Derivation of 3 posterior distributions :

① Posterior for μ_s for $s=1,\cdots,10$

$$P(\mu_s \mid \log m_i, z_i = s) \propto \prod_{i=1}^{n} P(\log m_i \mid z_i = s, \mu_s)\cdot P(\mu_s \mid z_i = s)$$ species s

Note that n is not 500 or simply the size of data, instead, n is # of beetles of species s

And z_i is the latent variable which is the species of a certain beetle.

(cont') know $\mu_s \sim N(1, 2^2)$, $\log m_i \sim N(\mu_s, 0.08^2)$

$$① \propto N(1,2^2)\cdot \prod_{i=1}^{n} N(\mu_s, 0.08^2)$$

$$= \frac{1}{2\sqrt{2}}\exp(-\tfrac{1}{2}(\mu_s-1)^2/4)\prod_{i=1}^{n}\frac{1}{0.08\sqrt{2\pi}}\exp(-\tfrac{1}{2}(\log m_i-\mu_s)^2/0.08^2)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{\mu_s^2-2\mu_s+1}{4}+\frac{\sum_{i=1}^{n}\log m_i^2-2\mu_s\sum_{i=1}^{n}\log m_i+n\mu_s^2}{0.08^2}\right)\right)$$

$$\propto \exp\left\{-\frac{1}{2}\left(\mu_s^2\underbrace{\left(\frac{1}{4}+\frac{n}{0.08^2}\right)}_{A}-\mu_s\underbrace{\left(\frac{1}{2}+\frac{2\sum_{i=1}^{n}\log m_i}{0.08^2}\right)}_{B}\right)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}(A\mu_s^2-\mu_s B)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left(\sqrt{A}-\mu_s-\frac{B}{2\sqrt{A}}\right)^2/1^2\right\}\cdot\frac{1}{1\sqrt{2\pi}}$$

$$\Rightarrow \sqrt{A}\,\mu_s \sim N\left(\frac{B}{2\sqrt{A}}, 1^2\right)$$

$$\mu_s \sim \frac{1}{\sqrt{A}}N\left(\frac{B}{2\sqrt{A}}, 1^2\right)$$

$$\frac{1}{\sqrt{\frac{1}{4}+\frac{n}{0.08^2}}}N\left(\frac{\frac{1}{2}+2\sum_{i=1}^{n}\log m_i/0.08^2}{2\sqrt{\frac{1}{4}+\frac{n}{0.08^2}}}, 1^2\right)$$

Similarily,

② Posterior of $\nu_s \propto \exp\{-\frac{1}{2}(C\nu_s^2-\nu_s D)\}$ for some C, D

&

$$\nu_s \sim \frac{1}{\sqrt{\frac{1}{4}+\frac{n}{0.1^2}}}N\left(\frac{\frac{1}{2}+2\sum_{i=1}^{n}\log r_i/0.1^2}{2\sqrt{\frac{1}{4}+n/0.1^2}}, 1^2\right)$$

③ Posterior of φ_s

$$= P(\varphi_s \mid sw_i, z_i = s)$$

$$\propto P(sw_i \mid z_i = s)\prod_{i=1}^{n} P(\varphi_s \mid z_i = s, sw_i)$$

$$\propto \text{Beta}(1,1)\cdot\text{Beta}(\alpha,\beta)$$

$$\propto \frac{\Gamma(1)^2}{\Gamma(2)}\cdot\varphi_s^{1-1}\cdot(1-\varphi_s)^{1-1}\cdot\varphi_s^{\alpha-1}\cdot(1-\varphi_s)^{\beta-1}$$

where $\alpha=\sum_{i=1}^{n}sw_i+1$, $\beta=n+1-\sum_{i=1}^{n}sw_i$

so $\varphi_s \sim 1 \cdot 1 \cdot 1 \cdot \text{Beta}(\alpha, \beta)$

$$\varphi_s \sim \text{Beta}\left(\sum_{i=1}^{n} sw_i + 1,\ n+1-\sum_{i=1}^{n} sw_i\right)$$