

# Type Systems in Programming Languages

1

## Typing in Programming Languages

- **Typing:** a mechanism for grouping values into types
  - ❖ Describe data effectively
  - ❖ Prevent nonsensical operations, *e.g.* multiply a string by a set
- **Components of this mechanism**
  - ❖ **Associating a type with every value:** values are grouped into types, and values of the same type exhibit similar behavior under similar operations
    - ◆ In complex type systems, certain values can be associated with more than one type.
    - ◆ Often, if there is more than one type associated with a value, there is an infinite number of types associated with this value.
  - ❖ **Checking rules:** Given a value and an operation, use the type of the value to check whether this operation is allowed on values of this type
    - This becomes more difficult if there are several types associated with the same value
  - ❖ **Inference rules:** Given the type of operands, determine:
    - ◆ the meaning of an operation
    - ◆ the type of the result

2

## Significance of Type

### ■ Type determines meaning (semantics):

What will be executed as a result of the following expression?

$a + b$

- ❖ Integer addition, if  $a$  and  $b$  are integers, or
- ❖ Floating point addition, if  $a$  and  $b$  are of a floating point type, or
- ❖ Conversion to a common type and then addition, if  $a$  and  $b$  are of different types.

### ■ Type determines what's allowed (semantically):

Is the following expression legal?

$X[i]$

- ❖ Yes, if  $X$  is an array type and  $i$  is of a discrete type.
  - ◆ In C, the above is legal also when  $x$  is a pointer (and  $i$  is of a discrete type).
- ❖ No, e.g. if  $X$  is a real number and  $i$  is a function.

## Classifying Type Systems

### ■ Existence

- ❖ Does the language include a type system at all?

### ■ Type equivalence and subtyping

- ❖ When can one type replace another?
- ❖ Structural/name/declaration type equivalence

### ■ Strength

- ❖ How strictly are the typing rules enforced?

### ■ Time of checking

- ❖ At what stage is type checking performed?
- ❖ Static vs. dynamic typing

### ■ Responsibility

- ❖ Is the programmer responsible for type declarations or is it the compiler?
- ❖ Explicit/implicit typing

### ■ Flexibility

- ❖ To what extent does the type system restrict the user's expressiveness?
- ❖ Polymorphic typing

## Existence of a Type System

A language can be

- **Typed:** The set of values can be broken into groups, with more or less uniform behavior under the same operation of values in each group.
  - ❖ C, Pascal, ML, Ada, Java, and most other programming languages
- **Untyped:** Each value has its own unique set of permissible operations, and their semantics are particular to the value.
  - ❖ **Lisp** (list processing) :
    - ◆ All values are S-expressions, which are not much more than binary trees.
    - ◆ Elementary operations are: choosing the right and left subtrees of a tree, and combinations and inverse of these operations.
    - ◆ Legality of operations is determined by tree structure and values
  - ❖ **Mathematica** (a language for symbolic mathematics):
    - ◆ Values are mathematical expressions (represented as S-expressions)
    - ◆ Legality and semantics of a manipulation of an expression is determined by the structure and semantics of the expression.
- **Degenerate:** The set of values is so simple that it admits only one type, or a small number of types; appears in most scripting languages
  - ❖ **BCPL (C's ancestor):** the only data type is a machine word
  - ❖ **DOS Batch Language:** the only data type is a string
  - ❖ **C-shell, Bourne-shell, AWK, REXX:** two data types - strings and numbers - with little distinction between them; minimal support for arrays.
  - ❖ **Perl:** Several extensions to the type system beyond AWK.

5

## Type Equivalence and Subtyping

6

## Type Equivalence

- Suppose that an operation expects an operand of type  $T$ , but receives an operand of type  $T'$ . **Is this an error?**
  - ❖ No, if  $T'$  is a **subtype** of  $T$
- Two types that are subtypes of each other are called **equivalent**
- **Caveat:** because the notion of subtype is more refined than the notion of type equivalence, we will find languages where type equivalence is defined but subtyping is not
- Kinds of type equivalence
  - ❖ Structural equivalence
  - ❖ Name equivalence
  - ❖ Declaration equivalence

## Structural Equivalence

- Have the “same” values.... In Algol-68:
  - ❖ If  $T$  and  $T'$  are both primitive, then  $T \cong T'$  only if they are identical.
  - ❖ Else if
    - ◆  $T = A \times B$ ,  $T' = A' \times B'$ , or
    - ◆  $T = A + B$ ,  $T' = A' + B'$ , or  $T = A + B$ ,  $T' = B' + A'$ , or
    - ◆  $T = A \rightarrow B$ ,  $T' = A' \rightarrow B'$
 and  $A \cong A'$ ,  $B \cong B' \Rightarrow T \cong T'$
  - ❖ Otherwise?
- **Recursive types:**
  - ❖  $T = \text{Unit} + c(A \times T)$
  - ❖  $S = \text{Unit} + c(A \times S)$
  - ❖  $R = \text{Unit} + c(A \times S)$
  - ❖ Intuitively  $T$ ,  $S$  and even  $R$  are structurally equivalent, but structural equivalence is not easy to define and test for in recursive types.
- **Points of disagreement between languages:**
  - ❖ Do records require field name identity to be structurally equivalent?
  - ❖ Do arrays require index range identity to be structurally equivalent?

## An example for (hypothetical) structural equivalence

```
EmployeeData = record
  id: Integer;
  name: String;
  isManager: Boolean;
  ranking: array [1..3] of Character;
end;
```

```
YearlyEmployee = record
  d: EmployeeData;
  salary: Real;
end;
```

```
Employee1 = record
  d: EmployeeData;
  case Integer of
    0: yearly_salary: Real;
    1: hourly_rate: Integer;
  end;
end;
```

```
HourlyEmployee = record
  d: EmployeeData;
  salary: Integer;
end;
```

```
Employee2 = record
  case Integer of
    0: y: YearlyEmployee;
    1: h: HourlyEmployee;
  end;
end;
```

9

## Name Equivalence

- $T \cong T'$  iff  $T$  and  $T'$  were defined in the same place (original Pascal and Ada):

```
TYPE  T1 = File of Integer;
      T2 = File of Integer;
VAR   f1: T1;
      f2: T2;
Procedure p(Var f: T1);
....
p(f1); (* ok *)
p(f2); (* compile-time error *)
```

## Name equivalence across programs

```
program p1(f)
type T = file of Integer;
var f: T;
begin
  ...
  write(f, ...);
  ...
end;
```

```
program p2(f)
type T = file of Integer;
var f: T;
begin
  ...
  read(f, ...); (* Type error *)
  ...
end;
```

- By **definition** of Pascal, it follows that
  - ❖ two Pascal programs cannot communicate legally through files using any user-defined type, hence the type error above.
  - ❖ type `Text`, which is the only predefined type for files in Pascal, allows communication between programs.
- In practice, most **implementations** of Pascal do not type check files. Thus, these implementations *subvert* Pascal's type safety, but allow reasonable interfacing.

## Declaration Equivalence

- A later Pascal standard (ANSI 1983):  
 $T \cong T'$  only if  $T$  and  $T'$  have the same declaration

```
TYPE  T1 = File of Integer;
      T2 = File of Integer;
VAR   f1: T1;
      f2: T2;
Procedure p(Var f: T1);
  ....
p(f1); (* ok *)
p(f2); (* ok *)
```

## Subtyping in Pascal

- A weaker notion than type equivalence: an operation that expects an operand of type  $T$  but receives an operand of type  $T'$  is legal also if  $T$  and  $T'$  are not equivalent, but  $T'$  is a *subtype* of  $T$ .
- **Subtyping in Pascal** - only in one case:
  - ❖ if  $T=[a..b]$  and  $T'=[c..d]$  then  $T'$  is a subtype of  $T$  iff  $T'$  is a subrange of  $T$ .
- **Limitation:** there is no way to override this definition.  
For instance:

```
type
    age = 0..120;
    height = 0..250;
```

age will be a subtype of height even though we don't want to confuse the two notions.

13

## Derived Types in Ada

```
type
    age = derived integer range 0..120;
    height = derived integer range 0..250;
```

- age is not equivalent to 0..120
- height is not equivalent to 0..250
- Therefore: age is not a subtype of height
- **In Modula-3:** values of derived types are *branded*, so that storage in external files will not create communication problems between programs.

14

## Subtypes in ADA

```
type
  age = derived integer range 0..120;
  height = derived integer range 0..250;
subtype child_height is height range 0..120;
```

- age and child\_height not equivalent to 0..120
- height is not equivalent to 0..250
- Therefore: age is not a subtype of height
- But child\_height is!
- A *convert* function that expects a height in cm and converts it to inches, can be used with a variable of type child\_height, but not one of type age.

15

## More about Subtyping in ADA

- Subtyping of ordered primitive types - using subranges:

```
subtype Probability is Float range
0.0..1.0;
```

- Subtyping of array types: using subranges as well:

```
subtype String is array (Integer range
<>) of Character;
subtype String5 is String (1..5);
```

- Subtyping of record types:

```
type Sex is (f,m);
type Person (gender: Sex) is record ...
end record;
subtype Female is Person (gender => f);
```

16



## Branding

- Sometimes it is not desirable to define two instances of the same type as type equivalent.
  - ❖ **Example:** a string describing a person's name and another string describing his or her address are not type equivalent.
- To distinguish two values of the same type we can **brand** these values by adding a tag to it.
- Simple technique for branding – labeled fields:

```
User = Record
  name: String;
  address: String;
end
```

- ❖ In this example, we brand the string types: Address\_Type and Name\_Type
- Branding is similar to **typedef**'s in C. However, **typedef** merely gives an alias to an existing type and does not define a new type.
- In Modula-3, branding is performed automatically for values of derived types, so their type is maintained also when stored by one program and read by another program.

17

## Subtyping as Subclassing

Consider the *class* `Collection` that allows the following operations:

```
insert (what: integer)
is_member (what: integer) : boolean
remove (what: integer) : boolean
write ()
```

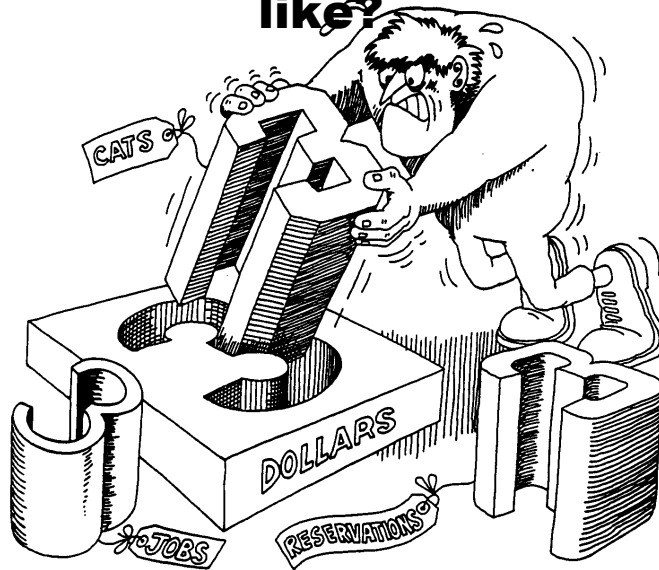
A *class* `Queue` that allows the same operations plus the following:

```
remove_first () : integer or Unit
```

would be a **subclass** of `Collection`.

18

## Strong Typing -- What does it look like?



Strong typing prevents mixing abstractions.

## Strength of Typing Systems

- **Strongly typed languages:** ML, Eiffel, Modula, Java
  - ❖ It is impossible to break the association of a value with a type from within the framework of the language.
  - ❖ It is impossible to subject a value to an operation which is not acceptable for its type.
  - ❖ **Consequence:** the representation of values can be concealed by the language
- **Weakly typed languages:** values have associated types, but it is possible for the programmer to break or ignore this association.
  - ❖ **C** – we can access the same memory cell in different ways:
    - ◆ In an array of Booleans, can do boolean operations on the values, or arithmetic ones, etc...
    - ◆ Parameter passing using pointers
    - ◆ Access beyond array dimensions (i.e, index value is not of the declared type)
    - ◆ Union type...misused disjoint union

## Spectrum of Strength

- Some languages are more strongly typed than others:
  - ❖ Pascal is more strongly typed than C, with the only ways of breaking the type rules being:
    - ◆ *Variant records*
    - ◆ *Illegal parameter passing with procedure-valued parameters*
      - *When a procedure B is passed as a parameter to a procedure A, the call in the declaration of A has parameters with types. The types of the parameters of B might not agree with those.*
    - ◆ *Through files (in some compilers that just don't check)*

21

## Loopholes in the Type System

Types *usually* hide the fact that a variable is just a box containing bits, however:

Type casting, as in

```
long i, j, *p = &i, *q = &j;
long ij = ((long) p) ^ ((long) q);
```

and *variant records*, as in

```
union {
    float f;
    long l;
} d;
d.f = 3.7;
printf("%ld\n", d.l);
```

allow one to peep into the implementation of types, by subjecting a value to operations not allowed for its type

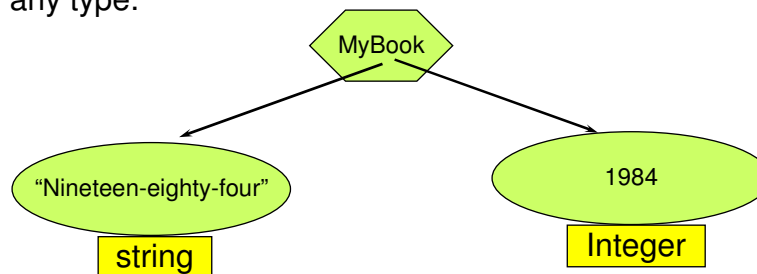
## Time of Enforcement

- **Type checking:** Language implementation must ensure that no nonsensical operations occur.
  - ❖ **Multiplication:** check that both operands are numeric.
  - ❖ **Logical operation:** check that both operands are truth values.
  - ❖ **Component selection:** possible only on arrays and records (different implementation in each).
- Type checking must be performed before the operation, but it could be done either at compile-time or at run-time:
  - ❖ **Statically typed languages:** type rules are enforced at compile time. Every variable and every formal parameter have an associated type. C, Pascal, Eiffel, ML, ...
  - ❖ **Dynamically typed languages:** type rules are enforced at run-time. *Variables*, and more generally – *expressions*, have no associated type. Only *values* have fixed types. Type checking must be done prior to each operation. Smalltalk, Prolog, Snobol, APL, Awk, Rexx, ...

23

## Dynamic Typing

- Types are associated with values.  
Each value carries a tag, identifying its type.
- **Polymorphic variables:** a variable may contain a value of any type.



- **Advantages:**
  - ❖ Arrays don't have to be of a homogeneous type
  - ❖ When data isn't uniformly typed, we don't need different variables

## Dynamic Typing - Example

```
procedure ReadLiteral(var item);  
begin  
    read a string of nonblanks;  
    if the string constitutes an integer literal  
    then  
        item := numeric value of the string  
    else  
        item := the string itself  
    end
```

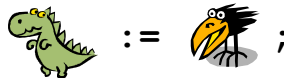
22,10,1996  
OR  
22,OCT,1996



```
read(day); ReadLiteral(month); read(year)  
if month is a string then  
    case month of {  
        "Jan": return 1;  
        "Feb": return 2;  
        ....  
    }  
else  
    return month;  
end
```

## Disadvantages of Dynamic Typing

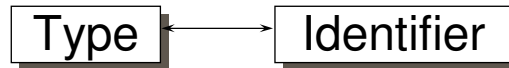
- Space overhead:
  - ❖ Each value is tagged with type information
- Slow-down:
  - ❖ Extra data to manipulate
  - ❖ Run-time checking
- Security:
  - ❖ Many bugs can be detected by static compile-time checks



Error: type mismatch

## Static Typing

- In static typing, each variable and parameter is associated with a type.



- ❖ This usually means that all identifiers should be declared before used. However this is not always the case.
- ❖ A variable may contain only values compatible with its associated type.
- All expressions are guaranteed to be type-consistent:
  - ❖ No value will be subject to operations it does not recognize.
  - ❖ This allows the compiler to engage in massive optimization.

## Why Static Typing?

- Computability theory teaches us that an automatic tool is limited as a programming aid to analyze code in advance
  - ❖ Cannot (always) determine if the program stops.
  - ❖ Cannot (always) determine if the program is correct.
  - ❖ Cannot decide almost any other interesting run time property of a program.
- One thing that *can* be done automatically by code analysis is to make sure that no run time *type error* occurs (for static typing).
- We can use every tiny bit of help in our struggle against the complexity of software!
  - ❖ A few other automatic aids are:
    - ◆ Garbage collection: automatic memory management (run time)
    - ◆ Const correctness: no modification of const parameters (compile time)
    - ◆ Pre and post (assert) conditions: partial specification of a function (run time)

## Benefits of Static Typing

- Enforce the design decisions
- Prevent run time crashes:
  - ❖ Mismatch in # of parameters
  - ❖ Mismatch in types of parameters
  - ❖ Sending an object an inappropriate message
- Early error detection reduces:
  - ❖ Development time
  - ❖ Cost
  - ❖ Effort
- More efficient and more compact object code
  - ❖ Values do not carry along the type tag
  - ❖ No need to conduct checks before each operation
  - ❖ `type SMALL_COUNTER is range 0 .. 255;`

## Limits of Static Typing

- Some senseless operations cannot be statically checked.
- Generic examples:
  - ❖ **Division by zero:** no typing system for the integers can prevent this without hitting the halting problem.
    - ◆ Checks are realized in run time by using machine-level exceptions. These checks carry no overhead.
  - ❖ **Exponentiation of certain pairs of real numbers:** similar problems to division by zero.
    - ◆ Realized at run time by the procedure that implements exceptions.
  - ❖ **Array references:** it is impossible to detect overflow of array indices.
    - ◆ Some hardware support, on some machines, but not without overhead.

## Other Times of Enforcement

- **Mixed typing:** There is a variety of possibilities between static and dynamic typing, the two extremes.
  - ❖ Mixed typing means that some typing rules are checked at compile time, others are checked during runtime.
  - ❖ **Pascal:** Most operations are checked at compile time. Array access is checked during runtime.
    - ◆ Type information is still with the variables, but checks must be deferred to run time.
    - ◆ Some Pascal compilers have a compilation option that drops array reference checks.
- **“Postmortem typing”:** A different name for weak typing.
  - ❖ No checking of type errors is made. If a program makes a type error, then it is allowed to merrily carry along with this error to the bitter end...
  - ❖ The programmer is then responsible to remove the type errors from his program

31

## Responsibility for Tagging

- **Type inference:** Compilers have the ability to apply type inference rules to determine the type of *expressions*
- Why not apply this also to variables and other entities?
  - ❖ **Explicit typing:** programmer is in charge for annotating variables and other entities with type information
    - ◆ **This is done by variable type declarations**
  - ❖ **Implicit typing:** the compiler infers type information of an entity from the way it is used
    - ◆ **No variable type declarations**
  - ❖ **Semi-implicit typing:** type is determined from the lexical structure of an identity
    - ◆ **No variable declarations**

32



## Explicit Typing

- **Example:** variable and parameter declarations in Pascal, Ada, C, etc.
- Type declarations help document programs
  - ❖ X: speed;                      (\* Good \*)
  - ❖ Y: real;                      (\* Bad \*)
  - ❖ Z = 3;                      (\* Worse \*)

33

## Implicit Typing

- The compiler infers the type of an entity from the way it is used
- **Risk:** Inadvertent creation of variables due to typos and spelling errors:
  - ❖ **ML's answer:** no variables!
    - ◆ **Value declaration:** just like CONST declaration in Pascal
      - ML: `val n = 100`
      - Pascal: `CONST n = 100`
    - ◆ **Formal parameters:** declared (without type) in the header of a function.
- **Risk:** Confusing error messages, and type errors
  - ❖ **ML's answer:** programmer is allowed to add *type constraints*
- **Risk:** Some complex (recursive and generic) type inference problems are undecidable
  - ❖ **ML's answer 1:** careful analysis of the type system to detect when this problem may occur
  - ❖ **ML's answer 2:** type constraints

34

## Semi-Implicit Typing

- **Fortran:** All variables which begin with one of the letters I, J, K, L, M and N are integers; all others are real.
  - ❖ **Risk:** inadvertent creation of variables.
  - ❖ **Fortran answer:** the declaration `implicit none` excludes automatic creation of variables. The programmer is required to explicitly declare all variables.
- **Basic (older versions):** Suffixes such as %, \$ etc. determine the variable's type.
- **Perl:** Essentially the same as Basic.

35

## Flexibility of Type System

- Type system should be an aide, not a hurdle:
  - ❖ Do not issue type error messages on programs which will not make run time type errors.
  - ❖ Allow the use of the same code for many different types.
- We refer to the extent to which a type system has these features as the *flexibility* of the type system.
- Too flexible: doesn't alert programmer to mistaken usage of variables, bad use of actual parameters, etc.
  - ❖ Adrienne rocket failed because an actual parameter in inches was used in a procedure with a formal parameter in centimeters....should have been a type error!
- Too inflexible: prevents reuse of code, especially in procedures
- Modern tendency: *polymorphism* – to be studied in great detail later...

36

## Inflexibility of Pascal

```
Type T = Integer;
Procedure sort (
  Function comp(a,b:T):Boolean,
  a: array[1..300] of T
); forward;
```

Could *not* be applied to ...

- ❖ **Arrays of real:** Procedure body and declaration has to be repeated with T1=Real
- ❖ **array[1..299] of T:** Array is too small.
- ❖ **array[1..500] of T:** Array is too big.
- ❖ **array[0..299] of T:** Mismatch in indices.
- ❖ **array[1..300] of T:** No name equivalence!!!!
  - ◆ Pascal is so fussy and inflexible in its type system that even two identical type declarations are considered distinct. A type declaration made at a certain point in a program is equivalent only to itself.

```
Type T = Integer;
  Tarray = array[1..300] of T
Procedure sort (
  Function comp(a,b:T):Boolean,
  a: Tarray;
); forward;
```

37

## Responses to Inflexibility

- **C camp:** weak typing
 

```
int qsort(char *base, int n, int width,int(*cmp)())
```
- **Smalltalk/Java camp:** dynamic typing overcomes complex inflexibility problems.
  - ❖ In Java dynamic typing is mostly geared towards this problem
- **Ada camp:** polymorphic type systems
 

```
generic
  type T is private
  with function comp(x: T, y: T)
  procedure sort(a: array(1..max) of T)
  ...
  procedure int_sort is new sort(int , "<");
```
- The backlash effect:
  - ❖ The C camp is leaning toward the Ada camp. ANSI-C, and even more so C++, tend towards strong, yet polymorphic type systems.
 

```
void qsort(void *base,size_t nel, size_t width,
  int (*compar) (const void *, const void *))
```
  - ❖ New additions to Java (e.g., Pizza) introduce generic programming into Java.
  - ❖ There were attempts to introduce static typing to Smalltalk!

38

## Summary: What's Best?

### ■ Depends on purpose...

- ❖ **Light-headed:** a scripting language, designed for small, not-to-be maintained, quick and dirty programs which are supposed to be run only a small number of times with little concern about efficiency:
- ❖ No typing or degenerate typing,
  - ◆ Weak typing to remove hassles
  - ◆ Dynamic typing to achieve flexibility
  - ◆ Semi-implicit typing to reduce programmer time
- ❖ **Software engineering oriented:** programs which are developed by several programmers, maintained and changed, run numerous times, and with efficiency concerns
  - ◆ Type system must exist to document and protect the program
  - ◆ Strong typing to reduce errors
  - ◆ Static typing to enhance efficiency, clarity and robustness
  - ◆ No semi-implicit typing to prevent inadvertent creation of variables
  - ◆ Flexible type system to allow the programmer to concentrate on the important stuff.