Analysis Module: Basic

Mathlib4 Documentation

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1 Module Overview

1.1

This file gathers basic facts of analytic nature on the complex numbers.

1.2

This file registers as a normed field, expresses basic properties of the norm, and gives tools on the real vector space structure of . Notably, it defines the following functions in the namespace .

—Name —Type —Description — ————————————————-—————————————————————— ——C L[R] R $\times\mathbb{R}|The natural from to|||\mathbb{C}[\mathbb{R}]\mathbb{R}|Real part function as a|||\mathbb{C}[\mathbb{R}]\mathbb{R}|$ $[\mathbb{R}]\mathbb{C}|Embedding of the reals as a|||\mathbb{C}L[\mathbb{R}]\mathbb{C}|Complex conjugation as a|||\mathbb{C}[\mathbb{R}]\mathbb{C}|Complex conjugation as a|$

We also register the fact that is an field.

2 Key Definitions

Definition 1 (continuous$_{n}ormSq$). A theorem defining `continuous`$_{n}ormSq$

Definition 2 (nnnorm$_{e}q_{o}ne_{o}f_{p}ow_{e}q_{o}ne$). A theorem defining `nnnorm`$_{e}q_{o}ne_{o}f_{p}ow_{e}q_{o}ne$

Definition 3 (norm$_{e}q_{o}ne_{o}f_{p}ow_{e}q_{o}ne$). A theorem defining `norm`$_{e}q_{o}ne_{o}f_{p}ow_{e}q_{o}ne$

Definition 4 (le$_{o}f_{e}q_{s}um_{o}f_{e}q_{s}um_{n}orm$). A lemma defining `le`$_{o}f_{e}q_{s}um_{o}f_{e}q_{s}um_{n}orm$

Definition 5 (equivRealProd$_{a}pply_{l}e$). A theorem defining `equivRealProd`$_{a}pply_{l}e$

Definition 6 (equivRealProd$_{a}pply_{l}e$). A theorem defining `equivRealProd`$_{a}pply_{l}e$

Definition 7 (lipschitz$_{e}quivRealProd$). A theorem defining `lipschitz`$_{e}quivRealProd$

Definition 8 (antilipschitz$_{e}quivRealProd$). A theorem defining `antilipschitz`$_{e}quivRealProd$

Definition 9 (isUniformEmbedding$_{e}quivRealProd$). A theorem defining `isUniformEmbedding`$_{e}quivRealProd$

Definition 10 (equivRealProdCLM). A def defining `equivRealProdCLM`

3 Main Theorems

Theorem 1 (tendsto$_{n}ormSq_{c}ocompact_{a}tTop$). The ‘normSq‘ function on ‘C‘ is proper.

Theorem 2 (ringHom$_{e}q_{o}fReal_{o}f_{c}ontinuous$). The only continuous ring homomorphism from ‘R‘ to ‘C‘ is the identity.

Theorem 3 (ball$_{o}ne_{s}ubset_{s}litPlane$). The slit plane includes the open unit ball of radius ‘1‘ around ‘1‘.

Theorem 4 (mem$_{s}litPlane_{o}f_{n}orm_{l}t_{o}ne$). The slit plane includes the open unit ball of radius ‘1‘ around ‘1‘.