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Control Theory assignment
Inverted pendulum

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Identification of motor

- Theoretical model: $\frac{\theta}{E} = \frac{\mathrm{K}_t/\mathrm{R}_a J_m}{s(s+\frac{1}{J_m}(D_m+\frac{K_t K_b}{J_m}))}$ [1]

Θ position– Output

K_t torque constant in Nm/A

J_m motor inertia in kg-m 2

R_a is armature resistance in ohms

E input voltage in volts - Input

K_b back e.m.f constant in Vs/rad,

D_m is motor damping in N-m s/rad,

- Discrete model using ZOH: $\frac{\Theta}{E} = \frac{b_1}{(z-1)(z+a_1)}$

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Identification of motors

- Excitation signal :

  pseudo-random binary sequence (PRBS)

  Matlab: U = idinput(1000,'PRBS',[0 0.1]);

  C:
```
#include <avr/pgmspace.h>
const float input [] PROGMEM = {/*data U*/}
//put data in flash.
```

- Identified model:

  After 1 iteration:

$$Motor1:\ g_1 = \frac{0.01831}{s(s+11.88)} \qquad Motor2:\ g_2 = \frac{0.02305}{s(s+10.79)}$$

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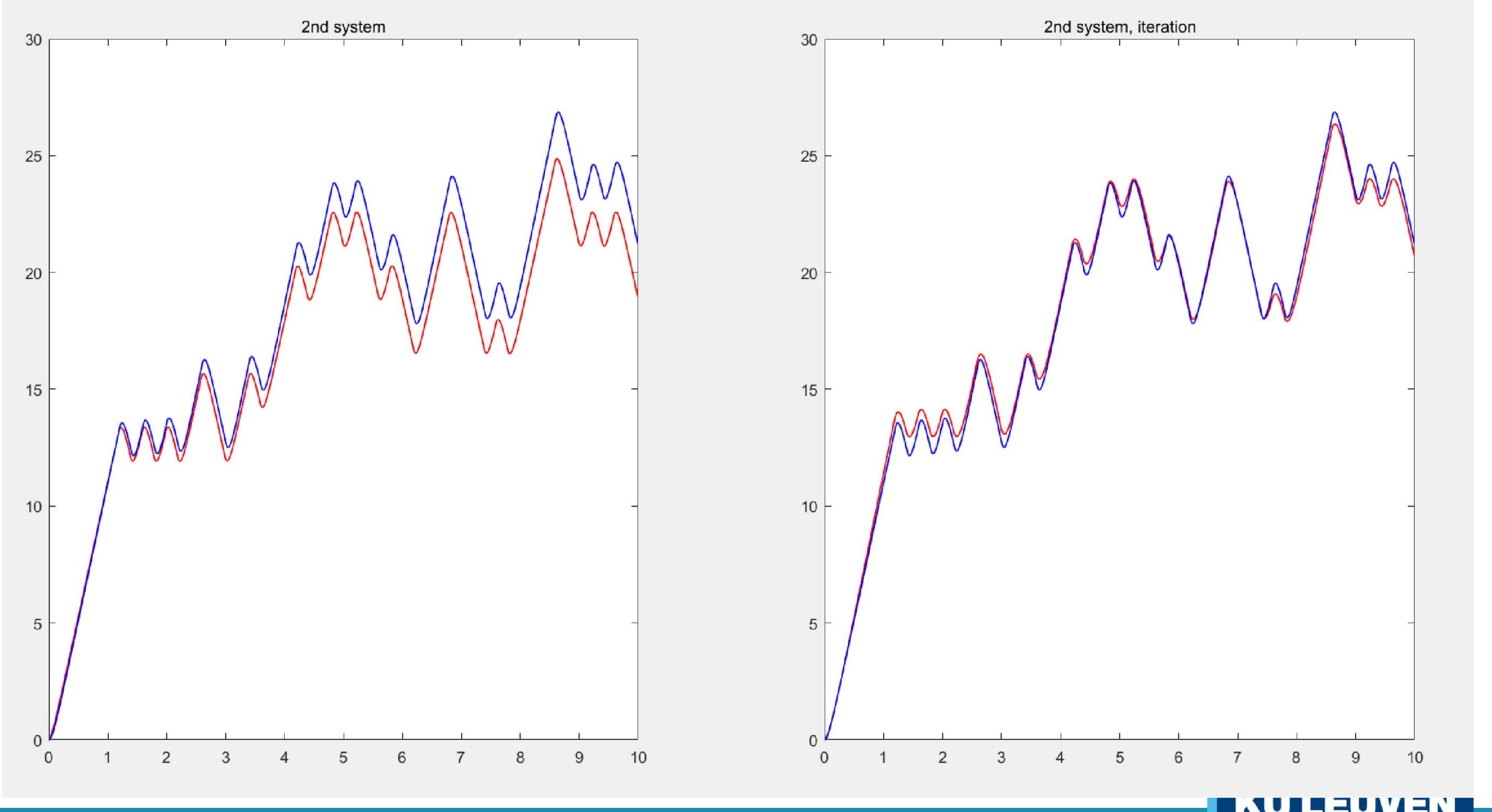
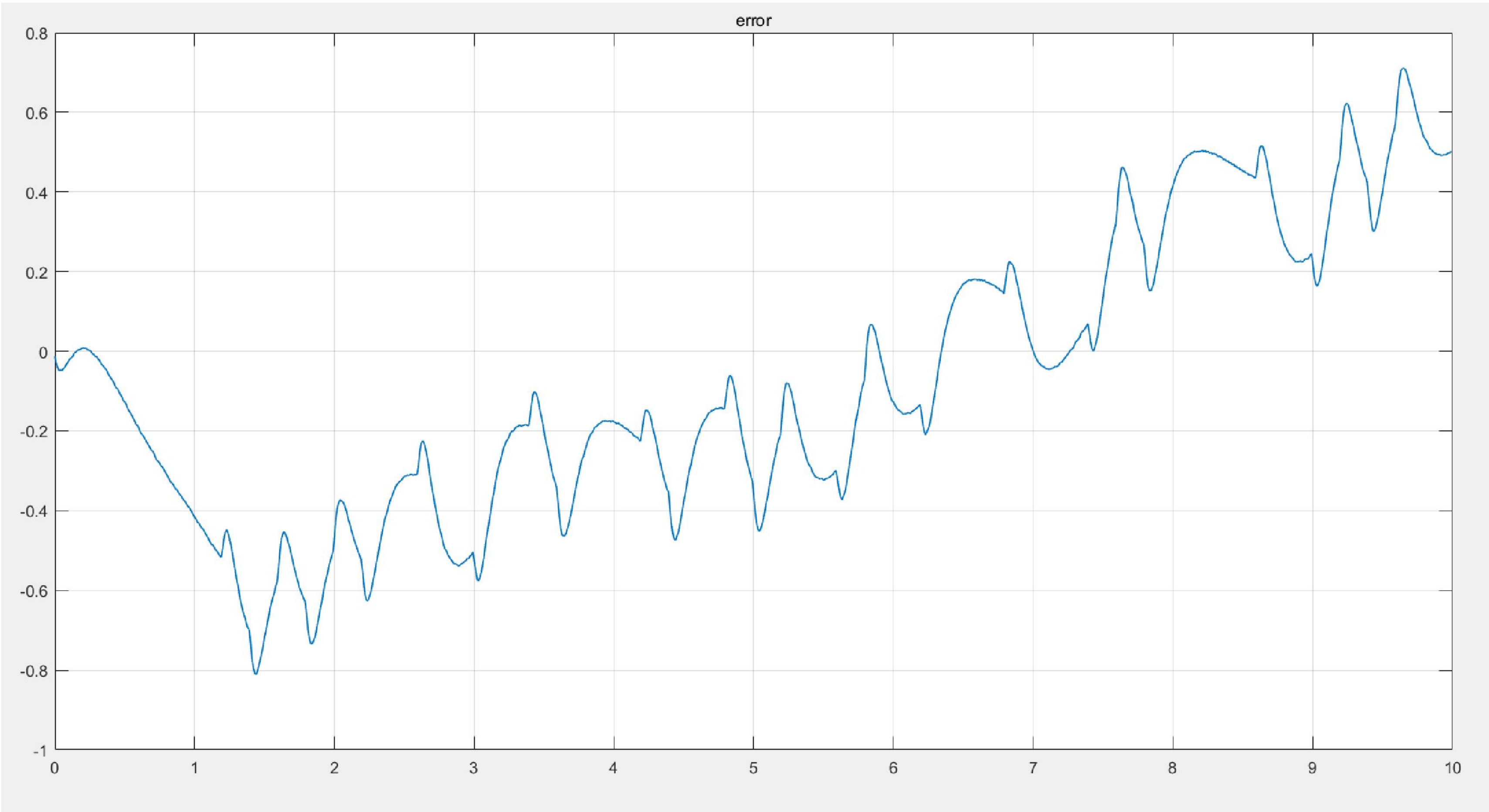
Identification of motors

- Test result and estimated model

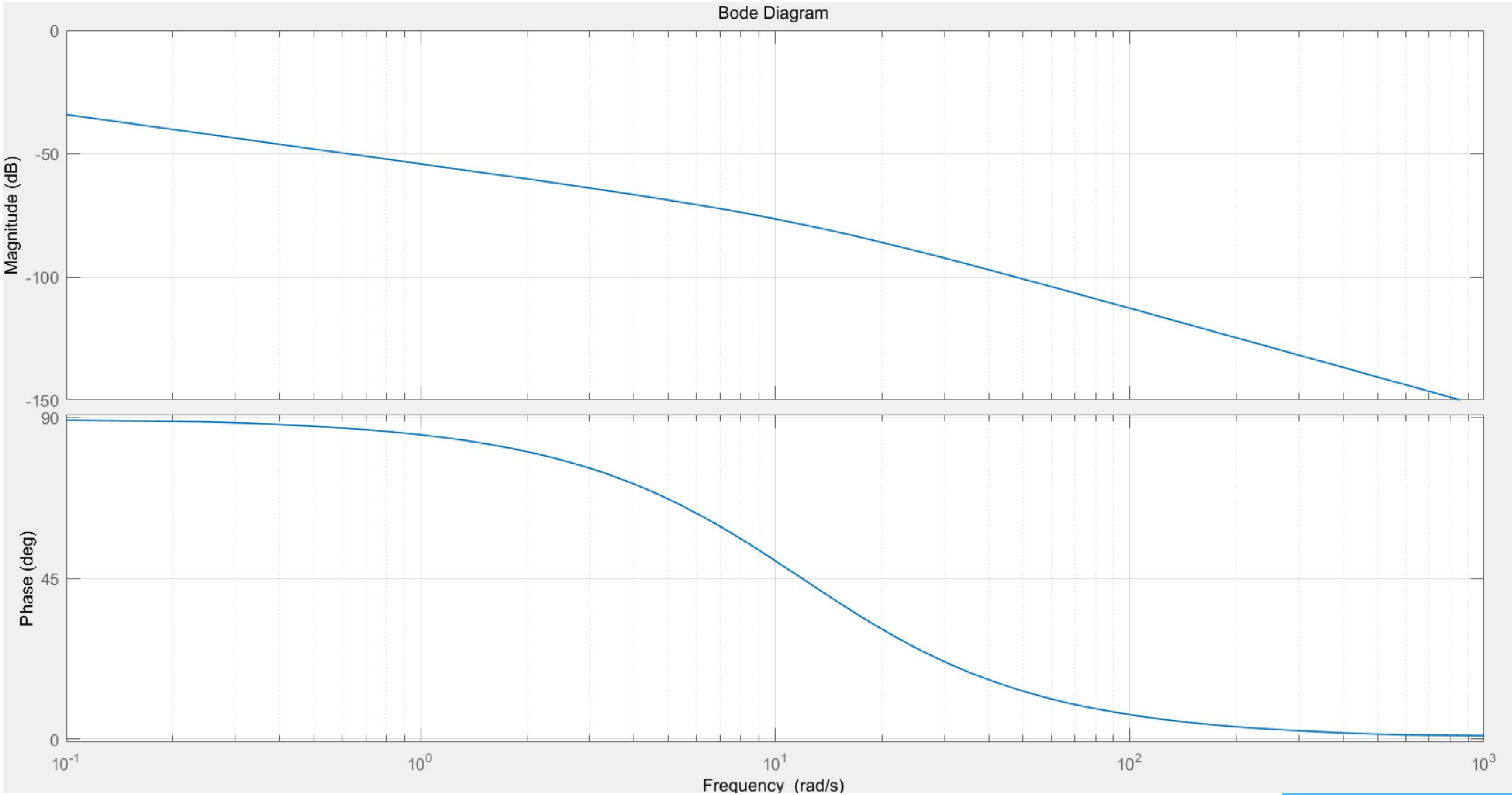


Identification of motors



Identification of motors

- Bode diagram:



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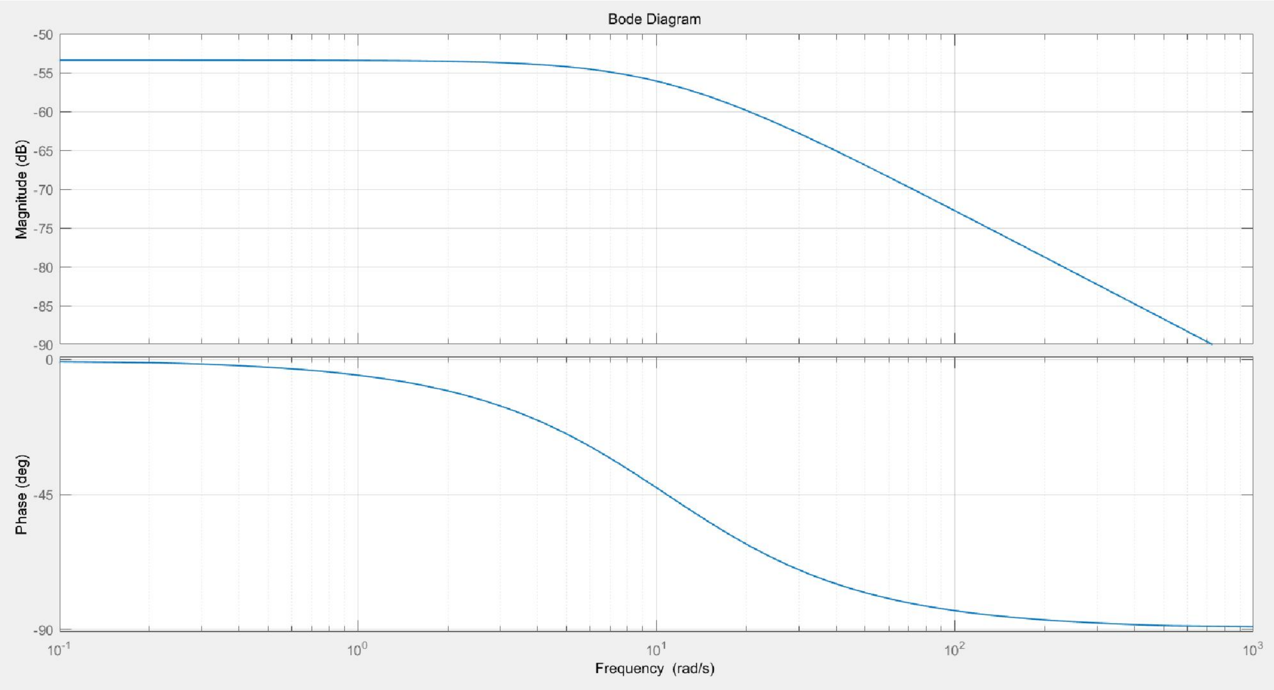
Velocity control – Motor

- The transfer function $\mathrm{H}(s) = \frac{\Omega(s)}{E(s)}$ of motor 2 is:

$$\mathrm{H(s)} = \frac{0.02305}{s+10.79}$$

The Bode diagram is shown on the right.



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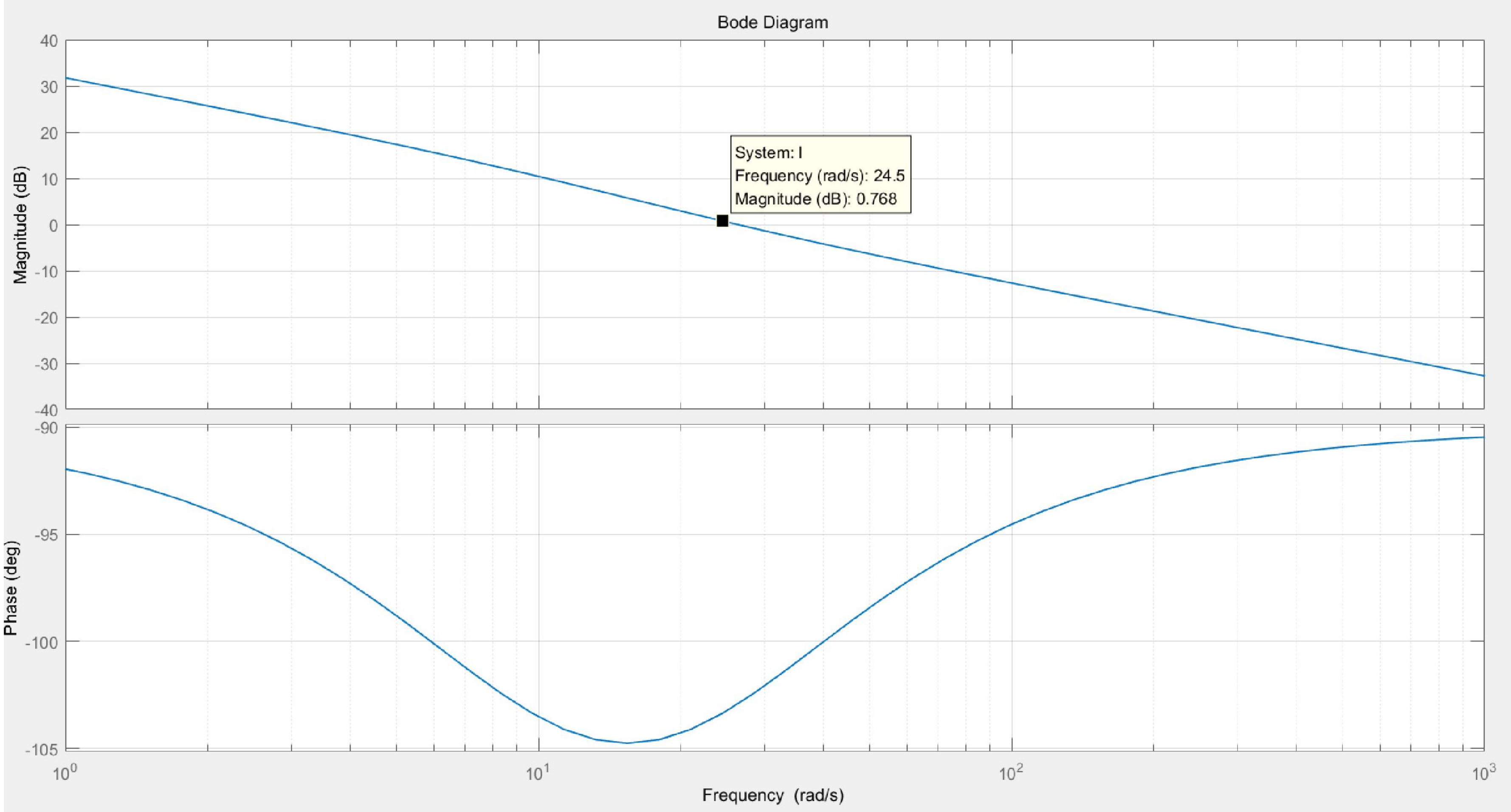
Velocity control – Motor

- PI controller to achieve zero steady-state error
- Controller design of motor 2:
  1. Close-loop bandwidth ω_b is chosen as (sampling frequency/20) = 5Hz (31rad/s) .
  2. CL bandwidth ω_b lies in $[\omega_c, 2\omega_c]$, cross over frequency was set at 20rad/s, which means K = 1000
  3. If we choose $\frac{1}{T_i} \ll \omega_c$, the step response would be very slow. Therefore we choose $\frac{1}{T_i} = \omega_c$. And such a choice results in a smaller PM, about 70, which means a faster response.
  4. Resulting PI controller D(s) $= \frac{1000(s+20)}{s}$

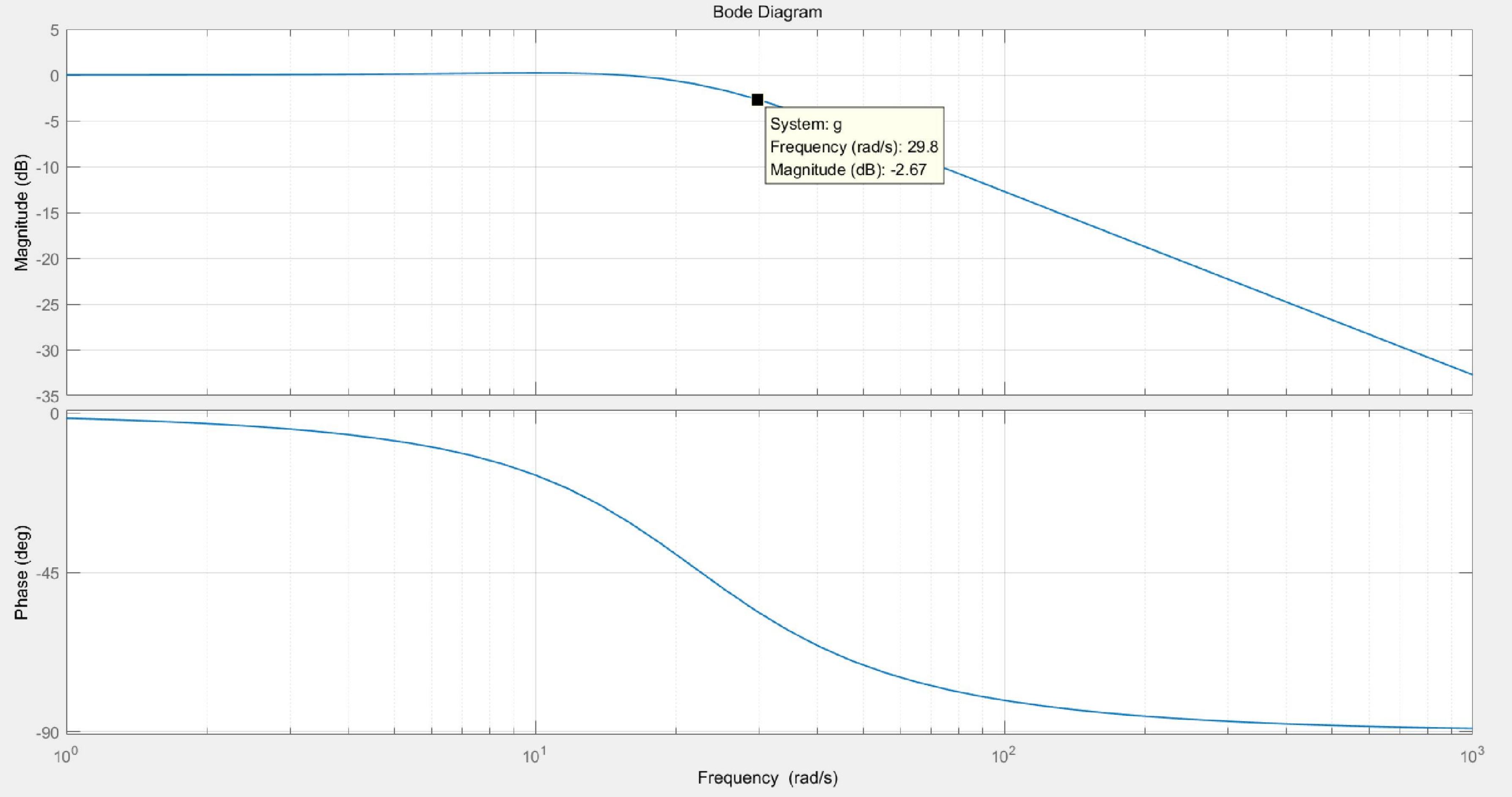
Velocity control - Motor

- Open loop system bode: PM = 77.1 degree, $\omega_c = 25\ rad/s$



Velocity control - Motor

- Closed loop bode $\omega_b = 32\,rad/s$



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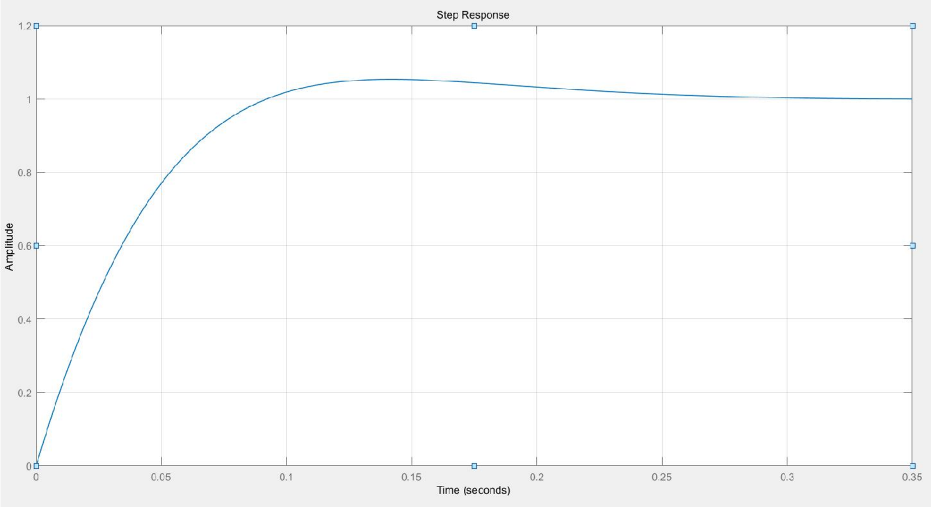
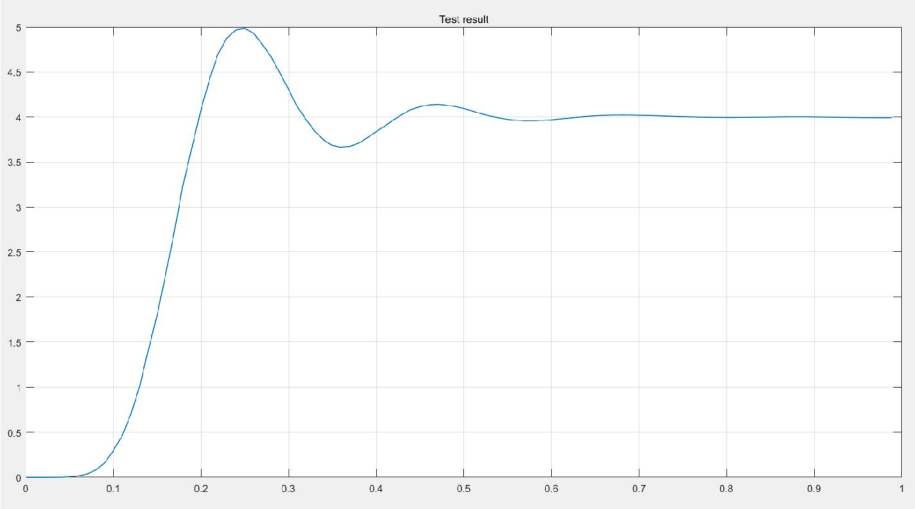
Velocity control - Motor

- Step response:



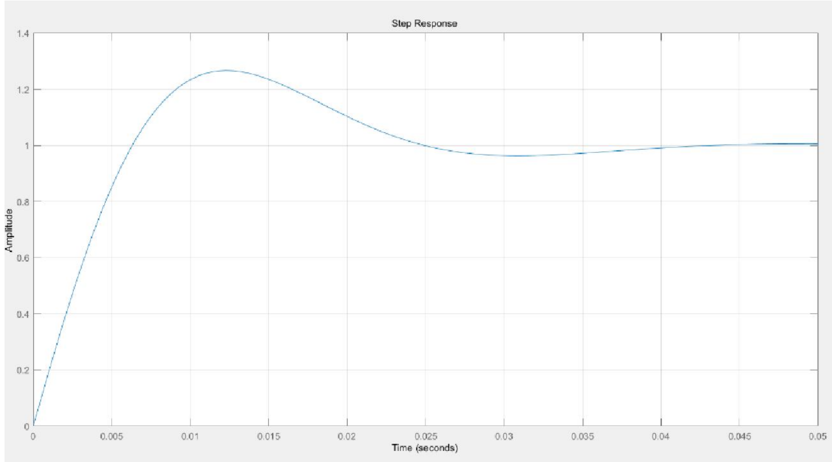


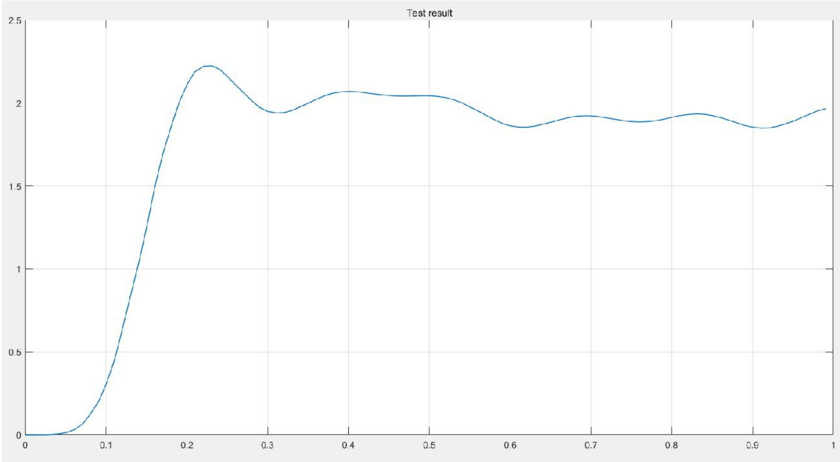
Theoretical step response:
Overshoot: about 5%
Settling time: about 0.3s

Test result:
Overshoot: 25%
Settling time: 0.6s

Velocity control - Motor

- If we try to set close loop bandwidth to 320 rad/s. In theory, we are going to have a faster response but the aliasing will prevent us from getting an acceptable result.





Theoretical response:

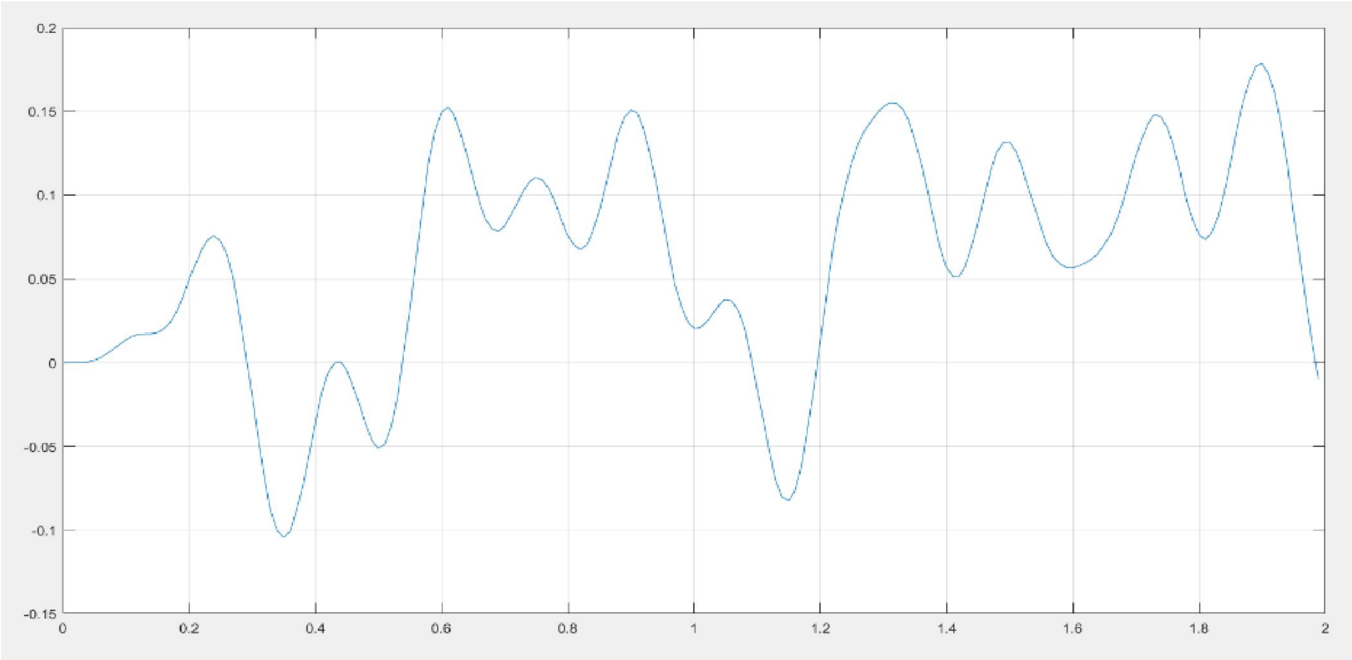
settling time: 0.04s

Test result:

The motor never stops.

Velocity control - Motor

- If we push the close loop bandwidth to a larger value, the result goes wild.



Velocity control - Cart

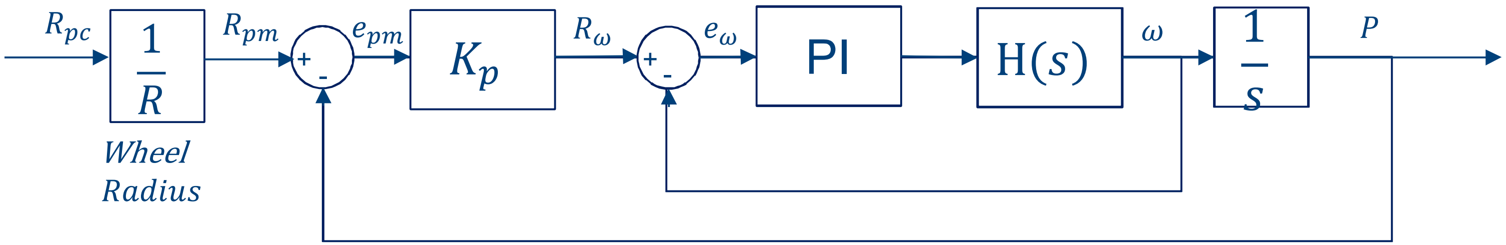
- Through the same procedures, we get the PI controller of motor 1: D(s) = $\frac{1245(s+20)}{s}$

- By multiplying the radius of wheels, we can now control the velocity of cart.

- If we put this cart on a slope, we can apply a constant drag to it. The cart can still follow the reference speed.

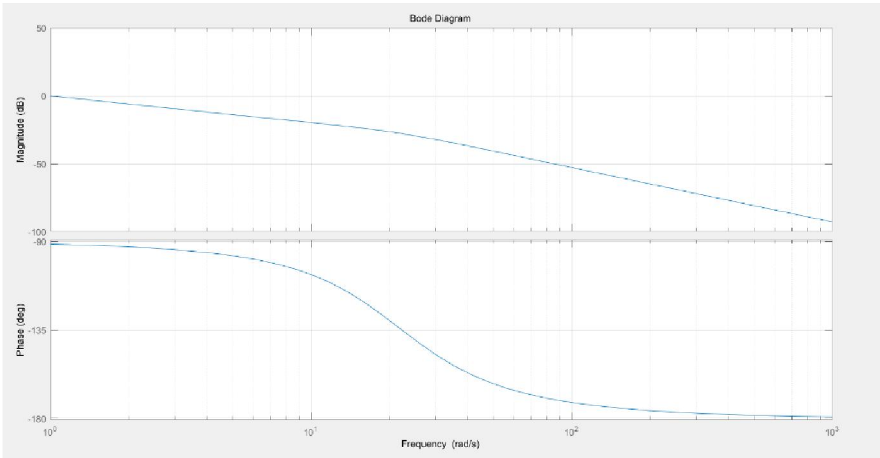
Position Control

- Block diagram



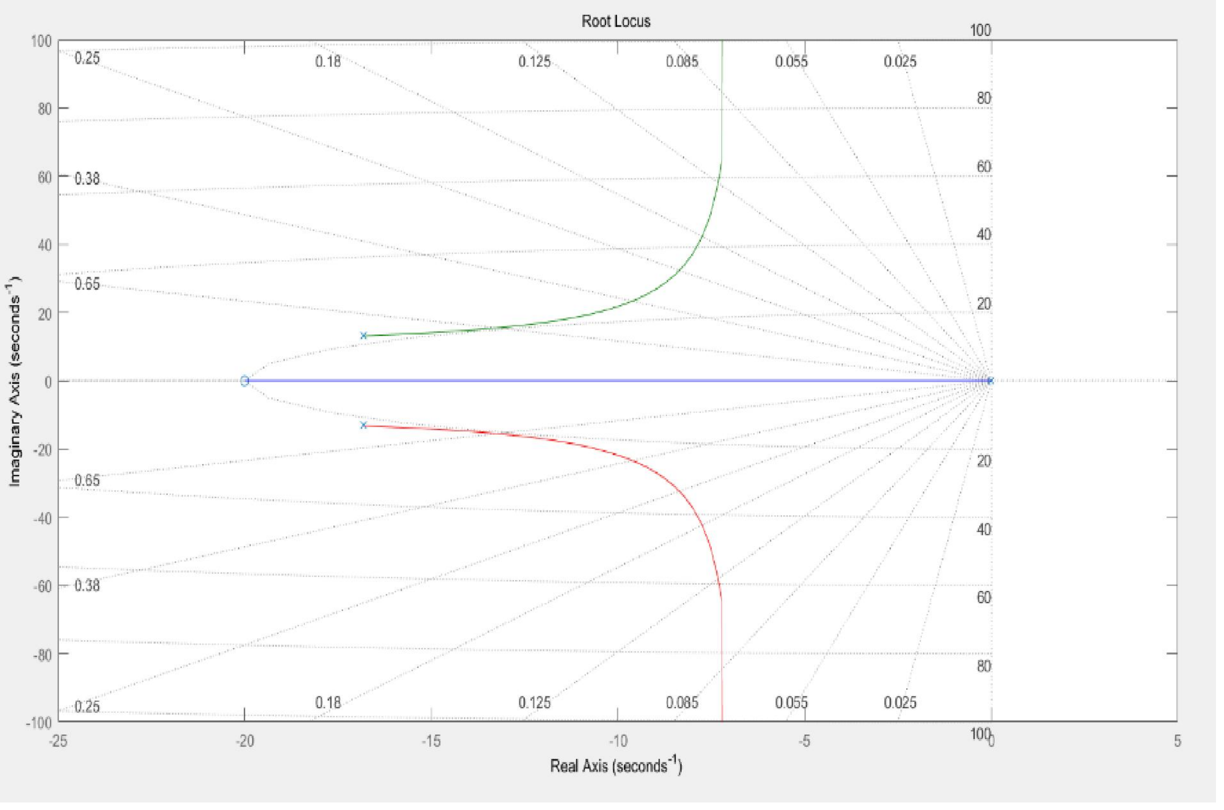
- The close-loop transfer function of velocity control is:

$$Gp = \frac{22.8s + 455.9}{s(s^2 + 33.59s + 455.9)}$$



Position Control

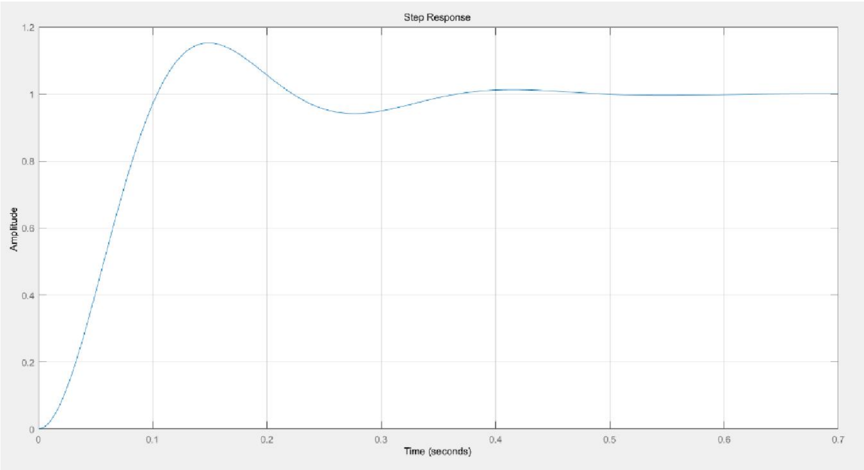
- We can draw the root locus of the system:

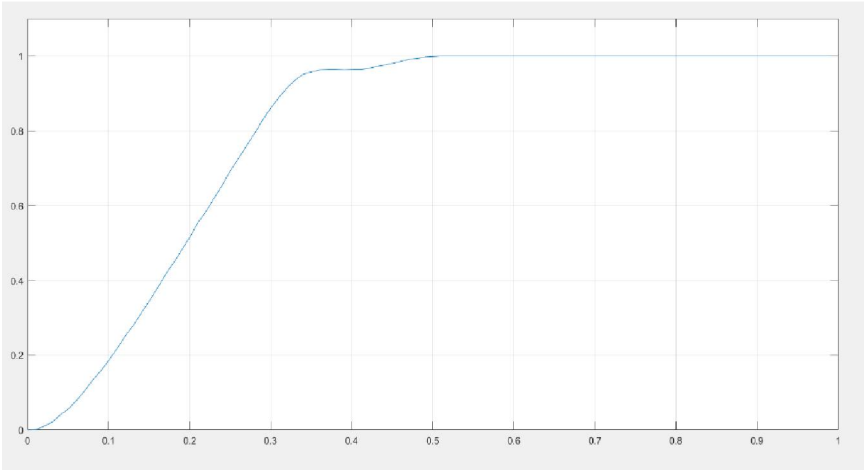


Through some tuning, we choose Kp = 20.

Position controller

The step response is shown below. We can find the test result is slower than the simulation.





Simulation:

settling time: <0.4s

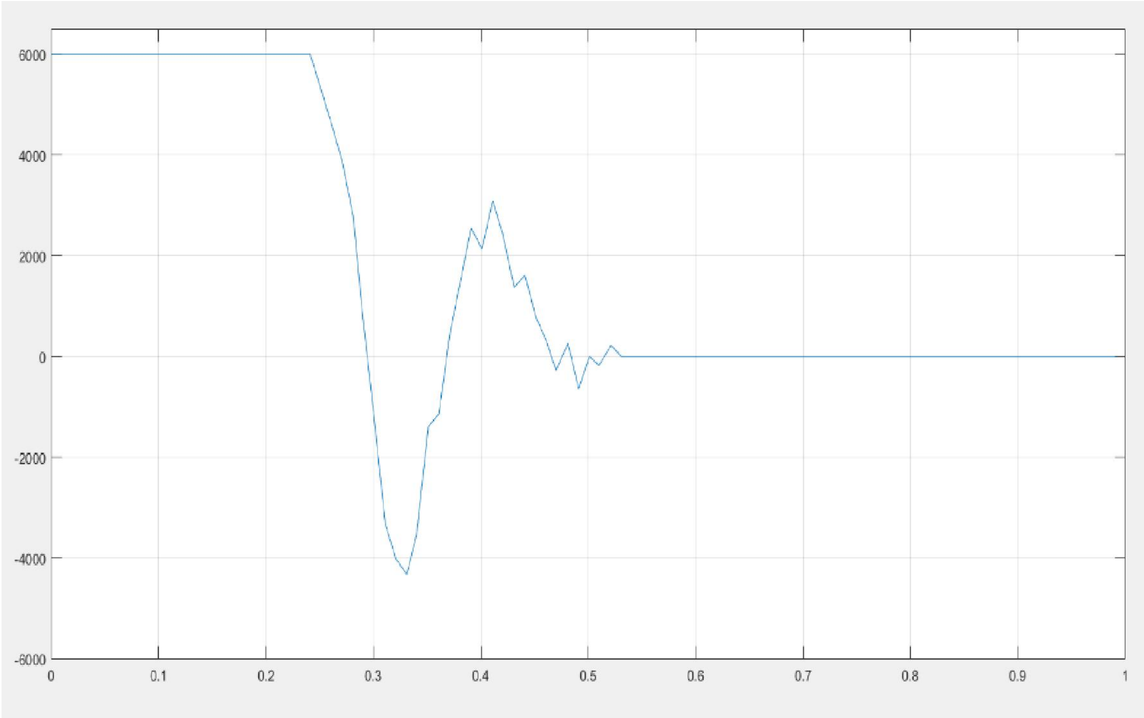
Test result:

settling time: 0.5s

Position control

- The test results is slower than theoretical simulation. This is caused by the saturation of motor. We can see this effect clearly from the input voltage.

- The motor is saturated in the first 0.25 second. Therefore the real result is slower than simulation.



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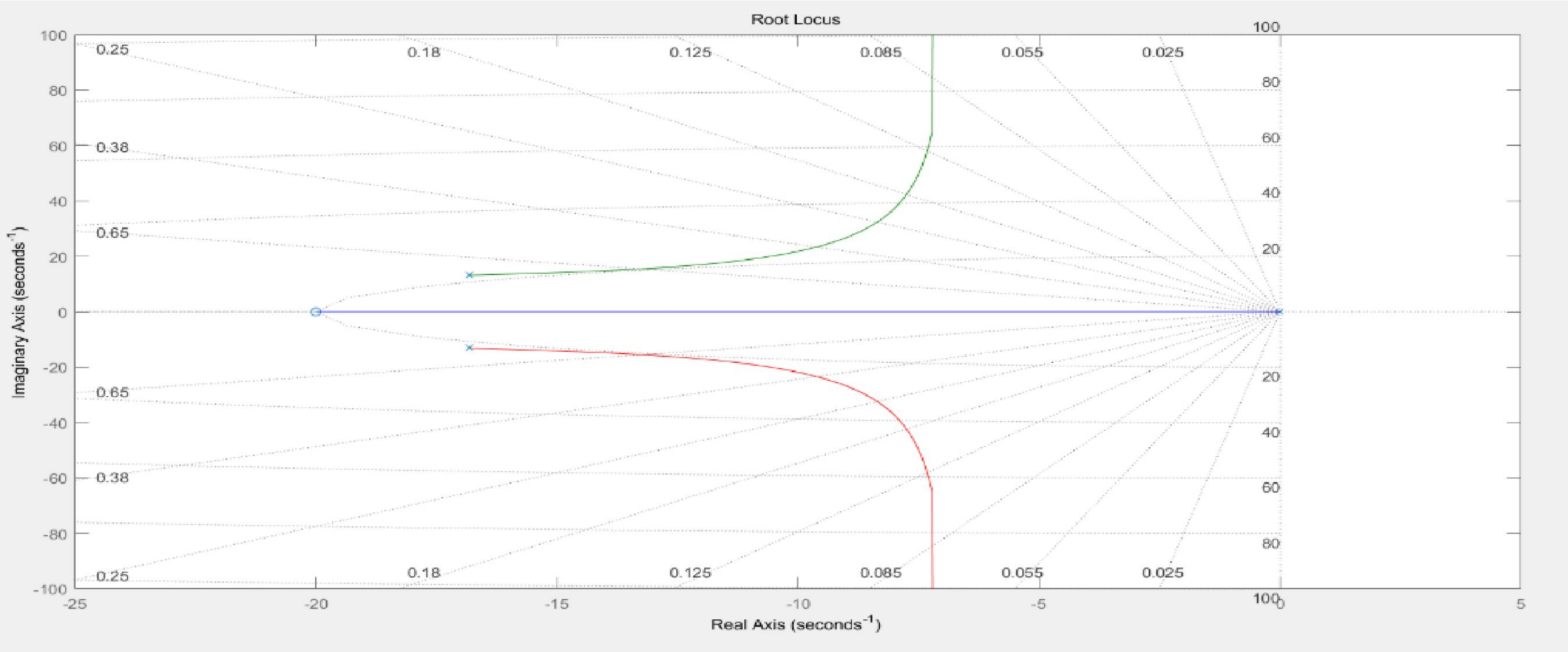
Influence of Non-ideal speed controller

The settling time of velocity controller is not zero. This effect has some influence on the position controller. We can see from the Nyquist diagram:



PI bandwidth = 32 rad/s

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Position controller

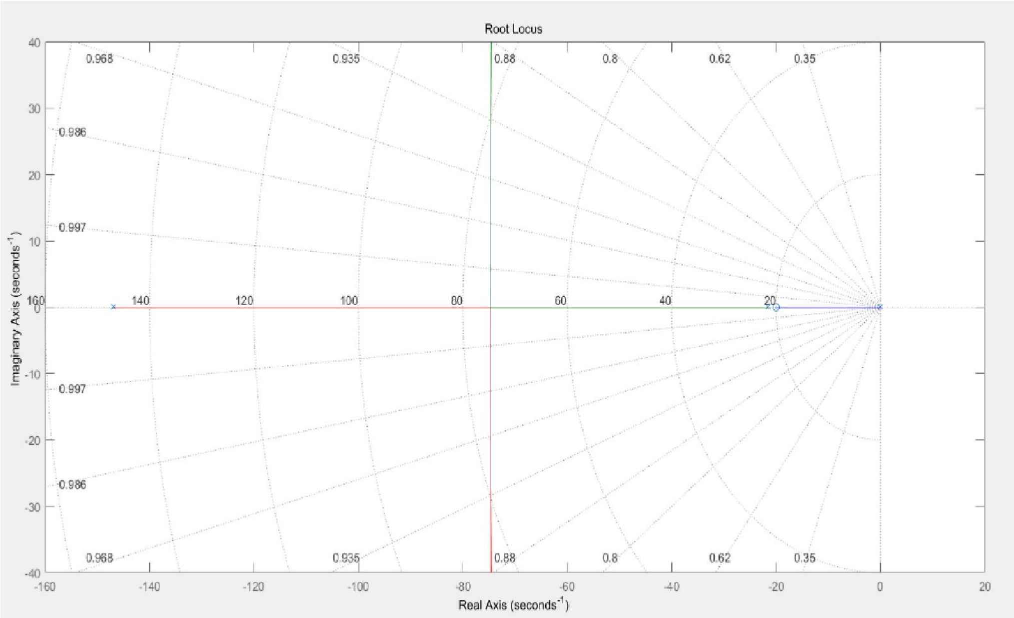
- No matter what Kp we choose, the real part of two conjugate poles, σ, is always larger than -18. And the imaginary parts $\pm\omega_d$ are always larger than 10(or smaller than -10). We can view this effect from two aspects.

- First, the settling time is limited. If we consider this system as a 2nd order system, the settling time $t_s = \frac{4.6}{\sigma}$ can never be smaller than 0.2s.

- Second, the damping ratio can never be 1, which means we always have overshoot.

Position controller

- If we choose the close loop bandwidth of velocity controller as 320 rad/s, which is 10 times larger than the original one, we can get the root locus as follows:



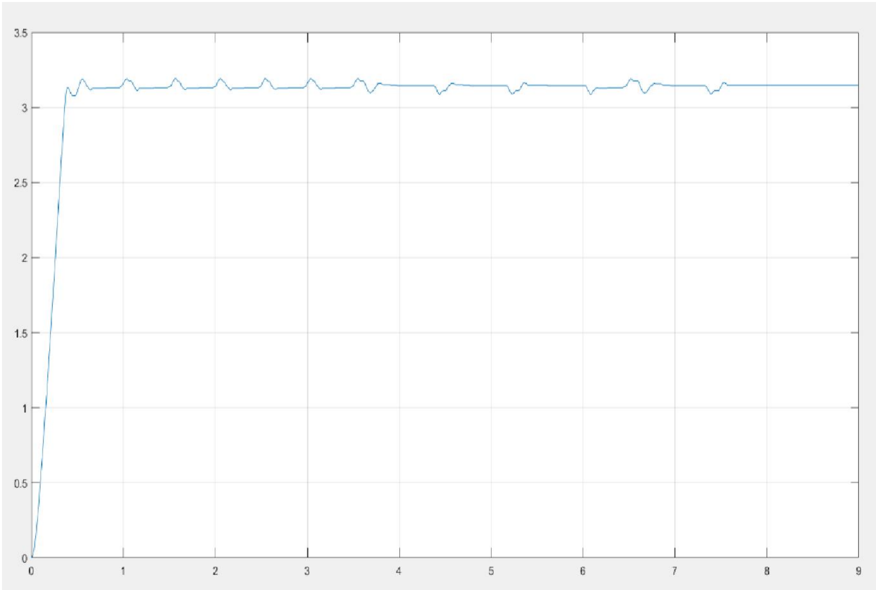
In this case, through some tuning, we can get a very fast system without overshoot.

However, the bandwidth is limited by our sampling frequency.

Non-linear behavior in motor 2

- If we apply the same proportional controller to motor 2, we will get such a result:

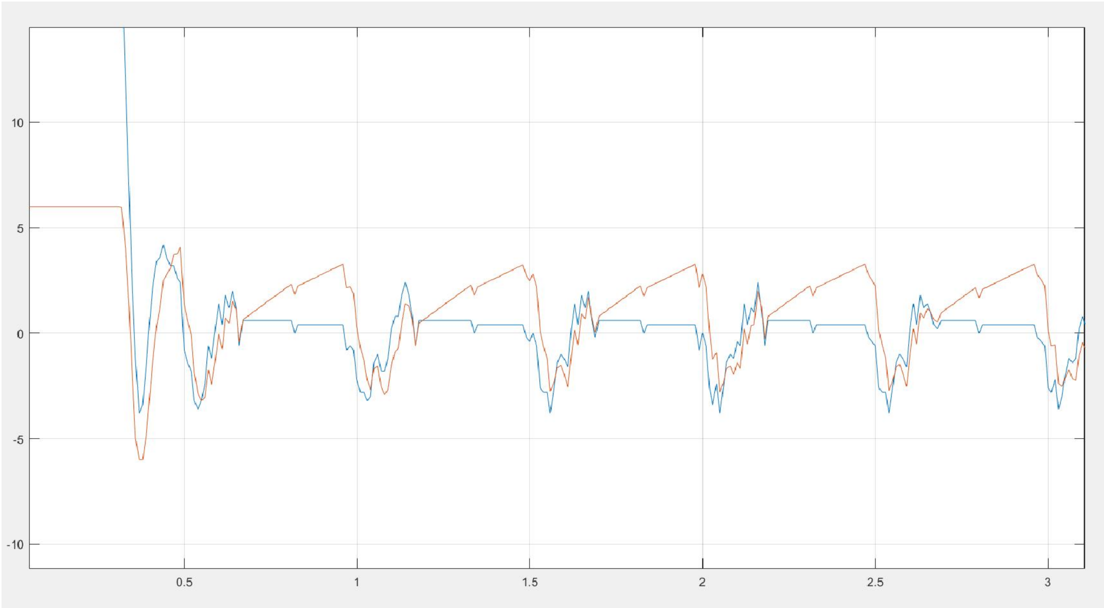
The motor can never stop. It might be a non-linear phenomenon, called stick slip which usually happens in low speed. Check the reference [2] for more details.



Non-linear behavior in motor 2

- We see it more clearly from the error.

We can see that the controller tries to drive the motor to reference position but the dry friction prevents motor2 from moving. Energy raises until the voltage is big enough. However, this causes the overshoot and the system goes back to phase 1. By the way, we can feel this effect when we turn the wheel of motor 2 by hand…



The blue line is error of velocity and the red line is voltage in volts

Non-linear behavior in motor 2

- By using a PI controller, we can solve this problem.

- We don't know the model of stick slip, so we choose Ki = 10 through tuning.

- To prevent the integral windup, we enable the integrator only after the error is smaller than 0.1 rad:

```
if(ep2<0.1&&ep2>-0.1)
{
   integral += ep2;
   rv2 += Ki*integral;
 }
```

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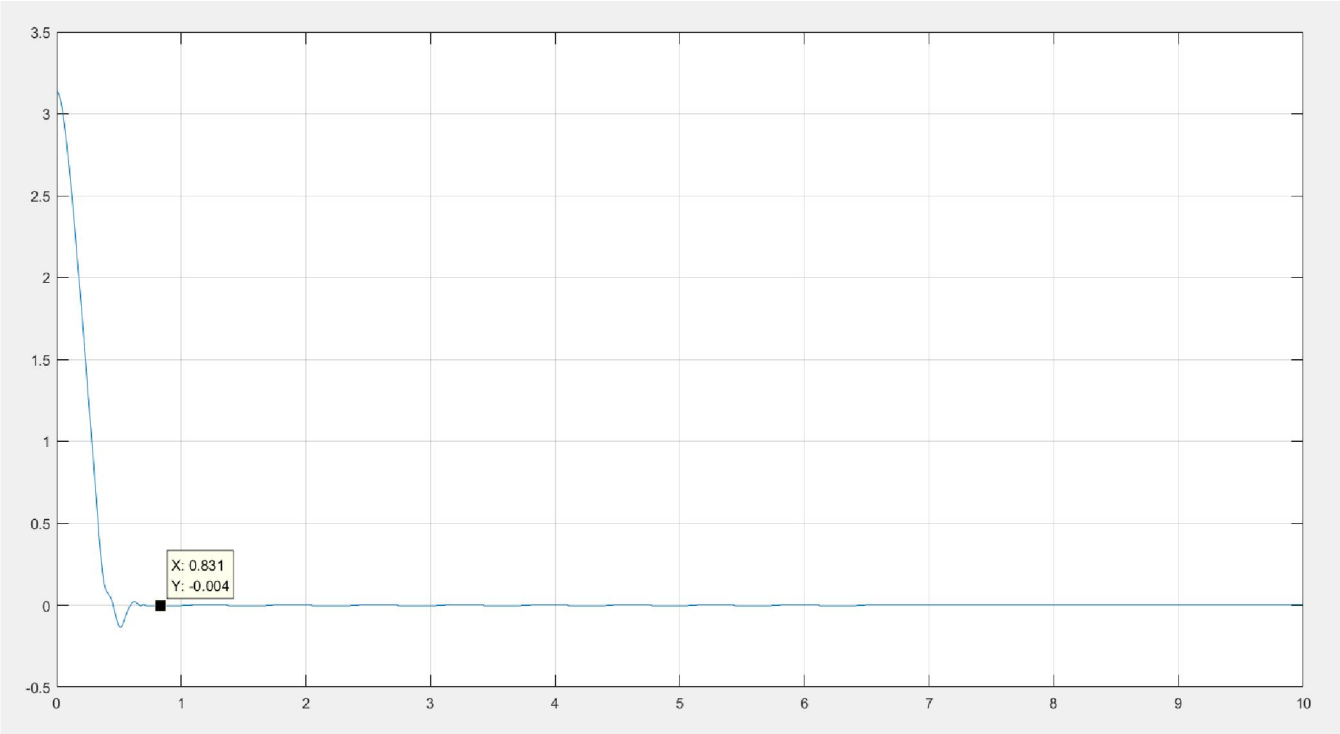
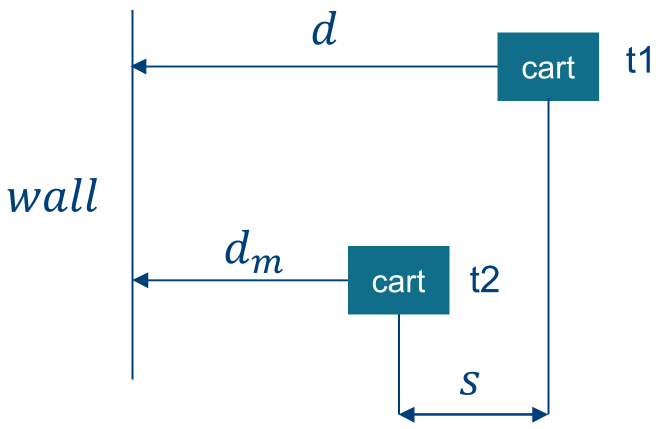
Non-linear behavior in motor 2

- The test result, the position error, is shown below. There is a small oscillation in our response with a magnitude of 0.004 rad. However, the resolution of our encoder is 0.004 rad which means such a small oscillation is acceptable.



Sensor fusion

- Measurement equations



At t1, encoder reads zero, distance to the wall is d, at t2, encoder reads m, distance to the wall is dm, which is the reading from distance sensor.

$$d_m = d - s$$
$$\theta_m = \frac{s}{R}$$

R = 0.0325m is the wheel radius

- State space

$$\begin{bmatrix} d \\ s \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix}_k + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} V_k \qquad \begin{bmatrix} d_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix} + [0] V_k$$

Sensor fusion

- We choose [0.9048, 0.9048] as the poles of estimator and we have L = $\begin{bmatrix} 0.0952 & 0.0031 \\ 0 & 0.0031 \end{bmatrix}$

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Inverted Pendulum

- Theoretical nonlinear model

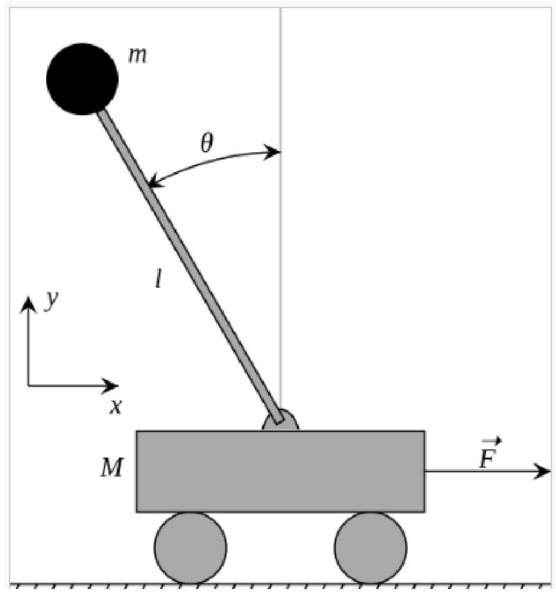
$$l\ddot{\theta} - gsin\theta = \dot{V}cos\theta$$

  - Input: velocity of the cart
  - Output: Pendulum angle
- Linearized model
  - Stable position $l\ddot{\theta} + g\theta = \dot{V}$

$$\frac{\theta(s)}{V(s)} = \frac{s}{ls^2+g} \quad \text{poles: } \pm\sqrt{\frac{g}{l}}i$$

  - Unstable position $l\ddot{\theta} - g\theta = \dot{V}$

$$\frac{\theta(s)}{V(s)} = \frac{s}{ls^2-g} \quad \text{poles : } \pm\sqrt{\frac{g}{l}}$$



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Inverted Pendulum

- State space model of $l\ddot{\theta} - g\theta = \dot{v}$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} - \frac{\dot{v}}{l} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{l} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix} + \begin{bmatrix} \frac{1}{l} \\ 0 \\ 1 \end{bmatrix} v$$

$$\begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix} + [0]v$$

Besides the dynamics of the pendulum, a third state, the position of cart is used. We can let the cart stay at the original position through this variable.

Inverted Pendulum

- In our measurement, $\frac{1}{\omega} = 7.27$
- Discrete model:
- $$\begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix}_{K+1} = \begin{bmatrix} 1.0034 & 0.01 & 0 \\ 0.6847 & 1.0034 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix}_{K} + \begin{bmatrix} 7.41 \\ 0 \\ 1 \end{bmatrix} v_K$$
- $$\begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix}_{K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix}_{K} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} v_K$$

Inverted Pendulum

- Controller design

  Place the poles of system at [-3, -3.1, -2.9], and the discrete poles are : [0.9704, 0.9695, 0.9714].

  The corresponding K = [1.2746, 0.1861, -0.3768]

- Estimator design

  Because of the sensor is noisy, we choose slower poles for estimator pe = [-9, -9.1, -8.9].

$$L = \begin{bmatrix} 0.0895 & 0.01 & 0 \\ 0.6847 & 0.0904 & 0 \\ 0 & 0 & 0.0852 \end{bmatrix}$$

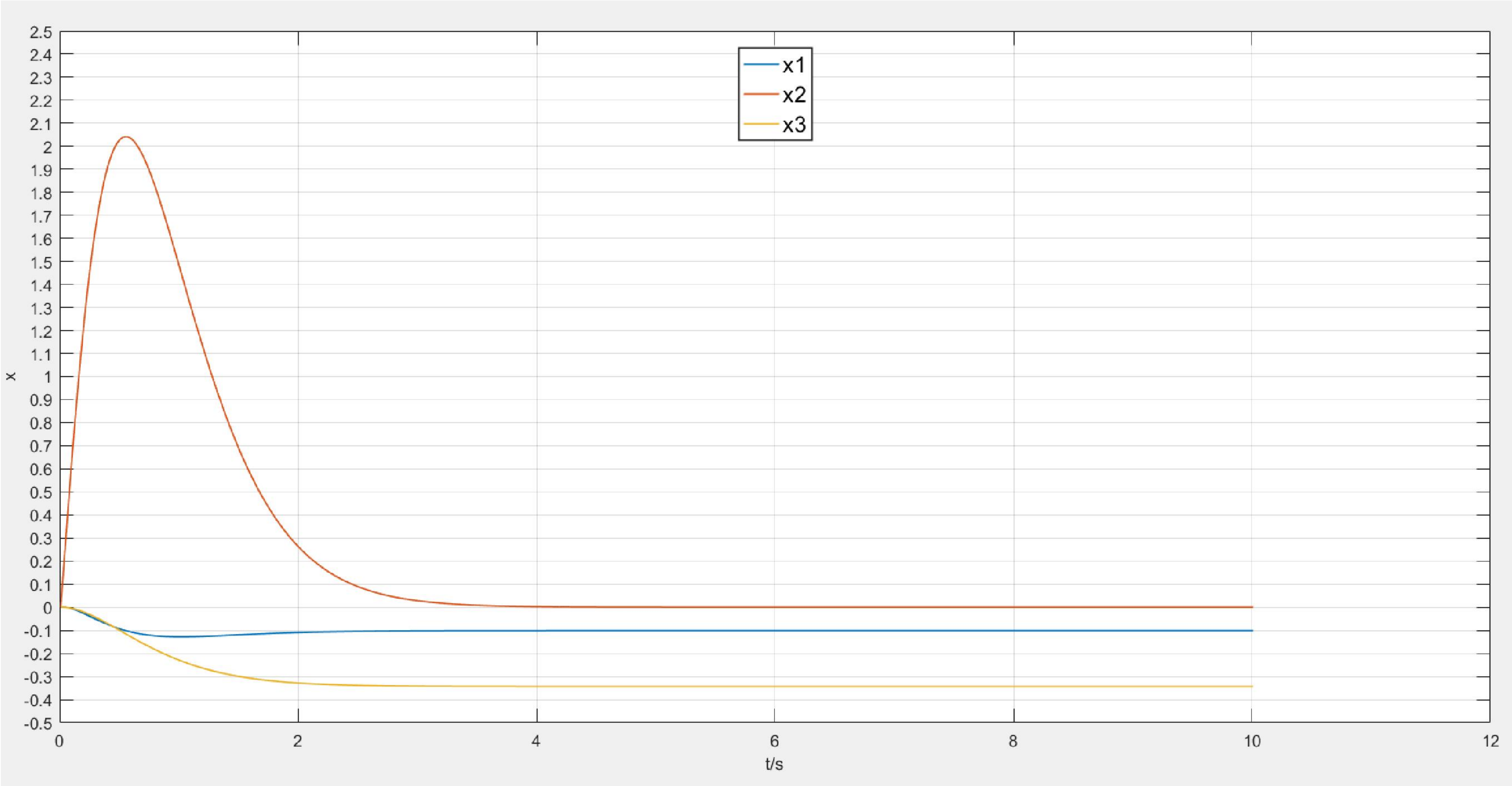
Inverted Pendulum

- If we choose a larger bandwidth, the system will response faster. But if the bandwidth is too large, the pendulum can’t remain stable. Instead, it will oscillate at beginning and fall down.

Inverted Pendulum

- Imperfect angle calibration
- $\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} - \frac{\dot{v}}{l} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{l} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta + \Delta\theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix} + \begin{bmatrix} \frac{1}{l} \\ 0 \\ 1 \end{bmatrix} v$ *it equals to:*
- $\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} - \frac{\dot{v}}{l} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{l} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} - \frac{v}{l} \\ x \end{bmatrix} + \begin{bmatrix} \frac{1}{l} & 0 \\ 0 & \frac{g}{l} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ \Delta\theta \end{bmatrix}$ *and K becomes:*
- $K' = \begin{bmatrix} K & 0 \\ \mathbf{0} & 1 \end{bmatrix}$
- *Let* $\Delta\theta = 0.1$*, the simulation is shown in next page.*

Inverted Pendulum



Inverted pendulum

- From the simulation, we find the pendulum can remain stable even if the calibration is not perfect. This is true if $\Delta\theta$ is very small. But for a larger $\Delta\theta$, say, 0.1rad, it can't be stable in practice.

- Integral control

  We have 3 additional state variables.

  Let p = [-3,-3.1,-3.2,-2.9, -2.8, -2.7];

  We tried to use place($\begin{bmatrix} \mathbf{0} & C \\ \mathbf{0} & A \end{bmatrix}, \begin{bmatrix} \mathbf{0} \\ B \end{bmatrix}$, p) in Matlab, but it is nearly uncontrollable…

Reference

- [1] Bature, A. A., Mustapha Muhammad, and Auwalu M. Abdullahi. "Identification and real time control of a dc motor." *IOSR J. Electr. Electron. Eng* 7.4 (2013): 54-58.
- [2] Wikipedia, https://en.wikipedia.org/wiki/Stick-slip_phenomenon

Thank you!

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