ME 491 Lecture 13

4.4 Optimization as the Corner Stone of Regression

We will fit our data to a 19-degree polynomial.
The function we want to fit to would be

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_{19} x^{19}$$

We can rewrite this equation in the form of $Ax = b$

$$\underbrace{\begin{bmatrix} | & | & | & | & & | \\ 1, & x, & x^2, & x^3, & \ldots, & x^{19} \\ | & | & | & | & & | \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{19} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_{100}) \end{bmatrix}}_{b}$$

Since we have 100 measurements, shape of A is $(100, 20)$, x is $(20, 1)$, b is $(100, 1)$, this is an over-determined problem.

If we only want to minimize the ℓ_2-norm,

$$x_{approx} = A^{+} b \quad (A^{+}\text{: pseudo-inverse of } A)$$