# GPU-parallel Gibbs sampling of a hierarchical model for hybrid vigor in gene expression

Will Landau

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Biological background DNA and RNA Central dogma Examples of gene regulation RNA-seq Hybrid vigor

The model

The Gibbs sampl Gibbs steps Estimated heterosis probabilities

## Outline

## Biological background

DNA and RNA

Central dogma

Examples of gene regulation

RNA-seq

Hybrid vigor

#### The model

#### The Gibbs sampler

Gibbs steps

Estimated heterosis probabilities

GPU parallelism

#### The software

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#### Biological background

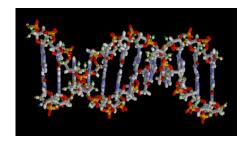
DNA and RNA Central dogma Examples of gene regulation RNA-seq Hybrid vigor

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## DNA



··· GTGCATCTGACTCCTGAGGAGAAG ··· CACGTAGACTGAGGACTCCTCTTC ···

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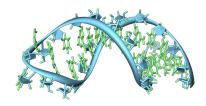
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Estimated betavesis

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## **RNA**



··· GUGCAUCUGACUCCUGAGGAGAAG ···

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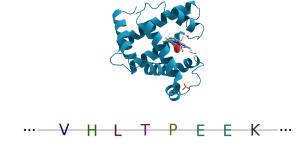
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## **Proteins**



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# Central dogma: how organisms make proteins

GTGCATCTGACTCCTGAGGAGAAG ... DNA GPU-parallel Gibbs sampling of a hierarchical model for hybrid vigor in gene expression

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Biological Central dogma

# Central dogma: how organisms make proteins

GTGCATCTGACTCCTGAGGAGAAG DNA
CACGTAGACTGAGGACTCCTCTTC

(transcription)
GUGCAUCUGACUCCUGAGGAGAAG RNA

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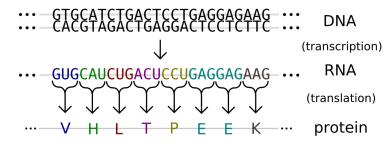
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## Central dogma of genetics



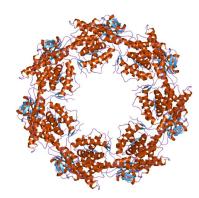
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Biological Central dogma

## HSP<sub>60</sub>

- ► HSP = heat shock protein.
- ▶ Prevent heat damage to other proteins.



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# Temperature spike triggers HSP60 production.

HSP60 Gene



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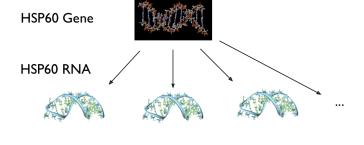
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# Temperature spike causes HSP60 expression.



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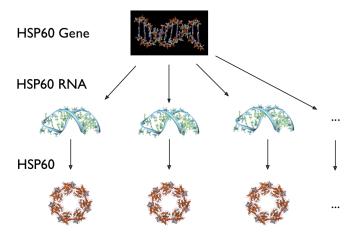
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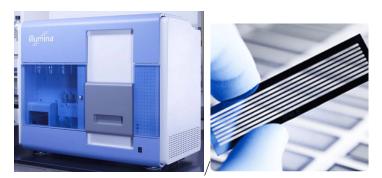
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## RNA-seq

- ► RNA sequencing: measure gene expression using relative abundance of RNA.
- ► Illumina Genome Analyzer:



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# RNA-seg data: counts of amplified RNA fragments

	Treatment I		Treatr	nent 2	Treatment 3		
Gene I	100	225	0	70	279	300	106
Gene 2	0	1	1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
Gene 34897	10	13	6	819	761	902	912

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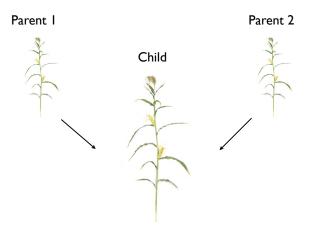
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Goal: use RNA-seq to study hybrid vigor (heterosis).

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# High-parent heterosis: child's trait surpasses both parents



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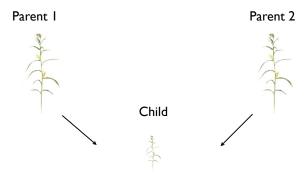
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# Low-parent heterosis: child's trait is weaker than in each parent



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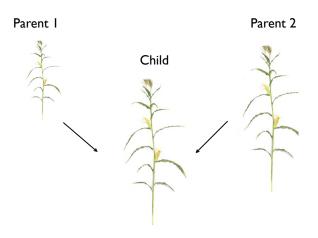
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# Mid-parent heterosis: child's trait is different than average of parents



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# High-parent heterosis in gene expression

	Parent I			Ch	ild	Parent 2	
Gene I	100	225	0	70	279	300	106
Gene 2	0	I	I	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
							•••
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## Mid-parent heterosis in gene expression

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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$$c_n \stackrel{\text{ind}}{\sim} \text{N}(c_n \mid 0, \sigma_c^2)$$

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 $c_n \stackrel{\text{ind}}{\sim} \mathsf{N}(c_n \mid 0, \sigma_c^2)$ 

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

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$$\begin{aligned} y_{g,n} &\overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ c_n &\overset{\text{ind}}{\sim} \mathsf{N}(c_n \mid 0, \sigma_c^2) \\ \sigma_c &\sim \mathsf{U}(\sigma_c \mid 0, \sigma_{c0}) \\ \varepsilon_{g,n} &\overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \end{aligned}$$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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$$\mu(n,\phi_g,\alpha_g,\delta_g) = \begin{cases} \phi_g - \alpha_g & \text{ sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{ sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{ sample } n \text{ from parent 3} \end{cases}$$

 $y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$ 

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

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$$\mu(n,\phi_{\it g},\alpha_{\it g},\delta_{\it g}) = \begin{cases} \phi_{\it g} - \alpha_{\it g} & \text{sample $n$ from parent 1} \\ \phi_{\it g} + \delta_{\it g} & \text{sample $n$ from child} \\ \phi_{\it g} + \alpha_{\it g} & \text{sample $n$ from parent 3} \end{cases}$$

$$\begin{aligned} \textit{y}_{\textit{g},\textit{n}} & \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\textit{y}_{\textit{g},\textit{n}} \mid \mathsf{exp}(\textit{c}_{\textit{n}} + \varepsilon_{\textit{g},\textit{n}} + \mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}))) \\ \phi_{\textit{g}} & \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_{\textit{g}} \mid \theta_{\textit{\phi}}, \sigma_{\phi}^{2}) \end{aligned}$$

$$\alpha_{\mathbf{g}} \overset{\mathsf{ind}}{\sim} \pi_{\alpha}^{1-l(\alpha_{\mathbf{g}})} [(1-\pi_{\alpha})\mathsf{N}(\alpha_{\mathbf{g}} \mid \theta_{\alpha}, \sigma_{\alpha}^2)]^{l(\alpha_{\mathbf{g}})}$$

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$$\begin{aligned} y_{g,n} &\stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ \phi_g &\stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \end{aligned}$$

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$$\delta_{\mathbf{g}} \overset{\text{ind}}{\sim} \pi_{\delta}^{1-l(\delta_{\mathbf{g}})} [(1-\pi_{\delta}) \mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^2)]^{l(\delta_{\mathbf{g}})}$$

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$$\alpha_{\mathbf{g}} \overset{\mathsf{ind}}{\sim} \pi_{\alpha}^{1-I(\alpha_{\mathbf{g}})} [(1-\pi_{\alpha}) \mathsf{N}(\alpha_{\mathbf{g}} \mid \theta_{\alpha}, \sigma_{\alpha}^2)]^{I(\alpha_{\mathbf{g}})}$$

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$$\mu(n,\phi_g,\alpha_g,\delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample $n$ from parent 1} \\ \phi_g + \delta_g & \text{sample $n$ from child} \\ \phi_g + \alpha_g & \text{sample $n$ from parent 3} \end{cases}$$

$$\begin{split} \mathbf{y}_{g,n} & \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g & \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ & \theta_\phi & \sim \mathsf{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ & \sigma_\phi & \sim \mathsf{U}(\sigma_\phi \mid 0,\sigma_{\phi 0}) \\ \alpha_g & \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-I(\alpha_g)} [(1-\pi_\alpha)\mathsf{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{I(\alpha_g)} \\ & \theta_\alpha & \sim \mathsf{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \end{split}$$

$$\begin{split} \delta_{\mathbf{g}} &\overset{\text{ind}}{\sim} \pi_{\delta}^{1-I(\delta_{\mathbf{g}})} [(1-\pi_{\delta})\mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^{2})]^{I(\delta_{\mathbf{g}})} \\ &\theta_{\delta} \sim \mathsf{N}(\theta_{\delta} \mid 0, \gamma_{\delta}^{2}) \end{split}$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 1} \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 3} \end{cases}$$

$$\begin{split} \mathbf{y}_{g,n} & \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g & \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ & \theta_\phi \sim \mathsf{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ & \sigma_\phi \sim \mathsf{U}(\sigma_\phi \mid 0,\sigma_{\phi 0}) \\ & \alpha_g & \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-I(\alpha_g)} [(1-\pi_\alpha)\mathsf{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{I(\alpha_g)} \\ & \theta_\alpha \sim \mathsf{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \\ & \sigma_\alpha \sim \mathsf{U}(\sigma_\alpha \mid 0,\sigma_{\alpha 0}) \end{split}$$

$$\begin{split} \delta_{\mathbf{g}} &\stackrel{\text{ind}}{\sim} \pi_{\delta}^{1-l(\delta_{\mathbf{g}})} [(1-\pi_{\delta}) \mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^{2})]^{l(\delta_{\mathbf{g}})} \\ &\theta_{\delta} \sim \mathsf{N}(\theta_{\delta} \mid 0, \gamma_{\delta}^{2}) \end{split}$$

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$$\mu(n,\phi_g,\alpha_g,\delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample $n$ from parent 1} \\ \phi_g + \delta_g & \text{sample $n$ from child} \\ \phi_g + \alpha_g & \text{sample $n$ from parent 3} \end{cases}$$

$$\begin{split} \mathbf{y}_{g,n} & \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g & \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ \theta_\phi & \sim \mathsf{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ \sigma_\phi & \sim \mathsf{U}(\sigma_\phi \mid 0,\sigma_{\phi0}) \\ \alpha_g & \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-I(\alpha_g)}[(1-\pi_\alpha)\mathsf{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{I(\alpha_g)} \\ \theta_\alpha & \sim \mathsf{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \\ \sigma_\alpha & \sim \mathsf{U}(\sigma_\alpha \mid 0,\sigma_{\alpha0}) \end{split}$$

$$\begin{split} \delta_{g} &\stackrel{\text{ind}}{\sim} \pi_{\delta}^{1-I(\delta g)} [(1-\pi_{\delta})\mathsf{N}(\delta_{g} \mid \theta_{\delta}, \sigma_{\delta}^{2})]^{I(\delta_{g})} \\ &\theta_{\delta} \sim \mathsf{N}(\theta_{\delta} \mid 0, \gamma_{\delta}^{2}) \\ &\sigma_{\delta} \sim \mathsf{U}(\sigma_{\delta} \mid 0, \sigma_{\delta 0}) \end{split}$$

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$$\mu(n,\phi_g,\alpha_g,\delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent } 1 \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

$$\theta_\phi \sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2)$$

$$\sigma_\phi \sim \text{U}(\sigma_\phi \mid 0, \sigma_{\phi 0})$$

$$\alpha_g \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-l(\alpha_g)} [(1 - \pi_\alpha) \text{N}(\alpha_g \mid \theta_\alpha, \sigma_\alpha^2)]^{l(\alpha_g)}$$

$$\theta_\alpha \sim \text{N}(\theta_\alpha \mid 0, \gamma_\alpha^2)$$

$$\sigma_\alpha \sim \text{U}(\sigma_\alpha \mid 0, \sigma_{\alpha 0})$$

$$\pi_\alpha \sim \text{Beta}(\pi_\alpha \mid a_\alpha, b_\alpha)$$

$$\delta_g \stackrel{\text{ind}}{\sim} \pi_\delta^{1-l(\delta_g)} [(1 - \pi_\delta) \text{N}(\delta_g \mid \theta_\delta, \sigma_\delta^2)]^{l(\delta_g)}$$

$$\theta_\delta \sim \text{N}(\theta_\delta \mid 0, \gamma_\delta^2)$$

$$\sigma_\delta \sim \text{U}(\sigma_\delta \mid 0, \sigma_{\delta 0})$$

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$$\begin{split} \mu(n,\phi_g,\alpha_g,\delta_g) &= \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent } 1 \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent } 3 \end{cases} \\ y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g \overset{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ \theta_\phi &\sim \text{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ \sigma_\phi &\sim \text{U}(\sigma_\phi \mid 0,\sigma_{\phi 0}) \\ \alpha_g \overset{\text{ind}}{\sim} \pi_\alpha^{1-l(\alpha_g)}[(1-\pi_\alpha)\text{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{l(\alpha_g)} \\ \theta_\alpha &\sim \text{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \end{split}$$

$$\begin{split} \delta_g & \stackrel{\text{ind}}{\sim} \pi_\delta^{1-l(\delta_g)}[(1-\pi_\delta)\mathsf{N}(\delta_g \mid \theta_\delta, \sigma_\delta^2)]^{l(\delta_g)} \\ & \theta_\delta \sim \mathsf{N}(\theta_\delta \mid 0, \gamma_\delta^2) \end{split}$$

 $\sigma_{\delta} \sim U(\sigma_{\delta} \mid 0, \sigma_{\delta 0})$ 

 $\sigma_{\alpha} \sim \mathsf{U}(\sigma_{\alpha} \mid 0, \sigma_{\alpha 0})$   $\pi_{\alpha} \sim \mathsf{Beta}(\pi_{\alpha} \mid a_{\alpha}, b_{\alpha})$ 

 $\pi_\delta \sim \mathsf{Beta}(\pi_\delta \mid a_\delta, b_\delta)$ 

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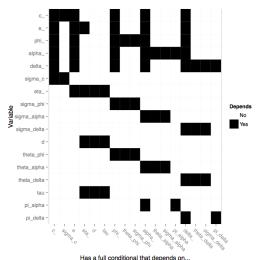
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# Partition parameters by conditional independence.



Has a full conditional that depends on...

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► From the appropriate full conditional distributions, sample the following:
- 1.  $c_1, \ldots, c_N$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► From the appropriate full conditional distributions, sample the following:
- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$

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- ► From the appropriate full conditional distributions, sample the following:
- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
- 4.  $\sigma_c$ ,  $\sigma_\phi$ ,  $\sigma_\alpha$ ,  $\sigma_\delta$ ,  $\eta_1^2$ , ...,  $\eta_G^2$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
- 4.  $\sigma_c$ ,  $\sigma_\phi$ ,  $\sigma_\alpha$ ,  $\sigma_\delta$ ,  $\eta_1^2$ , ...,  $\eta_c^2$
- 5.  $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
- 4.  $\sigma_c$ ,  $\sigma_{\phi}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\delta}$ ,  $\eta_1^2$ , ...,  $\eta_G^2$
- 5.  $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$
- 6.  $\phi_1, \ldots, \phi_G$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
- 4.  $\sigma_c$ ,  $\sigma_{\phi}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\delta}$ ,  $\eta_1^2$ , ...,  $\eta_G^2$
- 5.  $\varepsilon_{1,1}$ ,  $\varepsilon_{1,2}$ , ...,  $\varepsilon_{1,N}$ ,  $\varepsilon_{2,N}$ , ...,  $\varepsilon_{G,N}$
- 6.  $\phi_1, \ldots, \phi_G$
- 7.  $\alpha_1, \ldots, \alpha_G$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1.  $c_1, \ldots, c_N$
- 2.  $\tau$ ,  $\pi_{\alpha}$ ,  $\pi_{\delta}$
- 3. d,  $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
- 4.  $\sigma_c$ ,  $\sigma_{\phi}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\delta}$ ,  $\eta_1^2$ , ...,  $\eta_G^2$
- 5.  $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$
- 6.  $\phi_1, \ldots, \phi_G$
- 7.  $\alpha_1, \ldots, \alpha_G$
- 8.  $\delta_1, \ldots, \delta_G$
- ▶ and then repeat.

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 1} \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 3} \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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$$y_{g,n} \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with M iterations.

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with M iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$\mathbf{y_{g,n}} \overset{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y_{g,n}} \mid \exp(\mathbf{c_n} + \varepsilon_{g,n} + \mu(\mathbf{n}, \phi_g, \alpha_g, \delta_g)))$$

Consider one chain with M iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

$$P(\text{low-parent heterosis in gene }g\ ) \approx \frac{1}{M} \sum_{i=1}^M I(\delta_g^{(i)} < -|\alpha_g^{(i)}|)$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 1} \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 3} \end{cases}$$

$$y_{g,n} \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with M iterations.

$$P(\text{high-parent heterosis in gene } g) pprox rac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |lpha_g^{(i)}|)$$

$$P( ext{low-parent heterosis in gene } g \ ) pprox rac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} < -|lpha_g^{(i)}|)$$

$$P(\text{mid-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} \neq 0)$$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Sample in parallel:

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
  - $\blacktriangleright \phi_g$ 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
  - $ightharpoonup \phi_g$ 's
  - $ightharpoonup \alpha_{g}$ 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
  - $\blacktriangleright \phi_g$ 's
  - $ightharpoonup \alpha_{g}$ 's
  - $\triangleright \delta_g$ 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
  - $\triangleright \phi_{\sigma}$ 's
  - $ightharpoonup \alpha_{g}$ 's
  - $ightharpoonup \delta_g$ 's
  - $\triangleright$   $\varepsilon_{g,n}$ 's

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- ► Sample in parallel:
  - $\blacktriangleright \phi_{\sigma}$ 's
  - $ightharpoonup lpha_{
    m g}$ 's
  - $ightharpoonup \delta_g$ 's
  - $\triangleright$   $\varepsilon_{g,n}$ 's
  - $ightharpoonup \eta_{
    m g}$ 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
  - $\blacktriangleright \phi_{\sigma}$ 's
  - $ightharpoonup lpha_{
    m g}$ 's
  - $\triangleright$   $\delta_g$ 's
  - $\triangleright \varepsilon_{g,n}$ 's
  - $\triangleright \eta_g$ 's
- Use parallel reductions to calculate sufficient statistics for:

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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  - $\triangleright \varepsilon_{g,n}$ 's
  - η<sub>g</sub>'s
- Use parallel reductions to calculate sufficient statistics for:
  - $\triangleright$   $c_n$ 's

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  - $\vdash \eta_g$ 's
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  - ► Cn'S
  - ▶ τ, d

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  - η<sub>g</sub>'s
- Use parallel reductions to calculate sufficient statistics for:
  - ► Cn'S
  - ▶ τ, d
  - $\blacktriangleright$   $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$

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- Use parallel reductions to calculate sufficient statistics for:
  - ► Cn'S
  - ▶ τ, d
  - $\bullet$   $\theta_{\phi}$ ,  $\theta_{\alpha}$ ,  $\theta_{\delta}$
  - $\triangleright$   $\sigma_{\phi}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\delta}$ ,  $\sigma_{c}$

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  - ► Cn'S
  - ▶ τ. d
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  - $\triangleright$   $\sigma_{\phi}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\delta}$ ,  $\sigma_{c}$
  - $\blacktriangleright$   $\pi_{\alpha}$ ,  $\pi_{\delta}$

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### Example: $\phi_g$ 's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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### Example: $\phi_g$ 's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$
  
 $\phi_g \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$ 

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### Example: $\phi_{\varrho}$ 's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

$$\theta_\phi \sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2)$$

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# Example: $\phi_{\varrho}$ 's

$$\begin{aligned} y_{g,n} &\overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ \phi_g &\overset{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\ \theta_\phi &\sim \mathsf{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\ \sigma_\phi &\sim \mathsf{U}(\sigma_\phi \mid 0, \sigma_{\phi 0}) \end{aligned}$$

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## Example: $\phi_g$ 's

$$\begin{aligned} y_{g,n} &\overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ \phi_g &\overset{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\ \theta_\phi &\sim \mathsf{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\ \sigma_\phi &\sim \mathsf{U}(\sigma_\phi \mid 0, \sigma_{\phi 0}) \end{aligned}$$

• Using parallel random walk Metropolis steps, sample the  $\phi_g$ 's from their full conditional distributions,

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# Example: $\phi_{\sigma}$ 's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

$$\theta_\phi \sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2)$$

$$\sigma_\phi \sim \text{U}(\sigma_\phi \mid 0, \sigma_{\phi 0})$$

▶ Using parallel random walk Metropolis steps, sample the  $\phi_{\sigma}$ 's from their full conditional distributions.

$$p(\phi_g \mid \cdots) \propto \exp\left(\sum_{n=1}^{N} \left[y_{g,n} \cdot \mu(n, \phi_g, \alpha_g, \delta_g)\right] - \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))\right] - \frac{(\phi_g - \theta_\phi)^2}{2\sigma_\phi^2}$$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_\sigma^2)$$

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Biological

### Example: $\tau^2$

$$\begin{split} y_{g,n} &\stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ &\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ &\eta_g^2 \stackrel{\text{ind}}{\sim} \mathsf{Inv\text{-}Gamma}\left(\eta_g^2 \ \middle| \ \mathsf{shape} = \frac{d}{2} \ , \ \mathsf{rate} = \frac{d \cdot \tau^2}{2} \right) \end{split}$$

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#### Example: $\tau^2$

$$\begin{split} y_{g,n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ & \varepsilon_{g,n} \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 \overset{\text{ind}}{\sim} \mathsf{Inv\text{-}Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;,\; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \mathsf{U}(d \mid 0,d_0) \\ & \tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau,\mathsf{rate} = b_\tau) \end{split}$$

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$$\begin{aligned} y_{g,n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ & \varepsilon_{g,n} \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 \overset{\text{ind}}{\sim} \mathsf{Inv}\text{-}\mathsf{Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;, \; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \mathsf{U}(d \mid 0,d_0) \\ & \tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau,\mathsf{rate} = b_\tau) \\ p(\tau^2 \mid \cdots) \\ & = \mathsf{Gamma}\left(\tau^2 \mid \mathsf{shape} = a_\tau + \frac{Gd}{2} \;, \; \mathsf{rate} = b_\tau + \frac{d}{2}\sum_{g=1}^G \frac{1}{\eta_g^2}\right) \end{aligned}$$

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$$\begin{split} & p(\tau^2 \mid \cdots) \\ &= \mathsf{Gamma}\left(\tau^2 \mid \mathsf{shape} = \mathsf{a}_\tau + \frac{\mathsf{G} \mathsf{d}}{2} \;,\; \mathsf{rate} = \mathsf{b}_\tau + \frac{\mathsf{d}}{2} \sum_{g=1}^G \frac{1}{\eta_g^2} \right) \end{split}$$

Using a parallel reduction (NVIDIA's CUDA C/C++ Thrust library),
 calculate the sufficient statistic:

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#### Example: $\tau^2$

$$\begin{aligned} y_{g,n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ & \varepsilon_{g,n} \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 \overset{\text{ind}}{\sim} \mathsf{Inv}\text{-}\mathsf{Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;, \; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \mathsf{U}(d \mid 0,d_0) \\ & \tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau,\mathsf{rate} = b_\tau) \\ p(\tau^2 \mid \cdots) \\ & = \mathsf{Gamma}\left(\tau^2 \mid \mathsf{shape} = a_\tau + \frac{Gd}{2} \;, \; \mathsf{rate} = b_\tau + \frac{d}{2}\sum_{i=1}^G \frac{1}{\eta_g^2}\right) \end{aligned}$$

Using a parallel reduction (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistic.

$$\sum_{g=1}^{G} \frac{1}{\eta_g^2}$$

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### Example: $\tau^2$

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$$=\mathsf{Gamma}\left(\tau^2 \; \left| \; \mathsf{shape} = \mathsf{a}_\tau + \frac{\mathsf{G} \mathsf{d}}{2} \; , \; \mathsf{rate} = \mathsf{b}_\tau + \frac{\mathsf{d}}{2} \sum_{g=1}^G \frac{1}{\eta_g^2} \right)$$

Using a parallel reduction (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistic:

$$\sum_{g=1}^{G} \frac{1}{\eta_g^2}$$

▶ Use an efficient rejection sampler to sample  $\tau^2$ .

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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$$\begin{split} \mathbf{y}_{\mathbf{g},n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{\mathbf{g},n} \mid \exp(c_n + \varepsilon_{\mathbf{g},n} + \mu(n,\phi_{\mathbf{g}},\alpha_{\mathbf{g}},\delta_{\mathbf{g}}))) \\ & \varepsilon_{\mathbf{g},n} \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{\mathbf{g},n} \mid 0,\eta_{\mathbf{g}}^2) \\ & \eta_{\mathbf{g}}^2 \overset{\text{ind}}{\sim} \mathsf{Inv\text{-}Gamma}\left(\eta_{\mathbf{g}}^2 \ \middle| \ \mathsf{shape} = \frac{d}{2} \ , \ \ \mathsf{rate} = \frac{d \cdot \tau^2}{2} \right) \end{split}$$

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$$\begin{split} \mathbf{y}_{\mathbf{g},n} &\stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{\mathbf{g},n} \mid \mathsf{exp}(\mathbf{c}_n + \varepsilon_{\mathbf{g},n} + \mu(n,\phi_{\mathbf{g}},\alpha_{\mathbf{g}},\delta_{\mathbf{g}}))) \\ &\varepsilon_{\mathbf{g},n} &\stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{\mathbf{g},n} \mid 0,\eta_{\mathbf{g}}^2) \\ &\eta_{\mathbf{g}}^2 &\stackrel{\text{ind}}{\sim} \mathsf{Inv-Gamma}\left(\eta_{\mathbf{g}}^2 \ \middle| \ \mathsf{shape} = \frac{d}{2} \ , \ \mathsf{rate} = \frac{d \cdot \tau^2}{2} \right) \\ &d \sim \mathsf{U}(d \mid 0,d_0) \end{split}$$

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$$p(d \mid \cdots) \propto \Gamma(d/2)^{-G} \left(\frac{d \cdot \tau^2}{2}\right)^{Gd/2} \left(\prod_{g=1}^{G} \eta_g^2\right)^{-(d/2+1)} \exp\left(-\frac{d \cdot \tau^2}{2} \sum_{g=1}^{G} \frac{1}{\eta_g^2}\right) I(0 < d < d_0)$$

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$$\begin{split} y_{g,n} &\overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ &\varepsilon_{g,n} \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ &\eta_g^2 \overset{\text{ind}}{\sim} \mathsf{Inv\text{-}Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;,\; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ &d \sim \mathsf{U}(d \mid 0,d_0) \\ &\tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau,\mathsf{rate} = b_\tau) \\ &p(d \mid \cdots) \propto \Gamma(d/2)^{-G} \left(\frac{d \cdot \tau^2}{2}\right)^{Gd/2} \left(\prod_{g=1}^G \eta_g^2\right)^{-(d/2+1)} \exp\left(-\frac{d \cdot \tau^2}{2}\sum_{g=1}^G \frac{1}{\eta_g^2}\right) I(0 < d < d_0) \end{split}$$

▶ Using parallel reductions (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistics:

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$$\begin{split} \mathbf{y}_{g,n} & \stackrel{\text{ind}}{\sim} \operatorname{Poisson}(\mathbf{y}_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ & \varepsilon_{g,n} & \stackrel{\text{ind}}{\sim} \operatorname{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 & \stackrel{\text{ind}}{\sim} \operatorname{Inv-Gamma}\left(\eta_g^2 \mid \operatorname{shape} = \frac{d}{2} \;,\; \operatorname{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \operatorname{U}(d \mid 0,d_0) \\ & \tau^2 \sim \operatorname{Gamma}(\tau^2 \mid \operatorname{shape} = a_\tau, \operatorname{rate} = b_\tau) \\ & p(d \mid \cdots) \propto \Gamma\left(d/2\right)^{-G} \left(\frac{d \cdot \tau^2}{2}\right)^{Gd/2} \left(\prod_{g=1}^G \eta_g^2\right)^{-(d/2+1)} \exp\left(-\frac{d \cdot \tau^2}{2}\sum_{g=1}^G \frac{1}{\eta_g^2}\right) I(0 < d < d_0) \end{split}$$

▶ Using parallel reductions (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistics:

$$\prod_{g=1}^{G} \eta_g^2 \qquad \qquad \sum_{g=1}^{G} \frac{1}{\eta_g^2}$$

GPU-parallel Gibbs sampling of a hierarchical model for hybrid vigor in gene expression

Will Landau

Biological background DNA and RNA Central dogma Examples of gene regulation RNA-seq Hybrid vigor

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$$\begin{split} y_{g,n} &\stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ &\varepsilon_{g,n} &\stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ &\eta_g^2 &\stackrel{\text{ind}}{\sim} \mathsf{Inv\text{-}Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;,\; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ &d \sim \mathsf{U}(d \mid 0,d_0) \\ &\tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau,\mathsf{rate} = b_\tau) \\ &p(d \mid \cdots) \propto \Gamma(d/2)^{-G} \left(\frac{d \cdot \tau^2}{2}\right)^{Gd/2} \left(\prod_{g=1}^G \eta_g^2\right)^{-(d/2+1)} \exp\left(-\frac{d \cdot \tau^2}{2}\sum_{g=1}^G \frac{1}{\eta_g^2}\right) I(0 < d < d_0) \end{split}$$

Using parallel reductions (NVIDIA's CUDA C/C++ Thrust library), calculate the sufficient statistics:

$$\prod_{g=1}^{G} \eta_g^2 \qquad \qquad \sum_{g=1}^{G} \frac{1}{\eta_g^2}$$

▶ Use a random-walk metropolis step to sample *d*.

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#### The software

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#### The software

Ordinary C and GPU-accelerated versions, along with an R package wrapper, are available for download at https://github.com/wlandau/heterosis. GPU-parallel Gibbs sampling of a hierarchical model for hybrid vigor in gene expression

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- Time for a demo...

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