MATH 567 – Lecture 24 (04/14/2015)

Receiver location problem – Heuristic algorithm

Let $\underbrace{C_i}_{\text{coverage info for meter } i} = \{ j \in P \mid j \text{ covers } i \}$, and $\underbrace{C}_{\text{all coverage info}} = \{ C_i \}_i$.

HEURISTIC_COVER $(P, M, C, k, \varepsilon)$

set of poles: P ; set of meters: M ; $k \in \mathbb{Z}_{>0}$; ε: tolerance for comparing score

INPUT: set of poles P, set of meters M, Coverage C, parameter k, tolerance ε;

OUTPUT: $P' \subseteq P$ that covers all $i \in M$.

Initialization: $P' = \emptyset$, $M' = \emptyset$ (where $M' \subseteq M$ is the set of meters covered by P').

while $M' \neq M$ do

  PREPROCESS $(P \setminus P', M \setminus M')$; (set difference)

  Compute Score$(\cdot)$ for $(P \setminus P', M \setminus M')$; (after preprocessing)

  Select $j \in P \setminus P'$ with Score$(j) = \max\limits_{\ell \in P \setminus P'} (\text{Score}(\ell)) \pm \varepsilon$;

  $P' = P' \cup \{j\}$;

  update M';

  CLEANUP P'; ← could do CLEANUP only after every 10^{th} pole is selected, say.

end-while

IP formulation

Let $M = \{1, 2, \ldots, m\}$ (set of meters) and $P = \{1, \ldots, p\}$ (set of poles).

d.v.'s: Let $x_j = \begin{cases} 1 & \text{if pole } j \text{ is selected (for locating a receiver)} \\ 0 & \text{otherwise.} \end{cases}$

IP:

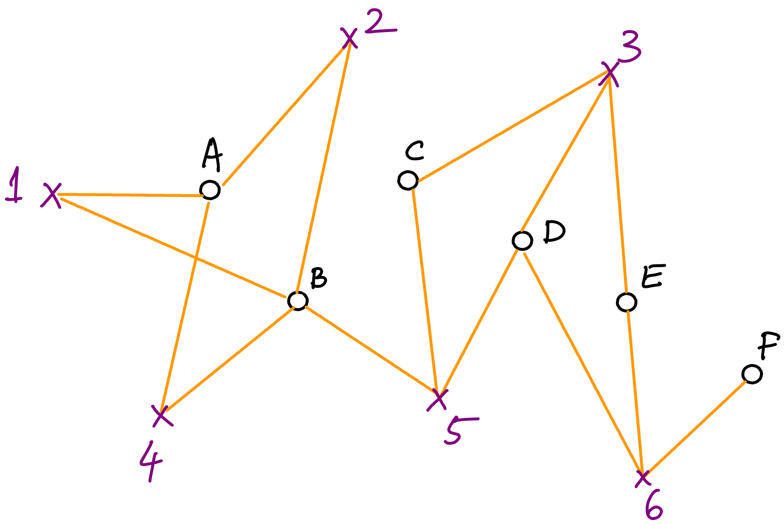
$$\min \sum_{j=1}^{p} x_j$$

$$\text{s.t.} \quad \sum_{j \in C_i} x_j \geq 1 \quad \forall i \in M \quad \text{———} \; \triangle$$

$$x_j \in \{0,1\}, \; j \in P.$$

These IPs are often too large to handle efficiently by packages like CPLEX.

Example (revisited)



$$\min \; x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$\text{s.t.} \quad A = \boxed{\begin{array}{llllll} x_1 + x_2 & & + x_4 & & & \\ x_1 + x_2 & & + x_4 & + x_5 & & \\ & x_3 & & + x_5 & & \\ & x_3 & & + x_5 & + x_6 & \\ & x_3 & & & + x_6 & \\ & & & & x_6 & \end{array}} \begin{array}{ll} \geq 1 & (A) \\ \geq 1 & (B) \\ \geq 1 & (C) \\ \geq 1 & (D) \\ \geq 1 & (E) \\ \geq 1 & (F) \end{array}$$

$$x_j \in \{0,1\}, \; j = 1, \ldots, 6.$$

Steps in Preprocessing

$$\min \quad x_1 + x_2 + \overset{\text{(ii)}}{\cancel{x_3}} + x_4 + x_5 + x_6$$

$$\text{s.t.} \quad A = \boxed{\begin{array}{llllllll}
x_1 + x_2 & & + x_4 & & & \geq 1 & (A) & \\
\cancel{x_1 + x_2} & & \cancel{+ x_4} & \cancel{+ x_5} & & \cancel{\geq 1} & \cancel{(B)} & \text{(iii)} \\
 & \cancel{x_3} & & + x_5 & & \geq 1 & (C) & \\
 & \cancel{x_3} & & \cancel{+ x_5} & \cancel{+ x_6} & \cancel{\geq 1} & \cancel{(D)} & \text{(i)} \\
 & \cancel{x_3} & & & \cancel{+ x_6} & \cancel{\geq 1} & \cancel{(E)} & \text{(i)} \\
 & & & & x_6 & \geq 1 & (F) \checkmark & \text{(i)}
\end{array}}$$

$$x_j \in \{0, 1\}, \ j = 1, \ldots, 6.$$

(i) Singleton rows: Row-6 $\Rightarrow$ set $x_6 = 1$, delete all rows containing x_6, i.e., delete (D), (E).

(ii) After Step (i), Column x_5 dominates Column x_3, so set $x_3 = 0$ (i.e., delete Column x_3).

(iii) Row (B) dominates Row (A), eliminate (or delete) Row (B).

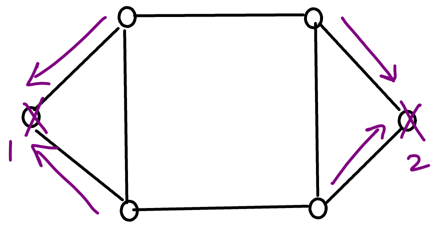
"Domination" in the context of binary vectors:

Let $\bar{u}, \bar{v} \in \{0,1\}^n$.

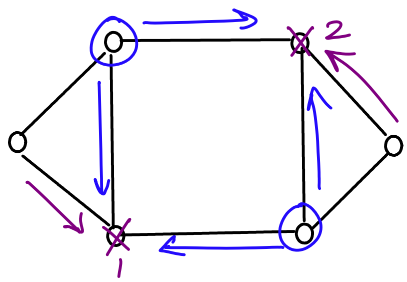
$\bar{u}$ (strictly) dominates $\bar{v}$ if $u_i \geq v_i \ \forall i$, and $u_j > v_j$ for at least one j.

Variants of the set-covering model for facility location

Variant 1 'o' = 'x', i.e., facility locations and customer locations are same.



optimal objective function value = 2.



This optimal solution is better, as two customers are covered twice.

Objectives: (1) minimize total # facilities

(2) among all optimal solutions for (1), pick the one that maximizes the # customers covered twice or more.

Define $y_i = \begin{cases} 1 & \text{if customer } i \text{ is covered twice or more} \\ 0 & \text{o.w.} \end{cases}$

IP is **NOT!** $\min \sum_{j=1}^{p} x_j - \sum_{i=1}^{m} y_i$

$$\text{s.t. } \sum_{j \in C_i} x_j - y_i \geq 1 \quad \forall i \in M$$

$$x_j, y_i \in \{0,1\} \quad \forall i, j.$$

This objective function might not work!

If $\bar{x}, \bar{y}$ are optimal for this IP, we could find $\bar{x}', \bar{y}'$ such that

$$\sum_{j \in C_i} x_j' = \sum_{j \in C_i} x_j + 1 \quad \text{and}$$

$$\sum_{i=1}^{m} y_i' \geq \sum_{i=1}^{m} y_i + 2 \; !$$

Instead, we can use the following objective function:

$$\min \sum_{j=1}^{p} x_j - \varepsilon \underbrace{\sum_{i=1}^{m} y_i}, \quad \text{where } 0 < \varepsilon \leq \frac{1}{m+1}.$$

The contribution of the second sum term is hence < 1.

<u>Variant 2</u> Let w_i = importance of covering customer i, $(w_i \geq 0)$,

and let p_0 be the max # facilities (or poles picked)

$(p_0 \in \mathbb{Z}_{>0}$, represents a budget constraint).

Let $s_i = \begin{cases} 1 & \text{if } i \text{ is covered}, \; i \in M \\ 0 & \text{otherwise} \end{cases}$

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} w_i s_i \\ \text{s.t.} \quad & \sum_{j \in C_i} x_j - s_i \geq 0 \quad \forall i \in M \\ & \sum_{j=1}^{p} x_j \leq p_0 \\ & x_j, s_i \in \{0,1\} \quad \forall i, j. \end{aligned}$$

Variant 3 : Existing and new facilities

Let $P = \{1, 2, \ldots, p_{ex}, p_{ex}+1, \ldots, p_{ex}+p_{new}\}$, where

p_{ex} = # existing facilities and

p_{new} = # new facilities

Objectives (1) minimize total # facilities;

(2) Among the optimal solutions to (1), pick one that minimizes the # new facilities.

We want to use as many of the existing facilities first, before using new ones. We could also incorporate the costs for establishing the new facilities.

$$\min \quad \sum_{j=1}^{p_{ex}} x_j + (1+\varepsilon) \sum_{j=p_{ex}+1}^{p_{new}} x_j$$

$$\text{s.t.} \quad \sum_{j \in C_i} x_j \geq 1 \quad \forall\, i \in M$$

$$x_j \in \{0,1\}, \ \forall j.$$

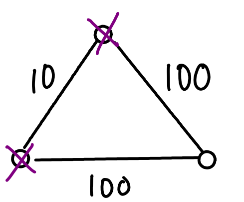
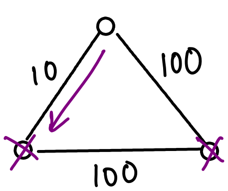
Need $0 < \varepsilon \leq \dfrac{1}{p_{new}+1}$, in the same way as in Variant 1, so that the contribution of the extra sum term is < 1.

p-center and p-median Problems

IDEA: Locate at most p facilities, and assign every customer to one facility.

minimize
- → total distance of all customers to their facility } p-median
- → max distance of any customer to its facility } p-center

Example

	2-center	2-median
	100	100
	10	10