1. Affine Space

1.1. Overview

Defines affine space $\mathbb{A}(n; S)$ over a scheme S and morphisms into it.

1.2. Main Definitions

1.2.1. Affine Space

`AffineSpace (n : Type v) (S : Scheme)`: The affine n-space over S

- Defined as pullback of terminal morphisms from S and Spec $\mathbb{Z}[n]$
- Notation: `𝔸(n; S)`
- Note: n is an arbitrary index type (e.g., `Fin m`)

1.2.2. Canonical Structure

`AffineSpace.over`: Instance making $\mathbb{A}(n; S)$ canonically over S

- `hom := pullback.fst`

`AffineSpace.toSpecMvPoly`: Map $\mathbb{A}(n; S) \to$ Spec $\mathbb{Z}[n]$

- Given by `pullback.snd`

1.3. Coordinate Functions

1.3.1. Standard Coordinates

`AffineSpace.coord`: The standard coordinate functions on affine space

- `coord i`: The i-th coordinate function
- Global sections of the structure sheaf

1.4. Morphisms into Affine Space

1.4.1. Vector of Functions

`AffineSpace.homOfVector`: Constructs morphism $X \to \mathbb{A}(n; S)$

- Input: Morphism $X \to S$ and n coordinate functions on X
- Output: The corresponding morphism to affine space

1.4.2. Equivalence for Morphisms

`AffineSpace.toSpecMvPolyIntEquiv`:

- Morphisms $X \to$ Spec $\mathbb{Z}[n] \simeq n$-tuples of global sections
- `toFun`: Extracts coordinates via Γ-Spec adjunction
- `invFun`: Constructs morphism via evaluation

`AffineSpace.homOverEquiv`: For X over S

- S-morphisms $X \to \mathbb{A}(n; S) \simeq n$-tuples of global sections
- Combines pullback structure with `toSpecMvPolyIntEquiv`

1.5. Isomorphisms

1.5.1. Affine Space over Spec

`AffineSpace.SpecIso`: $\mathbb{A}(n; \text{Spec } R) \cong \text{Spec } R[n]$

- Natural isomorphism
- Identifies affine space over affine base with polynomial ring spectrum

1.6. Properties

1.6.1. Finiteness

`AffineSpace.finite`: The projection $\mathbb{A}(n; S) \to S$ is finite when n is finite

`AffineSpace.finitePresentation`: The projection is of finite presentation when n is finite

1.6.2. Universal Property

The affine space satisfies the universal property:

- Morphisms into $\mathbb{A}(n; S)$ over S correspond to n-tuples of functions
- This makes it the scheme-theoretic product $\mathbb{A}^1 \times \ldots \times \mathbb{A}^1$ (n times)

1.7. Implementation Notes

- Uses `MvPolynomial n (ULift ℤ)` for universe polymorphism
- Local notation: `ℤ[n]` for the polynomial ring
- Universe parameters carefully managed for pullback construction
- Equivalences use `Equiv` for computational content