STA410 Assignment 2

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Problem 1: Minecraft Revisited

- The following is a partial output of EM algorithm:

iter	est.p1	est.p2	log-likelihood
82	0.83007546584433323	0.28329283584105358	-8.0379597929241164
83	0.83007546584433356	0.28329283584105336	-8.0379597929240667
84	0.83007546584433367	0.28329283584105319	-8.0379597929240454
85	0.83007546584433378	0.28329283584105308	-8.0379597929240330
86	0.83007546584433378	0.28329283584105303	-8.0379597929240241
87	0.83007546584433378	0.28329283584105303	-8.0379597929240241
88	0.83007546584433378	0.28329283584105303	-8.0379597929240241

- Discussion:
    - By observation, we find out that in our test case, the values of estimated p1, estimated p2 and log-likelihood all become stable after 86 iterations.
    - And actually it should converge from all the starting points.
- Easiness to do EM:
    - The EM algorithm is straightforward, namely, to do expectation in E-step and do maximization in M-step, and iterate finitely many times until we get some satisfying results.
    - The most important advantage is that EM involves no computation of first or second derivative, thus saving us lots of time.
- How fast it converges.

- So we take the estimated p1 from iteration 82, 83, 84:

  ```
  > final <- 0.830075465844333378
  > p1.82 <- 0.830075465844333323
  > p1.83 <- 0.830075465844333356
  > p1.84 <- 0.830075465844333367
  > (p1.82-final)/(p1.83-final)
  [1] 2.5
  > (p1.82-final)^2/(p1.83-final)
  [1] -1.38777878078e-15
  > (p1.83-final)/(p1.84-final)
  [1] 2
  > (p1.83-final)^2/(p1.84-final)
  [1] -4.4408920985e-16
  ```

- This is the behaviour expected with linear convergence -- the error after t+1 iterations is about 1/5 the after t iterations.
- So the rate of convergence is 1.

Problem 2: The Beetles

- The final output is shown below:

```
            mu        vu       rho      alpha
sp1  0.8942923 1.0342643 0.2657339 0.06729892
sp2  1.3899676 0.9623014 0.0993474 0.19440148
sp3  1.7168589 1.0546248 0.1551782 0.07671346
sp4  0.5312401 1.2620517 0.8176918 0.07347779
sp5  1.5094600 1.4086777 0.9183584 0.05511000
sp6  1.7233369 0.9994049 0.1464464 0.10582222
sp7  1.3208239 0.9380040 0.4295160 0.04366880
sp8  1.0519352 1.2553052 0.4406961 0.15757814
sp9  1.0239227 1.2366704 0.4606217 0.19213491
sp10 0.9046637 1.7304230 0.5328304 0.03379428
```

- Although we belive different initial guesses (parameter matrices) should lead to the same convergence, somehow when I test with mean value of different parameters, the likelihood goes to `NaN` immediately.
- How fast it converges.
  - The results are stable after 63 iterations.

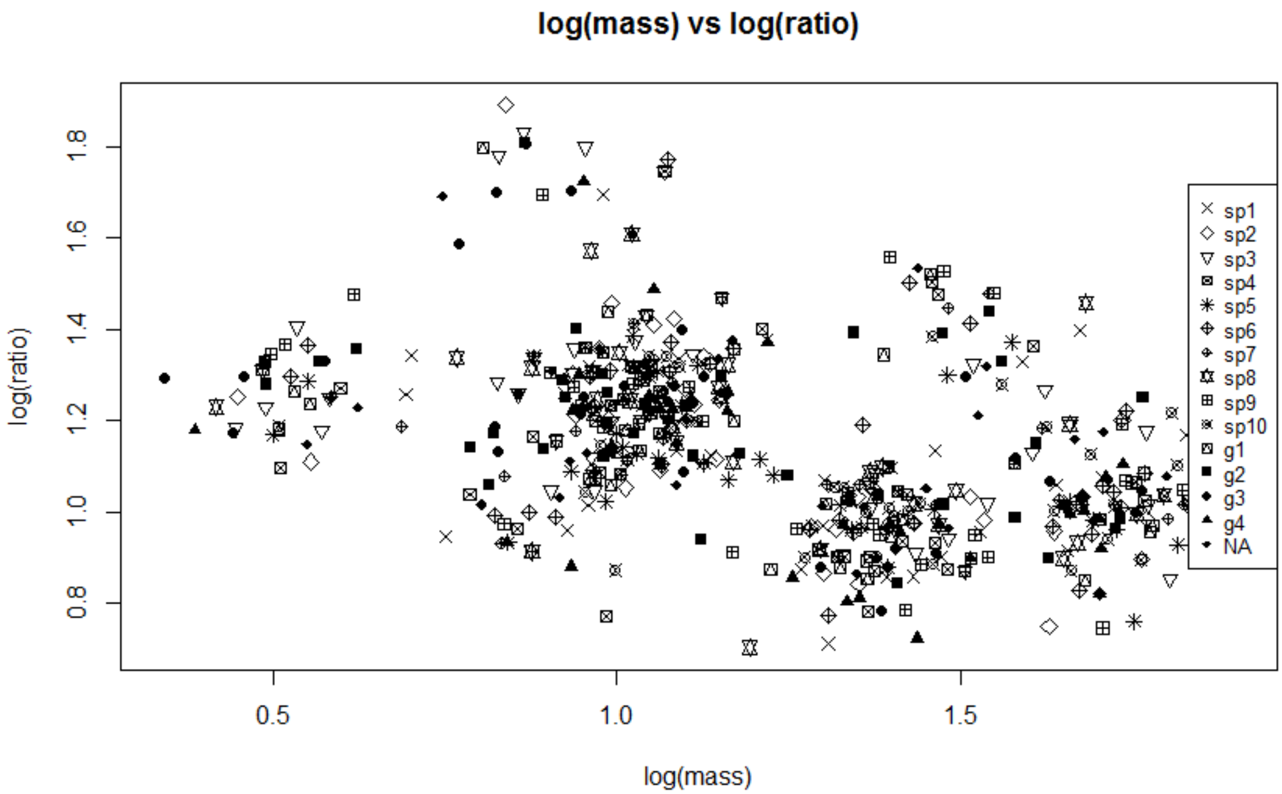
- And we take the estimated mu1 from iteration 59, 60, 61:

  ```
  > final <- 0.8942923
  > mu1.59 <- 0.8942927
  > mu1.60 <- 0.8942927
  > mu1.61 <- 0.8942926
  > (mu1.59-final)/(mu1.60-final)
  [1] 1
  > (mu1.60-final)/(mu1.61-final)
  [1] 1.333333
  > (mu1.59-final)^2/(mu1.60-final)
  [1] 4e-07
  > (mu1.60-final)^2/(mu1.61-final)
  [1] 5.333333e-07
  ```

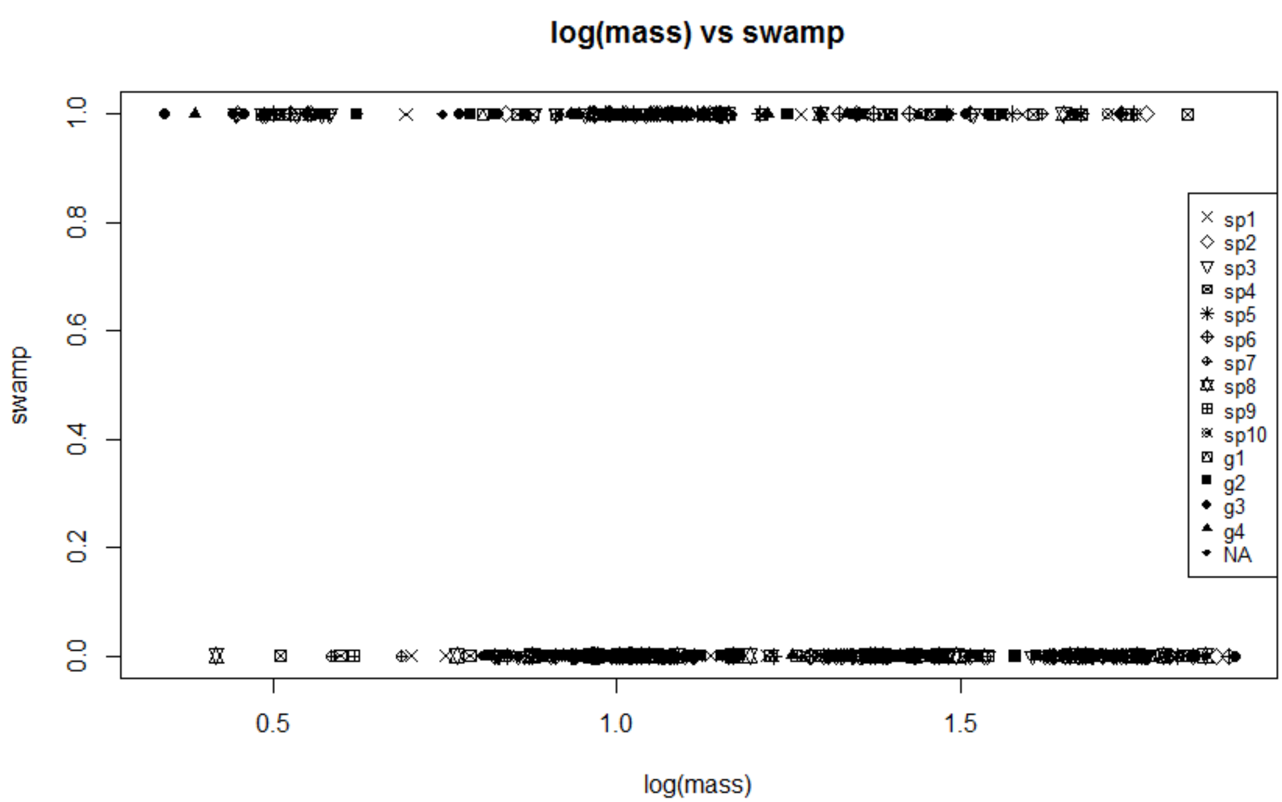
- Similarly, the rate of convergence should be 1. This is a linear convergence case.

* Alternative way to EM
  - Easy way: sometimes in practice, we could just ignore the data entry with `NA`, taking it just as an invalid entry.
  - Hard way: In order to guarantee feasibility and consistency in high dimensions, we could use moment-based approaches or so-called "spectral techniques". Reference: https://en.wikipedia.org/wiki/Expectation%E2%80%93maximizationalgorithm#Alternativesto_EM

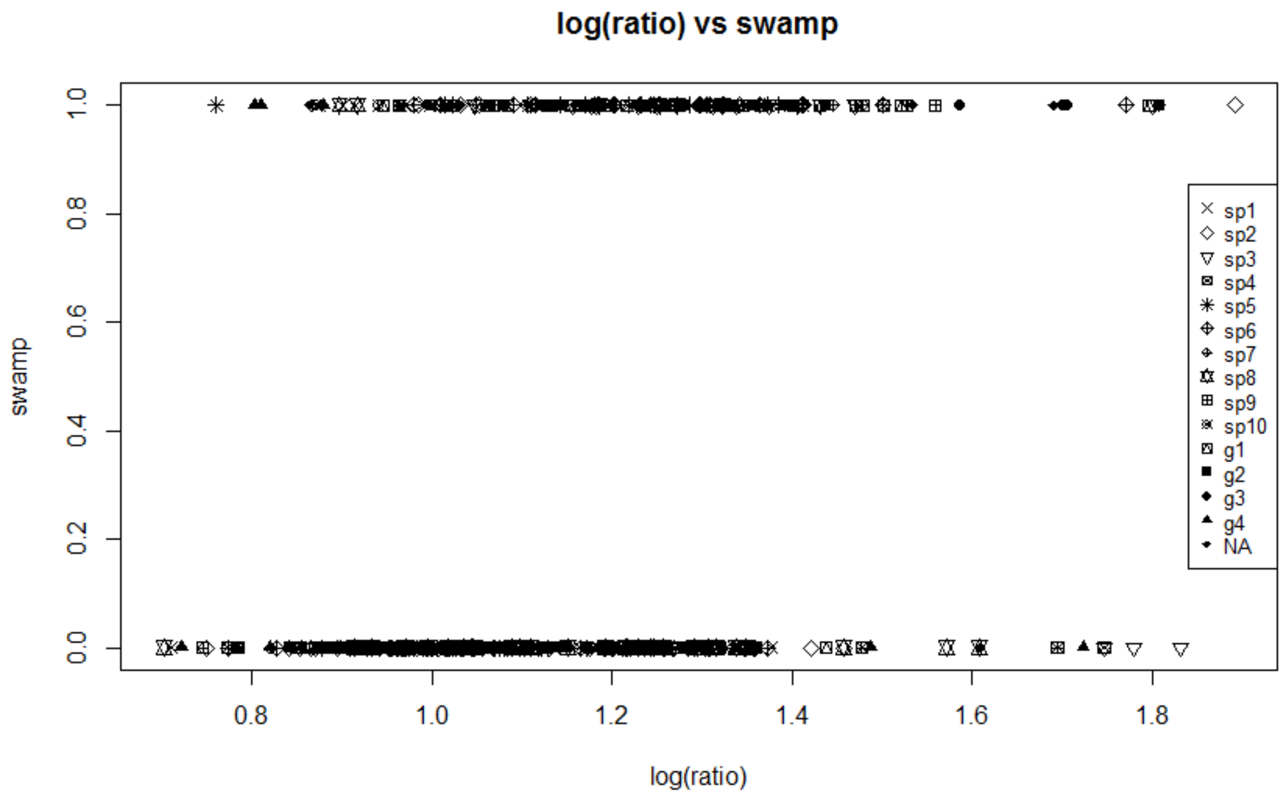
* Plotting
  - log(mass) vs log(ratio) plot



- log(mass) vs swamp plot



- log(ratio) vs swamp plot


log(ratio) vs swamp
swamp
log(ratio)
0.0
0.2
0.4
0.6
0.8
1.0
0.8
1.0
1.2
1.4
1.6
1.8
sp1
sp2
sp3
sp4
sp5
sp6
sp7
sp8
sp9
sp10
g1
g2
g3
g4
NA