

# Analysis Module: AsymptoticEquivalent

Mathlib4 Documentation

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## 1 Module Overview

### 1.1

In this file, we define the relation  $\sim$ , which means that  $f$  is little o of  $g$  along the filter  $\mathcal{F}$ .

Unlike  $\mathcal{O}$  relations, this one requires  $f$  and  $g$  to have the same codomain. While the definition only requires  $f$  to be a  $\mathcal{O}$ , most interesting properties require it to be a  $\mathcal{O}$ .

### 1.2

We introduce the notation  $\sim$ , which you can use by opening the `locale`.

### 1.3

If  $f$  is a  $\mathcal{O}$  :

- $\sim$  is an equivalence relation - Equivalent statements for  $f$  : - If  $f$  is  $\mathcal{O}$ , this is true iff (see ) - For  $f$ , this is true iff (see )

If  $f$  is a  $\mathcal{O}$  :

- Alternative characterization of the relation (see ) :

- Provided some non-vanishing hypothesis, this can be seen as (see ) - For any constant  $c$ ,  $c \cdot f$  implies (see ) -  $\mathcal{O}$  and  $\mathcal{O}$  are compatible with (see and )

If  $f$  is a  $\mathcal{O}$  :

- If  $f$  is  $\mathcal{O}$ , we have (see )

### 1.4

Note that  $\sim$  takes the parameters in that order. This is to enable  $\sim$ , as  $\sim$  requires that the last two explicit arguments are  $\mathcal{O}$ .

## 2 Key Definitions

**Definition 1** (IsEquivalent). A def defining IsEquivalent

**Definition 2** (IsEquivalent). A theorem defining IsEquivalent

**Definition 3** (IsEquivalent). A theorem defining IsEquivalent

**Definition 4** (IsEquivalent). A theorem defining IsEquivalent

**Definition 5** (IsEquivalent). A theorem defining IsEquivalent

**Definition 6** (IsEquivalent). A theorem defining IsEquivalent

**Definition 7** (IsEquivalent). A theorem defining IsEquivalent

**Definition 8** (IsEquivalent). A theorem defining IsEquivalent

**Definition 9** (IsEquivalent). A theorem defining IsEquivalent

**Definition 10** (IsEquivalent). A theorem defining IsEquivalent