GPU-parallel Gibbs sampling of a hierarchical model for hybrid vigor in gene expression

Will Landau

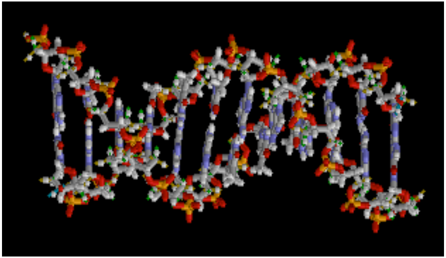
Iowa State University

September 29, 2013

Outline

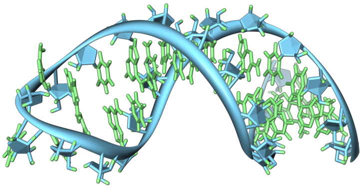
Outline

DNA

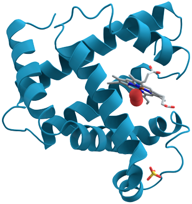


... GTGCATCTGACTCCTGAGGAGAAG ...
... CACGTAGACTGAGGACTCCTCTTC ...

RNA



Proteins



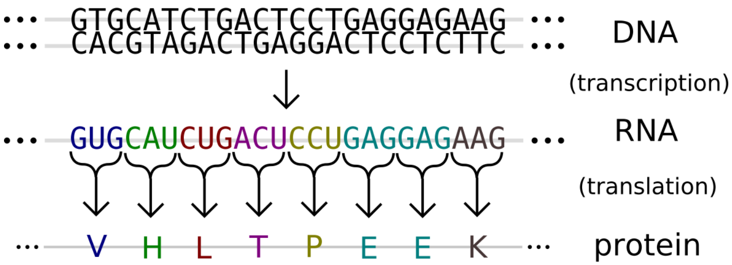
Central dogma: how organisms make proteins

… GTGCATCTGACTCCTGAGGAGAAG …
… CACGTAGACTGAGGACTCCTCTTC …

DNA

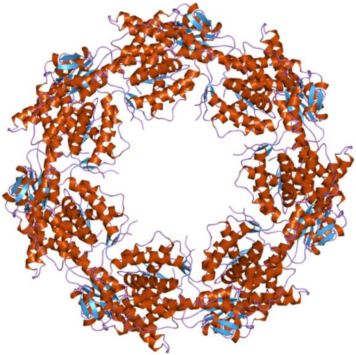
Central dogma: how organisms make proteins

Central dogma of genetics



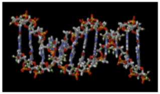
HSP60

- HSP = heat shock protein.
- Prevent heat damage to other proteins.

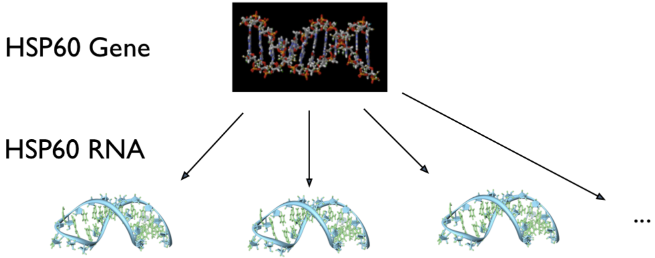


Temperature spike triggers HSP60 production.

HSP60 Gene



Temperature spike causes HSP60 expression.



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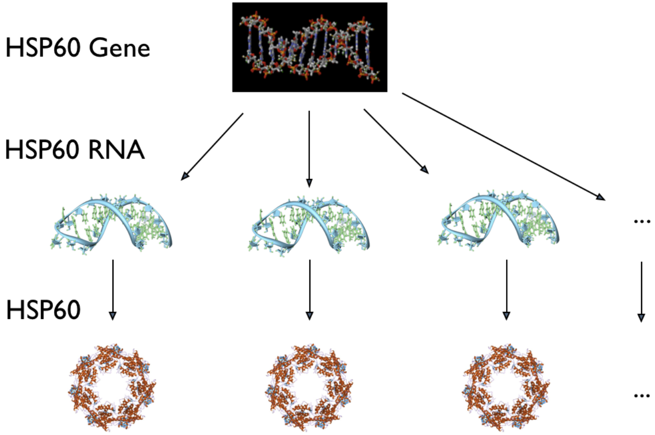
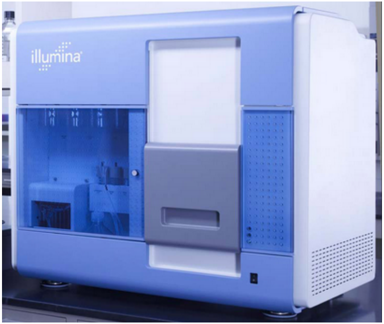
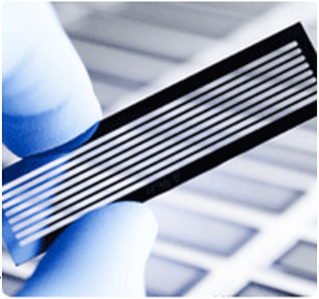


RNA-seq

- RNA sequencing: measure gene expression using relative abundance of RNA.
- Illumina Genome Analyzer:



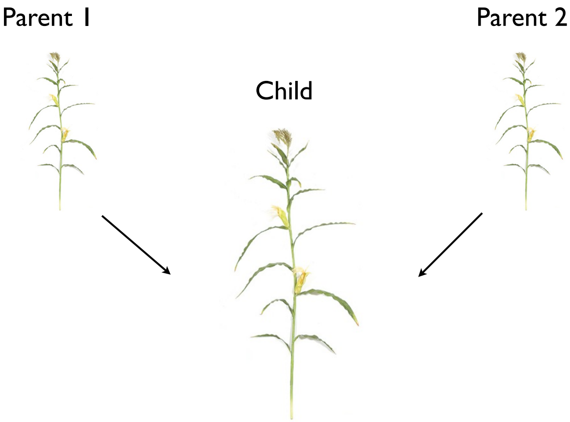


RNA-seq data: counts of amplified RNA fragments

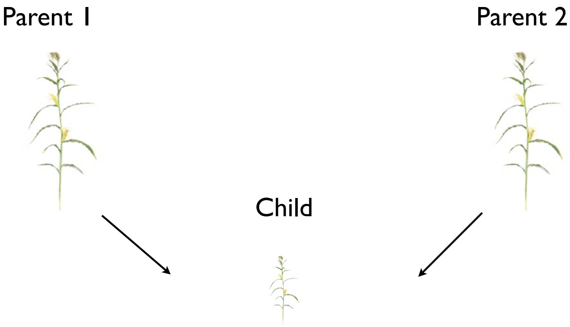
	Treatment 1			Treatment 2		Treatment 3	
Gene 1	100	225	0	70	279	300	106
Gene 2	0	1	1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
...	...	...	...	...	...	...	...
Gene 34897	10	13	6	819	761	902	912

- Goal: use RNA-seq to study hybrid vigor (heterosis).

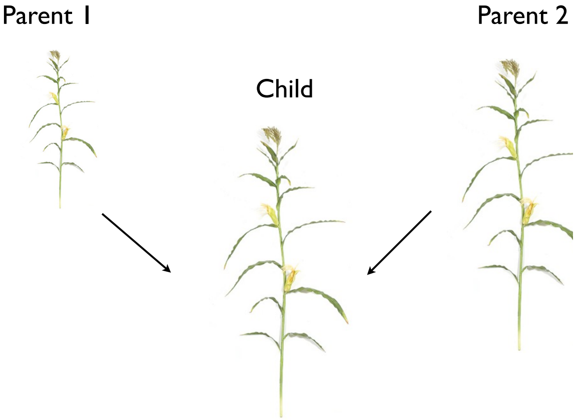
High-parent heterosis: child's trait surpasses both parents



Low-parent heterosis: child's trait is weaker than in each parent



Mid-parent heterosis: child's trait is different than average of parents



High-parent heterosis in gene expression

	Parent 1			Child		Parent 2	
Gene 1	100	225	0	70	279	300	106
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$$\eta_g^2 \overset{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \,\middle|\, \text{shape} = \frac{d}{2}, \ \text{rate} = \frac{d \cdot \tau^2}{2}\right)$$

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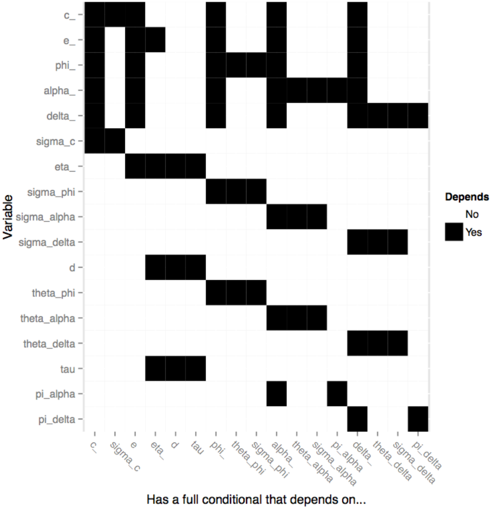
$$\sigma_\delta \sim \text{U}(\sigma_\delta \mid 0, \sigma_{\delta 0})$$

$$\pi_\delta \sim \text{Beta}(\pi_\delta \mid a_\delta, b_\delta)$$

Outline

Partition parameters by conditional independence.



Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- From the appropriate full conditional distributions, sample the following:

1. $c_1, \ldots, c_N$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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1. $c_1, \ \ldots, \ c_N$
2. $\tau, \ \pi_\alpha, \ \pi_\delta$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- From the appropriate full conditional distributions, sample the following:

1. $c_1, \ldots, c_N$
2. $\tau, \pi_\alpha, \pi_\delta$
3. $d, \theta_\phi, \theta_\alpha, \theta_\delta$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- From the appropriate full conditional distributions, sample the following:

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3. $d, \ \theta_\phi, \ \theta_\alpha, \ \theta_\delta$
4. $\sigma_c, \ \sigma_\phi, \ \sigma_\alpha, \ \sigma_\delta, \ \eta_1^2, \ \ldots, \ \eta_G^2$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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3. $d, \theta_\phi, \theta_\alpha, \theta_\delta$
4. $\sigma_c, \sigma_\phi, \sigma_\alpha, \sigma_\delta, \eta_1^2, \ldots, \eta_G^2$
5. $\varepsilon_{1,1}, \varepsilon_{1,2}, \ldots, \varepsilon_{1,N}, \varepsilon_{2,N}, \ldots, \varepsilon_{G,N}$

Use these partitions as Gibbs steps.

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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4. $\sigma_c, \sigma_\phi, \sigma_\alpha, \sigma_\delta, \eta_1^2, \ldots, \eta_G^2$
5. $\varepsilon_{1,1}, \varepsilon_{1,2}, \ldots, \varepsilon_{1,N}, \varepsilon_{2,N}, \ldots, \varepsilon_{G,N}$
6. $\phi_1, \ldots, \phi_G$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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4. $\sigma_c, \sigma_\phi, \sigma_\alpha, \sigma_\delta, \eta_1^2, \ldots, \eta_G^2$
5. $\varepsilon_{1,1}, \varepsilon_{1,2}, \ldots, \varepsilon_{1,N}, \varepsilon_{2,N}, \ldots, \varepsilon_{G,N}$
6. $\phi_1, \ldots, \phi_G$
7. $\alpha_1, \ldots, \alpha_G$

Use these partitions as Gibbs steps.

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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4. $\sigma_c, \ \sigma_\phi, \ \sigma_\alpha, \ \sigma_\delta, \ \eta_1^2, \ \ldots, \ \eta_G^2$
5. $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$
6. $\phi_1, \ \ldots, \ \phi_G$
7. $\alpha_1, \ \ldots, \ \alpha_G$
8. $\delta_1, \ \ldots, \ \delta_G$

- and then repeat.

Estimated heterosis probabilities

$$\mu(n, \phi_g, \alpha_g, \delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent 3} \end{cases}$$

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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Consider one chain with M iterations.

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Consider one chain with M iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

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Estimated heterosis probabilities

$$\mu(n, \phi_g, \alpha_g, \delta_g) = \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent 3} \end{cases}$$

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$$P(\text{mid-parent heterosis in gene } g\ ) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} \neq 0)$$

Tons of opportunity for GPU parallelism across genes!

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:

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- Sample in parallel:
  - ϕ_g's

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  - δ_g's
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  - η_g's
- Use parallel reductions to calculate sufficient statistics for:
  - c_n's
  - τ, d

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- Sample in parallel:
  - ϕ_g's
  - α_g's
  - δ_g's
  - $\varepsilon_{g,n}$'s
  - η_g's
- Use parallel reductions to calculate sufficient statistics for:
  - c_n's
  - τ, d
  - θ_ϕ, θ_α, θ_δ

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  - α_g's
  - δ_g's
  - $\varepsilon_{g,n}$'s
  - η_g's
- Use parallel reductions to calculate sufficient statistics for:
  - c_n's
  - τ, d
  - θ_ϕ, θ_α, θ_δ
  - σ_ϕ, σ_α, σ_δ, σ_c

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  - α_g's
  - δ_g's
  - $\varepsilon_{g,n}$'s
  - η_g's
- Use parallel reductions to calculate sufficient statistics for:
  - c_n's
  - τ, d
  - θ_ϕ, θ_α, θ_δ
  - σ_ϕ, σ_α, σ_δ, σ_c
  - π_α, π_δ

Example: ϕ_g's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Example: ϕ_g's

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

Example: ϕ_g's

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\phi_g &\overset{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\
\theta_\phi &\sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2)
\end{aligned}
$$

Example: ϕ_g's

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\begin{aligned}
y_{g,n} &\stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\phi_g &\stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\
\theta_\phi &\sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\
\sigma_\phi &\sim \text{U}(\sigma_\phi \mid 0, \sigma_{\phi 0})
\end{aligned}
$$

Example: ϕ_g's

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\phi_g &\overset{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\
\theta_\phi &\sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\
\sigma_\phi &\sim \text{U}(\sigma_\phi \mid 0, \sigma_{\phi 0})
\end{aligned}
$$

- Using parallel random walk Metropolis steps, sample the ϕ_g's from their full conditional distributions,

Example: ϕ_g's

$$\begin{aligned} y_{g,n} &\stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ \phi_g &\stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2) \\ \theta_\phi &\sim \text{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\ \sigma_\phi &\sim \text{U}(\sigma_\phi \mid 0, \sigma_{\phi 0}) \end{aligned}$$

- Using parallel random walk Metropolis steps, sample the ϕ_g's from their full conditional distributions,

$$p(\phi_g \mid \cdots) \propto \exp\left(\sum_{n=1}^{N} \left[y_{g,n} \cdot \mu(n, \phi_g, \alpha_g, \delta_g) - \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)) \right] - \frac{(\phi_g - \theta_\phi)^2}{2\sigma_\phi^2} \right)$$

Example: τ^2

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Example: τ^2

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

Example: τ^2

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$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

$$\eta_g^2 \stackrel{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \,\middle|\, \text{shape} = \frac{d}{2}, \ \text{rate} = \frac{d \cdot \tau^2}{2}\right)$$

Example: τ^2

$$
\begin{aligned}
y_{g,n} &\stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\varepsilon_{g,n} &\stackrel{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\
\eta_g^2 &\stackrel{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \;\middle|\; \text{shape} = \frac{d}{2},\ \text{rate} = \frac{d \cdot \tau^2}{2}\right) \\
d &\sim \text{U}(d \mid 0, d_0)
\end{aligned}
$$

Example: τ^2

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\varepsilon_{g,n} &\overset{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\
\eta_g^2 &\overset{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \,\middle|\, \text{shape} = \frac{d}{2}\ , \ \text{rate} = \frac{d \cdot \tau^2}{2}\right) \\
d &\sim \text{U}(d \mid 0, d_0) \\
\tau^2 &\sim \text{Gamma}(\tau^2 \mid \text{shape} = a_\tau, \text{rate} = b_\tau)
\end{aligned}
$$

Example: τ^2

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
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\eta_g^2 &\overset{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \;\middle|\; \text{shape} = \frac{d}{2},\ \text{rate} = \frac{d \cdot \tau^2}{2}\right) \\
d &\sim \text{U}(d \mid 0, d_0) \\
\tau^2 &\sim \text{Gamma}(\tau^2 \mid \text{shape} = a_\tau, \text{rate} = b_\tau)
\end{aligned}
$$

$$
p(\tau^2 \mid \cdots)
$$

$$
= \text{Gamma}\left(\tau^2 \;\middle|\; \text{shape} = a_\tau + \frac{Gd}{2},\ \text{rate} = b_\tau + \frac{d}{2}\sum_{g=1}^{G} \frac{1}{\eta_g^2}\right)
$$

Example: τ^2

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\varepsilon_{g,n} &\overset{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\
\eta_g^2 &\overset{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \;\middle|\; \text{shape} = \frac{d}{2}\ ,\ \text{rate} = \frac{d \cdot \tau^2}{2}\right) \\
d &\sim \text{U}(d \mid 0, d_0) \\
\tau^2 &\sim \text{Gamma}(\tau^2 \mid \text{shape} = a_\tau, \text{rate} = b_\tau)
\end{aligned}
$$

$$
p(\tau^2 \mid \cdots) = \text{Gamma}\left(\tau^2 \;\middle|\; \text{shape} = a_\tau + \frac{Gd}{2}\ ,\ \text{rate} = b_\tau + \frac{d}{2}\sum_{g=1}^{G} \frac{1}{\eta_g^2}\right)
$$

- Using a parallel reduction (NVIDIA's CUDA C/C++ `Thrust` library), calculate the sufficient statistic:

Example: τ^2

$$
\begin{aligned}
y_{g,n} &\overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\varepsilon_{g,n} &\overset{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\
\eta_g^2 &\overset{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \,\middle|\, \text{shape} = \frac{d}{2}, \text{ rate} = \frac{d \cdot \tau^2}{2}\right) \\
d &\sim \text{U}(d \mid 0, d_0) \\
\tau^2 &\sim \text{Gamma}(\tau^2 \mid \text{shape} = a_\tau, \text{rate} = b_\tau)
\end{aligned}
$$

$$
\begin{aligned}
&p(\tau^2 \mid \cdots) \\
&= \text{Gamma}\left(\tau^2 \,\middle|\, \text{shape} = a_\tau + \frac{Gd}{2}, \text{ rate} = b_\tau + \frac{d}{2}\sum_{g=1}^{G} \frac{1}{\eta_g^2}\right)
\end{aligned}
$$

- Using a parallel reduction (NVIDIA's CUDA C/C++ `Thrust` library), calculate the sufficient statistic:

$$
\sum_{g=1}^{G} \frac{1}{\eta_g^2}
$$

Example: τ^2

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

$$\eta_g^2 \stackrel{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \,\middle|\, \text{shape} = \frac{d}{2}, \ \text{rate} = \frac{d \cdot \tau^2}{2}\right)$$

$$d \sim \text{U}(d \mid 0, d_0)$$

$$\tau^2 \sim \text{Gamma}(\tau^2 \mid \text{shape} = a_\tau, \text{rate} = b_\tau)$$

$$p(\tau^2 \mid \cdots) = \text{Gamma}\left(\tau^2 \,\middle|\, \text{shape} = a_\tau + \frac{Gd}{2}, \ \text{rate} = b_\tau + \frac{d}{2}\sum_{g=1}^{G} \frac{1}{\eta_g^2}\right)$$

- Using a parallel reduction (NVIDIA's CUDA C/C++ `Thrust` library), calculate the sufficient statistic:

$$\sum_{g=1}^{G} \frac{1}{\eta_g^2}$$

- Use an efficient rejection sampler to sample τ^2.

Example: d

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Example: d

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$$\varepsilon_{g,n} \overset{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

Example: d

$$
\begin{aligned}
y_{g,n} &\stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\
\varepsilon_{g,n} &\stackrel{\text{ind}}{\sim} \text{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\
\eta_g^2 &\stackrel{\text{ind}}{\sim} \text{Inv-Gamma}\left(\eta_g^2 \;\middle|\; \text{shape} = \frac{d}{2}, \ \text{rate} = \frac{d \cdot \tau^2}{2}\right)
\end{aligned}
$$

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$$p(d \mid \cdots) \propto \Gamma(d/2)^{-G} \left(\frac{d \cdot \tau^2}{2}\right)^{Gd/2} \left(\prod_{g=1}^{G} \eta_g^2\right)^{-(d/2+1)} \exp\left(-\frac{d \cdot \tau^2}{2} \sum_{g=1}^{G} \frac{1}{\eta_g^2}\right) I(0 < d < d_0)$$

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$$

- Using parallel reductions (NVIDIA's CUDA C/C++ `Thrust` library), calculate the sufficient statistics:

Example: d

$$y_{g,n} \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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- Using parallel reductions (NVIDIA's CUDA C/C++ `Thrust` library), calculate the sufficient statistics:

$$\prod_{g=1}^G \eta_g^2 \qquad \sum_{g=1}^G \frac{1}{\eta_g^2}$$

- Use a random-walk metropolis step to sample d.

Outline

The software

- Ordinary C and GPU-accelerated versions, along with an R package wrapper, are available for download at `https://github.com/wlandau/heterosis`.

The software

- Ordinary C and GPU-accelerated versions, along with an R package wrapper, are available for download at `https://github.com/wlandau/heterosis`.
- Time for a demo...

Sources

1. A. Gelman, J. B. Carlin, H. S. Stern, and D. S. Rubin. Bayesian Data Analysis. Chapman & Hall/CRC, 2 edition, 2004.
2. Prof. Jarad Niemi's STAT 544 lecture notes.
3. J. Sanders and E. Kandrot. *CUDA by Example.* Addison-Wesley, 2010.
4. http://www.astrochem.org/sci/Nucleobases.php
5. http://www.biologycorner.com/bio1/DNA.html
6. http://www.qualitysilks.com/images/products/artificial-corn-stalk.jpg
7. http://en.wikipedia.org/wiki/dna
8. http://en.wikipedia.org/wiki/rna
9. http://en.wikipedia.org/wiki/HSP60