise summation: an example reduction

5 2 -3 1 1 8 2 6

6 10 -1 7

5 17

Thread 1



Pairwise summation: an example reduction

5 2 -3 1 1 8 2 6

6 10 -1 7

5 17

Synchronize Threads

Pairwise summation: an example reduction

5 2 -3 1 1 8 2 6

6 10 -1 7

5 17
↓ ↙
22

Thread 0

Example: τ^2

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \text{N}(0, \eta_g^2)$$

$$\eta_g^2 \stackrel{\text{ind}}{\sim} \text{Inv-Gamma} \left(\text{shape} = \frac{d}{2}, \text{rate} = \frac{d \cdot \tau^2}{2} \right)$$

$$d \sim \text{U}(0, d_0)$$

$$\tau^2 \sim \text{Gamma}(\text{shape} = a_\tau, \text{rate} = b_\tau)$$

$$p(\tau^2 \mid \dots)$$

$$= \text{Gamma} \left(\text{shape} = a_\tau + \frac{Gd}{2}, \text{rate} = b_\tau + \frac{d}{2} \sum_{g=1}^G \frac{1}{\eta_g^2} \right)$$

- Using a parallel reduction (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistic:

$$\sum_{g=1}^G \frac{1}{\eta_g^2}$$

- Use an efficient rejection sampler to sample τ^2 .

Example: d

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(\exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} N(0, \eta_g^2)$$

$$\eta_g^2 \stackrel{\text{ind}}{\sim} \text{Inv-Gamma} \left(\text{shape} = \frac{d}{2}, \text{rate} = \frac{d \cdot \tau^2}{2} \right)$$

$$d \sim U(0, d_0)$$

$$\tau^2 \sim \text{Gamma}(\text{shape} = a_\tau, \text{rate} = b_\tau)$$

$$p(d \mid \dots) \propto \Gamma(d/2)^{-G} \left(\frac{d \cdot \tau^2}{2} \right)^{Gd/2} \left(\prod_{g=1}^G \eta_g^2 \right)^{-(d/2+1)} \exp \left(-\frac{d \cdot \tau^2}{2} \sum_{g=1}^G \frac{1}{\eta_g^2} \right) I(0 < d < d_0)$$

- Using parallel reductions (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistics:

$$\prod_{g=1}^G \eta_g^2$$

$$\sum_{g=1}^G \frac{1}{\eta_g^2}$$

- Use a random-walk metropolis step to sample d .

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The software

- In progress...

Thanks for coming.

- Slides and video will be available at
<http://will-landau.com/research.html>.

Sources

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