A Bayesian model for hybrid vigor

Will Landau

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Will Landau

Iowa State University

September 29, 2013

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Examples of gene regulation

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Estimated heterosis probabilities

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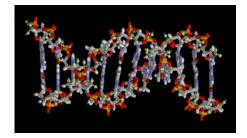
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DNA



··· GTGCATCTGACTCCTGAGGAGAAG ··· CACGTAGACTGAGGACTCCTCTTC ···

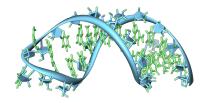
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··· GUGCAUCUGACUCCUGAGGAGAAG ···

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Proteins





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```
GTGCATCTGACTCCTGAGGAGAAG ...
                               DNA
```

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Central dogma

The Gibbs sampler

The Gibbs sampler

Central dogma: how organisms make proteins

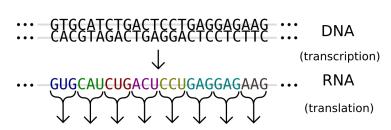
GTGCATCTGACTCCTGAGGAGAAG ··· CACGTAGACTGAGGACTCCTCTTC DNA (transcription) RNA GUGCAUCUGACUCCUGAGGAGAAG · · ·

Central dogma of genetics

Central dogma

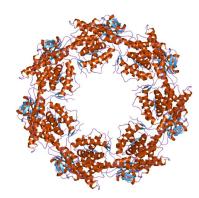
The Gibbs sampler





protein

- ► HSP = heat shock protein.
- ▶ Prevent heat damage to other proteins.



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Temperature spike triggers HSP60 production.

HSP60 Gene



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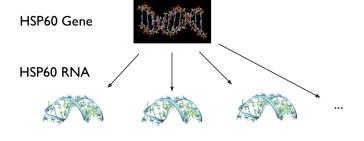
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Temperature spike causes HSP60 expression.

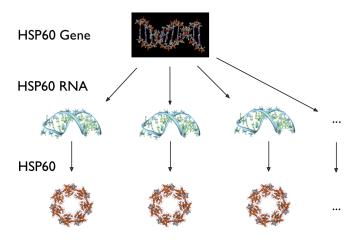


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Examples of gene regulation

Temperature spike causes HSP60 expression.



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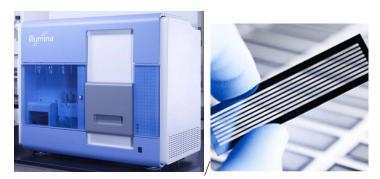
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- NA-seq
 - RNA sequencing: measure gene expression using relative abundance of RNA.
 - ► Illumina Genome Analyzer:



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RNA-seq data: counts of amplified RNA fragments

	Treatment I		Treatr	nent 2	Treatment 3		
Gene I	100	225	0	70	279	300	106
Gene 2	0	1	1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
Gene 34897	10	13	6	819	761	902	912

► Goal: use RNA-seq to study hybrid vigor (heterosis).

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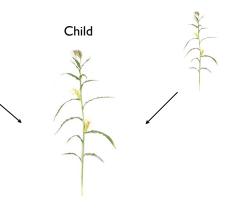
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High-parent heterosis: child's trait surpasses

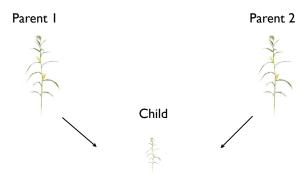


Parent 2

both parents

Parent I

Low-parent heterosis: child's trait is weaker than in each parent



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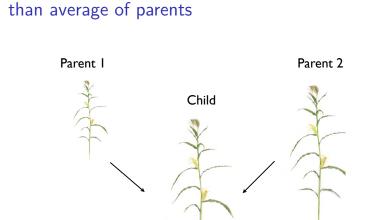
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Mid-parent heterosis: child's trait is different



High-parent heterosis in gene expression

	Parent I			Ch	nild	Parent 2	
Gene I	100	225	0	70	279	300	106
Gene 2	0	I	I	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
Gene 34897	10	13	6	819	761	902	912

Low-parent heterosis in gene expression

	Parent I			Ch	ild	Pare	ent 2
Gene I	100	225	0	70	279	300	106
Gene 2	0	-1	-1	50	501	2	7
Gene 3	3	4	2	700	900	0	0
Gene 4	893	400	760	5	5	1000	513
	•••						
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Mid-parent heterosis in gene expression

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

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$$c_n \stackrel{\text{ind}}{\sim} \text{N}(c_n \mid 0, \sigma_c^2)$$

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 $c_n \stackrel{\text{ind}}{\sim} \text{N}(c_n \mid 0, \sigma_c^2)$

$$\varepsilon_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_g^2)$$

$$\begin{aligned} y_{g,n} &\overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ c_n &\overset{\text{ind}}{\sim} \mathsf{N}(c_n \mid 0, \sigma_c^2) \\ \sigma_c &\sim \mathsf{U}(\sigma_c \mid 0, \sigma_{c0}) \\ \varepsilon_{g,n} &\overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \end{aligned}$$

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$$\begin{split} y_{g,n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ c_n & \overset{\text{ind}}{\sim} \mathsf{N}(c_n \mid 0,\sigma_c^2) \\ & \sigma_c \sim \mathsf{U}(\sigma_c \mid 0,\sigma_{c0}) \\ \varepsilon_{g,n} & \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 & \overset{\text{ind}}{\sim} \mathsf{Inv-Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;, \; \mathsf{rate} = \frac{d \cdot \tau^2}{2} \right) \end{split}$$

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$$\begin{split} y_{g,n} & \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ c_n & \overset{\text{ind}}{\sim} \mathsf{N}(c_n \mid 0,\sigma_c^2) \\ & \sigma_c \sim \mathsf{U}(\sigma_c \mid 0,\sigma_{c0}) \\ \varepsilon_{g,n} & \overset{\text{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0,\eta_g^2) \\ & \eta_g^2 & \overset{\text{ind}}{\sim} \mathsf{Inv-Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;, \; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \mathsf{U}(d \mid 0,d_0) \end{split}$$

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$$\begin{aligned} y_{g,n} & \overset{\mathsf{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g))) \\ c_n & \overset{\mathsf{ind}}{\sim} \mathsf{N}(c_n \mid 0, \sigma_c^2) \\ & \sigma_c \sim \mathsf{U}(\sigma_c \mid 0, \sigma_{c0}) \\ \varepsilon_{g,n} & \overset{\mathsf{ind}}{\sim} \mathsf{N}(\varepsilon_{g,n} \mid 0, \eta_g^2) \\ & \eta_g^2 & \overset{\mathsf{ind}}{\sim} \mathsf{Inv}\text{-}\mathsf{Gamma}\left(\eta_g^2 \mid \mathsf{shape} = \frac{d}{2} \;, \; \mathsf{rate} = \frac{d \cdot \tau^2}{2}\right) \\ & d \sim \mathsf{U}(d \mid 0, d_0) \\ & \tau^2 \sim \mathsf{Gamma}(\tau^2 \mid \mathsf{shape} = a_\tau, \mathsf{rate} = b_\tau) \end{aligned}$$

 $\mathbf{y_{g,n}} \overset{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y_{g,n}} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$

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$$\mu(n,\phi_g,\alpha_g,\delta_g) = \begin{cases} \phi_g - \alpha_g & \text{ sample } n \text{ from parent 1} \\ \phi_g + \delta_g & \text{ sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{ sample } n \text{ from parent 3} \end{cases}$$

 $y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_{\phi_g}, \sigma_{\phi}^2)$$

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$$\phi_g \stackrel{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi, \sigma_\phi^2)$$

$$\alpha_{g} \overset{\text{ind}}{\sim} \pi_{\alpha}^{1-l(\alpha_{g})} [(1-\pi_{\alpha}) \mathsf{N}(\alpha_{g} \mid \theta_{\alpha}, \sigma_{\alpha}^{2})]^{l(\alpha_{g})}$$

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$$\alpha_{\mathsf{g}} \overset{\mathsf{ind}}{\sim} \pi_{\alpha}^{1-I(\alpha_{\mathsf{g}})} [(1-\pi_{\alpha})\mathsf{N}(\alpha_{\mathsf{g}} \mid \theta_{\alpha}, \sigma_{\alpha}^2)]^{I(\alpha_{\mathsf{g}})}$$

$$\delta_{\mathbf{g}} \overset{\text{ind}}{\sim} \pi_{\delta}^{1-l(\delta_{\mathbf{g}})} [(1-\pi_{\delta}) \mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^2)]^{l(\delta_{\mathbf{g}})}$$

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$$\begin{split} \mathbf{y}_{\mathbf{g},n} &\stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{\mathbf{g},n} \mid \mathsf{exp}(c_n + \varepsilon_{\mathbf{g},n} + \mu(n,\phi_{\mathbf{g}},\alpha_{\mathbf{g}},\delta_{\mathbf{g}}))) \\ \phi_{\mathbf{g}} &\stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_{\mathbf{g}} \mid \theta_{\phi},\sigma_{\phi}^2) \\ \theta_{\phi} &\sim \mathsf{N}(\theta_{\phi} \mid 0,\gamma_{\phi}^2) \end{split}$$

$$\alpha_{\mathsf{g}} \overset{\mathsf{ind}}{\sim} \pi_{\alpha}^{1-I(\alpha_{\mathsf{g}})} [(1-\pi_{\alpha})\mathsf{N}(\alpha_{\mathsf{g}} \mid \theta_{\alpha}, \sigma_{\alpha}^{2})]^{I(\alpha_{\mathsf{g}})}$$

$$\delta_g \overset{\text{ind}}{\sim} \pi_\delta^{1-I(\delta_g)} [(1-\pi_\delta) \mathsf{N}(\delta_g \mid \theta_\delta, \sigma_\delta^2)]^{I(\delta_g)}$$

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$$\delta_{\mathbf{g}} \overset{\text{ind}}{\sim} \pi_{\delta}^{1-l(\delta_{\mathbf{g}})} [(1-\pi_{\delta})\mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^2)]^{l(\delta_{\mathbf{g}})}$$

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$$\begin{split} \delta_{\mathbf{g}} &\stackrel{\text{ind}}{\sim} \pi_{\delta}^{1-I(\delta_{\mathbf{g}})} [(1-\pi_{\delta}) \mathsf{N}(\delta_{\mathbf{g}} \mid \theta_{\delta}, \sigma_{\delta}^{2})]^{I(\delta_{\mathbf{g}})} \\ &\theta_{\delta} \sim \mathsf{N}(\theta_{\delta} \mid 0, \gamma_{\delta}^{2}) \end{split}$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 1} \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 3} \end{cases}$$

$$\begin{split} \mathbf{y}_{g,n} & \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(\mathbf{y}_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g & \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ & \theta_\phi \sim \mathsf{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ & \sigma_\phi \sim \mathsf{U}(\sigma_\phi \mid 0,\sigma_{\phi 0}) \\ & \alpha_g & \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-I(\alpha_g)} [(1-\pi_\alpha)\mathsf{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{I(\alpha_g)} \\ & \theta_\alpha \sim \mathsf{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \\ & \sigma_\alpha \sim \mathsf{U}(\sigma_\alpha \mid 0,\sigma_{\alpha 0}) \end{split}$$

$$\begin{split} \delta_g &\stackrel{\text{ind}}{\sim} \pi_{\delta}^{1-I(\delta_g)} [(1-\pi_{\delta})\mathsf{N}(\delta_g \mid \theta_{\delta}, \sigma_{\delta}^2)]^{I(\delta_g)} \\ &\theta_{\delta} \sim \mathsf{N}(\theta_{\delta} \mid 0, \gamma_{\delta}^2) \\ &\sigma_{\delta} \sim \mathsf{U}(\sigma_{\delta} \mid 0, \sigma_{\delta 0}) \end{split}$$

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$$\begin{split} \mu(n,\phi_g,\alpha_g,\delta_g) &= \begin{cases} \phi_g - \alpha_g & \text{sample } n \text{ from parent } 1 \\ \phi_g + \delta_g & \text{sample } n \text{ from child} \\ \phi_g + \alpha_g & \text{sample } n \text{ from parent } 3 \end{cases} \\ y_{g,n} & \overset{\text{ind}}{\sim} \text{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ \phi_g & \overset{\text{ind}}{\sim} \text{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ \theta_\phi & \sim \text{N}(\theta_\phi \mid 0,\gamma_\phi^2) \\ \sigma_\phi & \sim \text{U}(\sigma_\phi \mid 0,\sigma_{\phi 0}) \\ \alpha_g & \overset{\text{ind}}{\sim} \frac{1^{-l}(\alpha_g)}{\pi_\alpha} [(1-\pi_\alpha)\text{N}(\alpha_g \mid \theta_\alpha,\sigma_\alpha^2)]^{l(\alpha_g)} \\ \theta_\alpha & \sim \text{N}(\theta_\alpha \mid 0,\gamma_\alpha^2) \\ \sigma_\alpha & \sim \text{U}(\sigma_\alpha \mid 0,\sigma_{\alpha 0}) \\ \pi_\alpha & \sim \text{Beta}(\pi_\alpha \mid a_\alpha,b_\alpha) \\ \delta_g & \overset{\text{ind}}{\sim} \frac{1^{-l}(\delta_g)}{\pi_0^2} [(1-\pi_\delta)\text{N}(\delta_g \mid \theta_\delta,\sigma_\delta^2)]^{l(\delta_g)} \\ \theta_\delta & \sim \text{N}(\theta_\delta \mid 0,\gamma_\delta^2) \end{split}$$

 $\sigma_s \sim U(\sigma_s \mid 0, \sigma_{so})$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 1} \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent 3} \end{cases}$$

$$\begin{aligned} & y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g))) \\ & \phi_g \stackrel{\text{ind}}{\sim} \mathsf{N}(\phi_g \mid \theta_\phi,\sigma_\phi^2) \\ & \theta_\phi \sim \mathsf{N}(\theta_\phi \mid 0, \gamma_\phi^2) \\ & \sigma_\phi \sim \mathsf{U}(\sigma_\phi \mid 0, \sigma_{\phi 0}) \\ & \alpha_g \stackrel{\text{ind}}{\sim} \pi_\alpha^{1-I(\alpha_g)} [(1 - \pi_\alpha) \mathsf{N}(\alpha_g \mid \theta_\alpha, \sigma_\alpha^2)]^{I(\alpha_g)} \\ & \theta_\alpha \sim \mathsf{N}(\theta_\alpha \mid 0, \gamma_\alpha^2) \\ & \sigma_\alpha \sim \mathsf{U}(\sigma_\alpha \mid 0, \sigma_{\alpha 0}) \\ & \pi_\alpha \sim \mathsf{Beta}(\pi_\alpha \mid a_\alpha, b_\alpha) \\ & \delta_g \stackrel{\text{ind}}{\sim} \pi_\delta^{1-I(\delta_g)} [(1 - \pi_\delta) \mathsf{N}(\delta_g \mid \theta_\delta, \sigma_\delta^2)]^{I(\delta_g)} \\ & \theta_\delta \sim \mathsf{N}(\theta_\delta \mid 0, \gamma_\delta^2) \\ & \sigma_\delta \sim \mathsf{U}(\sigma_\delta \mid 0, \sigma_{\delta 0}) \\ & \pi_\delta \sim \mathsf{Beta}(\pi_\delta \mid a_\delta, b_\delta) \end{aligned}$$

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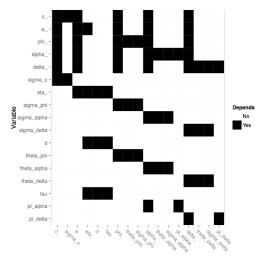
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Partition parameters by conditional independence.



Has a full conditional that depends on...

pends on...

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► From the appropriate full conditional distributions, sample the following:
- 1. c_1, \ldots, c_N

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Use these partitions as Gibbs steps.

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► From the appropriate full conditional distributions, sample the following:
- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}

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Use these partitions as Gibbs steps.

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}
- 4. σ_c , σ_ϕ , σ_α , σ_δ , η_1^2 , ..., η_G^2

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}
- 4. σ_c , σ_{ϕ} , σ_{α} , σ_{δ} , η_1^2 , ..., η_G^2
- 5. $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \mathsf{exp}(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}
- 4. σ_c , σ_{ϕ} , σ_{α} , σ_{δ} , η_1^2 , ..., η_G^2
- 5. $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$
- 6. ϕ_1, \ldots, ϕ_G

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Use these partitions as Gibbs steps.

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

► From the appropriate full conditional distributions, sample the following:

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}
- 4. σ_c , σ_{ϕ} , σ_{α} , σ_{δ} , η_1^2 , ..., η_G^2
- 5. $\varepsilon_{1,1}, \ \varepsilon_{1,2}, \ \ldots, \ \varepsilon_{1,N}, \ \varepsilon_{2,N}, \ \ldots, \ \varepsilon_{G,N}$
- 6. ϕ_1, \ldots, ϕ_G
- 7. $\alpha_1, \ldots, \alpha_G$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- 1. c_1, \ldots, c_N
- 2. τ , π_{α} , π_{δ}
- 3. d, θ_{ϕ} , θ_{α} , θ_{δ}
- 4. σ_c , σ_{ϕ} , σ_{α} , σ_{δ} , η_1^2 , ..., η_G^2
- 5. $\varepsilon_{1,1}$, $\varepsilon_{1,2}$, ..., $\varepsilon_{1,N}$, $\varepsilon_{2,N}$, ..., $\varepsilon_{G,N}$
- 6. ϕ_1, \ldots, ϕ_G
- 7. $\alpha_1, \ldots, \alpha_G$
- 8. $\delta_1, \ldots, \delta_G$
- and then repeat.

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \operatorname{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

$$I(\delta_{\mathsf{g}}^{(i)} > |\alpha_{\mathsf{g}}^{(i)}|)$$

$$I(\delta_{\mathbf{g}}^{(i)} < -|\alpha_{\mathbf{g}}^{(i)}|)$$

$$I(\delta_g^{(i)} \neq 0)$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \operatorname{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with *M* iterations.

$$I(\delta_{\mathsf{g}}^{(i)} > |\alpha_{\mathsf{g}}^{(i)}|)$$

$$I(\delta_{\mathbf{g}}^{(i)} < -|\alpha_{\mathbf{g}}^{(i)}|)$$

$$I(\delta_g^{(i)} \neq 0)$$

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 $\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$

$$y_{g,n} \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with *M* iterations.

$$P(\text{high-parent heterosis in gene }g) pprox rac{1}{M} \sum_{i=1}^{M} I(\delta_{g}^{(i)} > |lpha_{g}^{(i)}|)$$

$$I(\delta_{\mathbf{g}}^{(i)} < -|\alpha_{\mathbf{g}}^{(i)}|)$$

$$I(\delta_{\mathsf{g}}^{(i)} \neq 0)$$

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$$\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$$

$$y_{g,n} \stackrel{\text{ind}}{\sim} \operatorname{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with *M* iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

$$P(\text{low-parent heterosis in gene } g \) pprox rac{1}{M} \sum_{i=1}^{M} I(\delta_{g}^{(i)} < -|lpha_{g}^{(i)}|)$$

$$I(\delta_{\mathsf{g}}^{(i)} \neq 0)$$

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 $\mu(\textit{n}, \phi_{\textit{g}}, \alpha_{\textit{g}}, \delta_{\textit{g}}) = \begin{cases} \phi_{\textit{g}} - \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 1 \\ \phi_{\textit{g}} + \delta_{\textit{g}} & \text{sample } \textit{n} \text{ from child} \\ \phi_{\textit{g}} + \alpha_{\textit{g}} & \text{sample } \textit{n} \text{ from parent } 3 \end{cases}$

$$y_{g,n} \overset{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n,\phi_g,\alpha_g,\delta_g)))$$

Consider one chain with *M* iterations.

$$P(\text{high-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} > |\alpha_g^{(i)}|)$$

$$P(\text{low-parent heterosis in gene } g) pprox rac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} < -|lpha_g^{(i)}|)$$

$$P(\text{mid-parent heterosis in gene } g) \approx \frac{1}{M} \sum_{i=1}^{M} I(\delta_g^{(i)} \neq 0)$$

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

Sample in parallel:

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - $ightharpoonup \phi_g$'s

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
 - $ightharpoonup \phi_g$'s
 - $\sim \alpha_g$'s

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
 - $\blacktriangleright \phi_g$'s
 - $ightharpoonup \alpha_{g}$'s
 - \triangleright δ_g 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- Sample in parallel:
 - φ_g's
 - $ightharpoonup \alpha_{g}$'s
 - $ightharpoonup \delta_g$'s
 - $\triangleright \varepsilon_{g,n}$'s

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Tons of opportunity for GPU parallelism across genes!

$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup lpha_{
 m g}$'s
 - $ightharpoonup \delta_g$'s
 - $\triangleright \varepsilon_{g,n}$'s
 - $ightharpoonup \eta_{
 m g}$'s

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup lpha_{m{g}}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{g,n}$'s
 - η_g's
- Use parallel reductions to calculate sufficient statistics for:

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup lpha_{m{g}}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{g,n}$'s
 - $\triangleright \eta_g$'s
- Use parallel reductions to calculate sufficient statistics for:
 - \triangleright c_n 's

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup lpha_{f g}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{g,n}$'s
 - $\triangleright \eta_g$'s
- Use parallel reductions to calculate sufficient statistics for:
 - \triangleright c_n 's
 - ▶ τ, d

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup lpha_{
 m g}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{\sigma,n}$'s
 - $\vdash \eta_g$'s
- Use parallel reductions to calculate sufficient statistics for:
 - \triangleright c_n 's
 - ▶ τ, d
 - \blacktriangleright θ_{ϕ} , θ_{α} , θ_{δ}

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - φ_σ's
 - $ightharpoonup \alpha_{g}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{\sigma,n}$'s
 - $\vdash \eta_g$'s
- Use parallel reductions to calculate sufficient statistics for:
 - ► Cn'S
 - ▶ τ. d
 - \blacktriangleright θ_{ϕ} , θ_{α} , θ_{δ}
 - \triangleright σ_{ϕ} , σ_{α} , σ_{δ} , σ_{c}

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$$y_{g,n} \stackrel{\text{ind}}{\sim} \mathsf{Poisson}(y_{g,n} \mid \exp(c_n + \varepsilon_{g,n} + \mu(n, \phi_g, \alpha_g, \delta_g)))$$

- ► Sample in parallel:
 - $\rightarrow \phi_{\sigma}$'s
 - $ightharpoonup \alpha_{g}$'s
 - \triangleright δ_g 's
 - $\triangleright \varepsilon_{\sigma,n}$'s
 - $\vdash \eta_g$'s
- Use parallel reductions to calculate sufficient statistics for:
 - \triangleright c_n 's
 - ▶ τ, d
 - \bullet θ_{ϕ} , θ_{α} , θ_{δ}
 - \triangleright σ_{ϕ} , σ_{α} , σ_{δ} , σ_{c}
 - \blacktriangleright π_{α} , π_{δ}

Hybrid vigor

The Gibbs sample Gibbs steps Estimated heterosis

A Bavesian model

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The software

Ordinary C and GPU-accelerated versions, along with an R package wrapper, are available for download at https://github.com/wlandau/heterosis. A Bayesian model for hybrid vigor

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► Time for a demo...

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Sources

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