Person who catch an infectious disease either die almost at once during its initial phase or live an exponential time
denote the suvival time x_i and declare that x_i=0 if death occures in the initial phase
write the MLE

It is a mixture of 0-1 distribution and exponential distribution
Basic idea is modify Page 63 of testbook

Let denote following parameters we want to estimate
p : probabilty of to pass initial stage
θ : parameter in exponential distribution
which is in the form of

$$f(x) = \theta^{-1}\exp(-x/\theta)$$

probability to die in initial stage is, that is x_i==0 is

$(1-p)$ (1)

probability of time after that is pass initial stage and follow exponential distribution

$p\theta^{-1}\exp(-x/\theta)$ (2)

Use indicator to join this two into one function
y_i : indicator whether x_i>0

we get

$$p(x_i)=(1-p)^{1-y_i}\{p\theta^{-1}\exp(-x_i/\theta)\}^{y_i}$$

which is the some as
when y==0 => (1)
when y==1 => (2)

$L(p, \theta) = \Pi (1-p)^{1-y_i}\{p\theta^{-1}\exp(-x_i/\theta)\}^{y_i}$
$l(p, \theta) = \log(L(p, \theta))$
$= \sum(1-y_i)\log(1-p) + \log(\Pi p^{y_i}) + \log(\Pi \theta^{-1\ y_i}) + \sum \log(\exp(-x_i/\theta)^{y_i})$
$= \sum(1-y_i)\log(1-p) + \sum y_i \log p - \sum y_i \log\theta + \theta^{-1}\sum y_i x_i$ (3)

notice y_i=0 when x_i=0; y_i=1, otherwise, this means
$\sum x_i y_i = \sum x_i$ (4)
Let denote
$r = \sum y_i$ (5)
$s = \sum x_i$ (6)
N: number of data (7)

subsititue (4) (5) (6) (7) into (3)
we get
$l(p, \theta) = (N-r)\log(1-p) + r\log p - r\log\theta - \theta^{-1}s$

argmax

$\partial l/\partial p = (N-r)/(1-p) + r/p = 0$ => $p = r/2r-N$
$\partial l/\partial \theta = -r/\theta + s/\theta^2$ => $\theta = s/r$