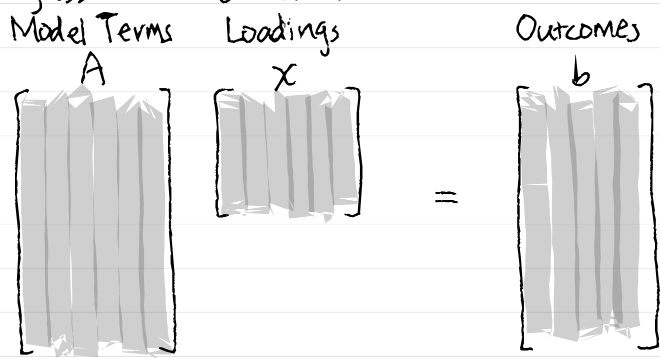
ME 491 Lecture 12

4.3. Regression and $Ax=b$: Over- and Under-Determined Systems

Over-determined Systems

more constraints than unknown variables, generally there is no solution that satisfies $Ax=b$.

- Regression Framework for over-determined system.



We want to solve the problem that minimize error, for example solving the equarion

$$\hat{x} = \underset{x}{\text{argmin}} \|Ax-b\|_2$$

minimizing ℓ_2 norm.

The basic architecture does not explicitly enforce any constraints on the loadings x. We can modify the basic optimization architecture to enforce a constraint

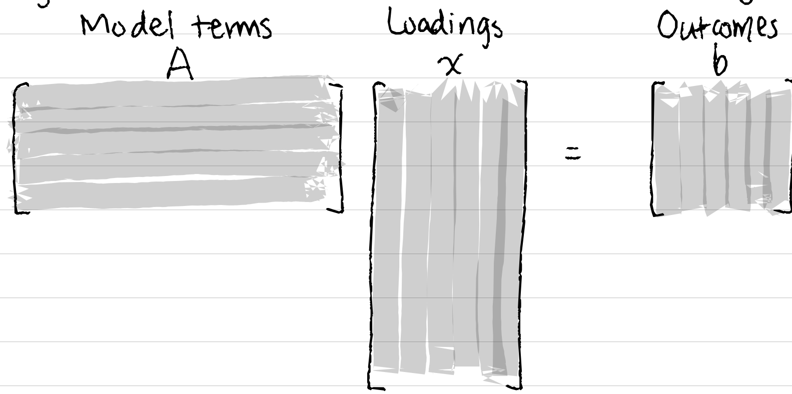
$$\hat{x} = \underset{x}{\text{argmin}} \left(\|Ax-b\|_2 + \lambda_1 \|x\|_1 + \lambda_2 \|x\|_2 \right)$$

λ_1, λ_2 controlls the penalization of the ℓ_1 and ℓ_2 norm

The ability to design the penalty by adding regularizing constraints is critical for understanding model selection.

Under-determined Systems

- Regression framework for under-determined system



To solve the under-determined problem

$$\min \|x\|_p \quad \text{subject to} \quad Ax = b$$

p denotes the p-norm of the vector x, a common optimization modifier is to consider only the ℓ_1 & ℓ_2 norm.

$$\min(\lambda_1 \|x\|_1 + \lambda_2 \|x\|_2) \quad \text{subject to} \quad Ax = b$$