ME 360 Lecture 2.

Ch.0 Fundamentals

0.1. Evaluating a polynomial

$$P(x) = 9x^4 - 2x^3 - 4x^2 + 3x - 1$$

Q: How to compute this polynomial fast for a given x?
Let's try for $x = 1/3$.

*. Method #1 Direct Method.

$$P(\tfrac{1}{3}) = 9 \times \tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3} - 2 \times \tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3} - 4 \times \tfrac{1}{3} \times \tfrac{1}{3} + 3 \times \tfrac{1}{3} + 1$$

$$= \frac{43}{27}$$

10 multiplication, 4 addition

Can we do better?

* Method #2 Finding Power First

- First, we compute the power of $1/3$.

$$\tfrac{1}{3} \times \tfrac{1}{3} = (\tfrac{1}{3})^2; \quad (\tfrac{1}{3})^2 \times \tfrac{1}{3} = (\tfrac{1}{3})^3; \quad (\tfrac{1}{3})^3 \times \tfrac{1}{3} = (\tfrac{1}{3})^4$$

- Then we add up the terms.

$$P(\tfrac{1}{3}) = 9 \times (\tfrac{1}{3})^4 - 2 \times (\tfrac{1}{3})^3 - 4 \times (\tfrac{1}{3})^2 + 3 \times (\tfrac{1}{3}) - 1$$

$$= \frac{43}{27}$$

3 multiplication from power
4 multiplication from computing prefactor
4 addition.

total: 7 multiplication, 4 addition.

*** Method #3 (Nested Multiplication) Horner's Method**

$$P(x) = -1 + x(3 - 4x - 2x^2 + 9x^3)$$
$$= -1 + x(3 - x(4 + 2x - 9x^2))$$
$$= -1 + x(3 - x(4 + x(2 - 9x)))$$

4 multiplication, 4 addition

General degree d polynomial: d multiplication, d addition

Exercise in class:

0.2 Binary Numbers

- Binary number system simplifies computer operations like addition & multiplication.
  We only use 0 & 1 in binary number system.

- Binary numbers are expressed as

  $$\ldots b_2 b_1 b_0 . b_{-1} b_{-2} \ldots$$

  To convert into base 10, we have

  $$\ldots b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0 + b_{-1} \cdot 2^{-1} + b_{-2} 2^{-2} + \ldots$$

0.2.1 Decimal to Binary

What is 59.6 in binary?

* Integer part.

$59 / 2 = 29$ R 1 finish
$29 / 2 = 14$ R 1
$14 / 2 = 7$ R 0
$7 / 2 = 3$ R 1
$3 / 2 = 1$ R 1
$1 / 2 = 0$ R 1 start

$\Rightarrow \boxed{(59)_{10} = (111011)_2}$

* Fractional part

$$\left.\begin{aligned} 0.6 \times 2 &= 0.2 + 1 \\ 0.2 \times 2 &= 0.4 + 0 \\ 0.4 \times 2 &= 0.8 + 0 \\ 0.8 \times 2 &= 0.6 + 1 \end{aligned}\right] \text{repeating unit}$$

$0.6 \times 2 = 0.2 + 1$
$0.2 \times 2 = 0.4 + 0$
$\vdots$

start
finish

$\therefore (59.6)_{10} = (111011.\overline{1001}1)$

Exercise in class

⋮

0.2.2 Binary to decimal

Convert $(1101.0011)_2$ to decimal.

* Integer part: $1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 = 8+4+1 = (13)_{10}$

* Fractional part: $0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4} = (3/16)_{10}$.

What happens when the fractional part is not finite?

*** A useful property of binary numbers.**

Multiply a binary number by 2 will shift all bits one place to the left.

Example: $2\times(10110.1011)_2 = (101101.011)_2$

Why this works:

$$(10110.1011)_2 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 + 1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4}$$

$$2\times(10110.1011)_2 = 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3}$$

$$\therefore\ 2\times(10110.1011)_2 = (101101.011)_2$$

Now let's go back to binary numbers that are not finite...

A not finite, repeating number:

$$x = (0.\overline{0101})_2$$

To convert this, we have

$$2^4 \times x = (101.\overline{0101})_2$$

$$x = (0.\overline{0101})_2$$

so $$(2^4 - 1)x = (101)_2 = (5)_{10}$$

$$\boxed{x = \left(\frac{5}{15}\right)_{10} = \left(\frac{1}{3}\right)_{10}}$$

Exercise in class