Integral Cheatsheet

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1 基本的积分公式

1. $\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1} + C. (\alpha \neq -1)$
2. $\int \frac{1}{x}dx = \ln|x| + C$
3. $\int a^{x}dx = \frac{a^{x}}{\ln a} + C. (0 < a \neq 1)$
4. $\int e^{x}dx = e^{x} + C$
5. $\int \sin x dx = -\cos x + C$
6. $\int \cos x dx = \sin x + C$
7. $\int \sec^2 x dx = \tan x + C$
8. $\int \sec x \tan x dx = \sec x + C$
9. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
10. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
11. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$
12. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$
13. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + C$
14. $\int \sec x dx = \ln|\sec x + \tan x| + C$

2 二级结论

2.1 幂函数

1. $\int \frac{1}{x}dx = \ln(x) + C$
2. $\int x^{a}dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$
3. $\int a^{x}dx = \frac{a^{x}}{\ln(a)} + C \quad (a > 0, a \neq 1)$
4. $\int \frac{1}{x^{a}}dx = -\frac{x^{(1-a)}}{1-a} + C \quad (a \neq 1)$
5. $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1) + C$
6. $\int x^{n}e^{ax}dx = \frac{x^{n}e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}dx$
7. $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$
8. $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$

2.2 三角函数

1. $\int \sin(x)dx = -\cos(x) + C$
2. $\int \frac{1}{\sin(x)}dx = \ln\left|\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}\right| + C$
3. $\int \sin(ax)\sin(bx)dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C$
4. $\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
5. $\int \sin^3(x)dx = \frac{1}{12}\Big(\cos(3x) - 9\cos(x)\Big) + C$
6. $\int \cos(x)dx = \sin(x) + C$
7. $\int \frac{1}{\cos(x)}dx = \int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$
8. $\int \cos(ax)\cos(bx)dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C$
9. $\int \cos mx \cos nx dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C$
10. $\int \sin mx \cos nx dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C$
11. $\int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
12. $\int \cos^3(x)dx = \frac{1}{12}\Big(9\sin(x) + \sin(3x)\Big) + C$
13. $\int \sin(x)\cos(x)dx = -\frac{\cos^2(x)}{2} + C$
14. $\int \sin^2(x)\cos^2(x)dx = \frac{1}{32}\Big(4x - \sin(4x)\Big) + C$
15. $\int \tan(x)dx = -\ln|\cos(x)| + C$
16. $\int \tan(x)dx = \ln|\sec(x)| + C$
17. $\int \tan^2(x)dx = \tan(x) - x + C$
18. $\int \tan^3(x)dx = \frac{\tan^2(x)}{2} + \ln|\cos(x)| + C$
19. $\int \frac{1}{\tan(x)}dx = \ln|\sin(x)| + C$
20. $\int \frac{1}{\tan^2(x)}dx = -\frac{1}{\tan(x)} - x + C$
21. $\int \frac{1}{\tan^3(x)}dx = \frac{\tan^2(x)}{2} - \ln|\sin(x)| + C$

2.3 倒数函数

1. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
2. $\int \frac{1}{x^2-a^2}dx = \frac{1}{a}\ln(a^2+x^2) + C$
3. $\int \frac{1}{1-x^2}dx = \frac{\ln|u+1| - \ln|u-1|}{2} + C$
4. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$
5. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$
6. $\int \frac{1}{\sqrt{a^2+u^2}}dx = \ln(u + \sqrt{a^2+x^2}) + C$