1. The term 'efficient frontier' refers to the portfolios that:

   (Choose all that apply)

   a. Yield the greatest return for a given level of risk
   b. Involve the least risk for a given level of expected return
   c. Yield the greatest return for maximum risk
   d. None of the above

   Answer:

   The correct answers are a and b.

2. An investor has total wealth of $100,000 and wants to invest in a portfolio with 3 securities A, B and C. If he chooses to invest $50,000 in security A, $25,000 in security B and $25,000 in security C, what will be the expected return of this portfolio? Round off your final answer to two digits after the decimal point. State your answer as a percentage rate (such as 5.55)

   The correct answer is 12.53.

   $E(R_p) = \sum w_i \cdot E(R_i)$, where w_i are the weights we choose for each investment

   Note: $\sum w_i = 1$

   We form a portfolio with:

   $w_A = 50{,}000 / 100{,}000 = 0.50$
   $w_B = 25{,}000 / 100{,}000 = 0.25$
   $w_C = 25{,}000 / 100{,}000 = 0.25$

   The expected return of the portfolio would be:

   $E(R_p) = 0.50 \cdot 15.5\% + 0.25 \cdot 14.5\% + 0.25 \cdot 4.6\% = 12.53\%$

3. Consider the following distribution of returns:

State of the economy	Probability	R_A	R_B	R_C
Depression	30%	10%	-5%	-3%
Normal	50%	15%	20%	5%
Expansion	20%	25%	30%	15%

   Which of the following statements are correct? (Multiple correct answers are allowed)

   a. $E(R_A) = 15.5\%$ and $\sigma_B = 13.31\%$
   b. $E(R_B) = 12.5\%$ and $\sigma_A = 5.22\%$

c. $E(R_C) = 4.6\%$ and $E(R_B) = 14.5\%$
d. $\sigma_A = 5.22\%$ and $\sigma_C = 6.25\%$

Answer:

The correct answers are a, c and d.

The expected return is calculated as the probability-weighted average of the returns. $P(R_i)$ is the probability of each scenario and R_i the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$\mathrm{E(R_A)} = 10\% \cdot 0.30 + 15\% \cdot 0.50 + 25\% \cdot 0.20 = \mathbf{15.5}\%$$

$$\mathrm{E(R_B)} = -5\% \cdot 0.30 + 20\% \cdot 0.50 + 30\% \cdot 0.20 = \mathbf{14.5}\%$$

$$\mathrm{E(R_C)} = -3\% \cdot 0.30 + 5\% \cdot 0.50 + 30\% \cdot 0.20 = \mathbf{4.6}\%$$

The standard deviation of the return is defined as the square root of variance, which is the expected value of the squared deviations from the expected return. The variance is calculated as:

$$\sigma_x^2 = \sum \{[\mathrm{R_{xi}} - \mathrm{E(R_x)}]^2 \cdot \mathrm{P_i}\}$$

The expected return is calculated as the probability-weighted average of the returns. P(Ri) is the probability of each scenario and Ri the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$\mathrm{E(R_A)} = 10\% \cdot 0.30 + 15\% \cdot 0.50 + 25\% \cdot 0.20 = \mathbf{15.5}\%$$

The variance for A is calculated as:

$$\sigma_A^2 = \sum \{[\mathrm{R_{Ai}} - \mathrm{E(R_A)}]^2 \cdot \mathrm{P_i}\}$$

$$\sigma_A^2 = (10 - 15.5)^2 \cdot 0.30 + (15 - 15.5)^2 \cdot 0.50 + (25 - 15.5)^2 \cdot 0.20 = 27.25$$

Hence the standard deviation for A is equal to $\sigma_\mathrm{A} = \sqrt{27.25} = \mathbf{5.22}\%$.

The variance for B is calculated as:

$$\sigma_B^2 = \sum\{[R_{Bi} - E(R_B)]^2 \cdot P_i\}$$

$$\sigma_B^2 = (-5 - 14.5)^2 \cdot 0.30 + (20 - 14.5)^2 \cdot 0.50 + (30 - 14.5)^2 \cdot 0.20 = 177.25$$

Hence the standard deviation for B is equal to $\sigma_B = \sqrt{177.25} = \mathbf{13.31}\%$

The variance for C is calculated as:

$$\sigma_C^2 = \sum\{[R_{Ci} - E(R_C)]^2 \cdot P_i\}$$

$$\sigma_C^2 = (-3 - 4.6)^2 \cdot 0.30 + (5 - 4.6)^2 \cdot 0.50 + (15 - 4.6)^2 \cdot 0.20 = 39.04$$

Hence the standard deviation for C is equal to $\sigma_C = \sqrt{39.04} = \mathbf{6.25}\%$

4. Consider the following distribution of returns:

State of the economy	Probability	R_A	R_B
Depression	30%	10%	-5%
Normal	50%	15%	20%
Expansion	20%	25%	30%

Based on the distribution above compute the covariance between A and B. Round off your final answer to two digits after the decimal point (such as 5.55). (Hint: Your answers to previous questions may be useful.)

Answer:

The answer is that $\sigma_{AB} = 60.25$.

The covariance between A and B is calculated as:

$$\sigma_{AB} = \sum\{[R_{Ai} - E(R_A)] \cdot [R_{Bi} - E(R_B)] \cdot P_i\}$$

Hence: $\sigma_{AB} = (10 - 15.5) \cdot (-5 - 14.5) \cdot 0.30 + (15 - 15.5) \cdot (20 - 14.5) \cdot 0.50$
$+(25 - 15.5) \cdot (30 - 14.5) \cdot 0.20 = \mathbf{60.25}$

5. Consider the following distribution of returns:

State of the economy	Probability	R_A	R_B
Depression	30%	10%	-5%
Normal	50%	15%	20%
Expansion	20%	25%	30%

Calculate the correlation coefficient between A and B. Round off your final answer to two digits after the decimal point (such as 5.55). (Hint: Your answers to previous questions may be useful.)

Answer:

The answer is that $\rho_{AB} = 0.87$.

The correlation coefficient between A and B is calculated as:

$\rho_{AB} = \sigma_{AB}/(\sigma_A \times \sigma_B)$

Previously, we computed the following measures:

$\sigma_{AB} = 60.25$

$\sigma_A = 5.22\%$

$\sigma_B = 13.31\%$

The correlation coefficient between A and B is calculated as:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \cdot \sigma_B} = \frac{60.25}{5.22 \cdot 13.31} = 0.87$$

6. Consider the following distribution of returns:

State of the economy	Probability	R_A	R_B
Depression	30%	10%	-5%
Normal	50%	15%	20%
Expansion	20%	25%	30%

What would be the expected return and standard deviation of a portfolio with 60% in A and 40% in B?

a. $E(R_P) = \mathbf{15.10}\%$ and $\sigma_P = \mathbf{8.19}\%$
b. $E(R_P) = \mathbf{16.23}\%$ and $\sigma_P = \mathbf{5.25}\%$
c. $E(R_P) = \mathbf{10.13}\%$ and $\sigma_P = \mathbf{7.50}\%$
d. $E(R_P) = \mathbf{20.20}\%$ and $\sigma_P = \mathbf{3.00}\%$

Answer:

The correct answer is a.

The expected return of a portfolio is calculated as:

$E(R_p) = \sum w_i \cdot E(R_i)$, where w_i are the weights we choose for each investment

Note: $\sum w_i = 1$

In this case we form a portfolio with 60% in A and 40% in B, whereas w_A=60%=0.60 and w_B=40%=0.40.

$$\mathrm{E}(\mathrm{R_P}) = w_A \cdot \mathrm{E}(\mathrm{R_A}) + w_B \cdot \mathrm{E}(\mathrm{R_B}) \Rightarrow$$
$$\mathrm{E}(\mathrm{R_P}) = 0.60 \cdot 15.5\% + 0.40 \cdot 14.5\% = \mathbf{15.1}\%$$

The risk of the portfolio is expressed by the standard deviation. In order to compute the standard deviation, we should first calculate the portfolio's variance. The variance of the portfolio is calculated by the following formula:

$$\sigma_P^2 = w_A^2 \cdot \sigma_A^2 + w_B^2 \cdot \sigma_B^2 + 2 \cdot w_A \cdot w_B \cdot \sigma_{AB} \Longrightarrow$$
$$\sigma_P^2 = 0.60^2 \cdot 27.25 + 0.40^2 \cdot 177.25 + 2 \cdot 0.60 \cdot 0.40 \cdot 60.25 = 67.09$$

Hence the standard deviation of the portfolio is equal to $\sigma_P = \sqrt{67.09} = \mathbf{8.19}\%$

7. What characteristics of a security are most important in the determination of the variance of a well-diversified portfolio?

    a. The expected return of a portfolio
    b. The selection of an equally weighted portfolio
    c. The correlation between the securities of a portfolio
    d. The number of the securities that form the portfolio

Answer:

The correct answer is c.

The most important characteristic in the determination of the variance of a well-diversified portfolio is the correlation between the securities. The lower the correlation between the securities of a portfolio, the greater the possibility of risk reduction.

8. Which of the following options can be classified as a case of unique risk?

    a. A sudden change at the exchange rate between US dollar and euro.
    b. Kraft Foods buys Cadbury.
    c. The economy is expanding.
    d. Oil prices fall.

Answer:

The correct answer is b.

While an acquisition may perhaps affect multiple companies, it is still industry specific.

9. Which of the following statements are true about the mean-variance frontier? Choose all that apply.

    a. Mean-variance frontier is the locus of the portfolios in expected return-standard deviation space that have the maximum variance for each expected return.

    b. For two assets, it consists simply of all possible portfolio combinations of the two assets.

    c. The left-most point on the minimum variance frontier is called the minimum variance portfolio.

    d. The mean variance frontier contracts as we add more assets to the mix.

Answer:

The correct answers are b and c.

Answer a is wrong because the mean-variance frontier is the locus of the portfolios in expected return-standard deviation space that have the minimum variance for each expected return and not the maximum.

Answer d is incorrect because the mean variance frontier shifts to the left and expands as we add more assets to the mix. This is because the additional assets improve the diversification opportunities.

10. Consider a portfolio of risky equities and Treasury bills. Suppose the expected return on equities is 12% per year with a volatility of 18%. Let's also suppose that T-bills offer a risk-free 7% rate of return. What would be the volatility of your portfolio if you have 60% in equities and 40% in Treasuries?

    a. 10.0%

    b. 13.60%

    c. 1.94%

    d. 10.8%

Answer:

The correct answer is d.

The volatility of a two-asset portfolio is given by:

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_{12}$$

Note, however, the risk-free asset has zero variance and zero covariance. Therefore,

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 \Rightarrow \sigma_P = w_1 \times \sigma_1 = 0.6 \text{ x } 18\% = 10.8\%$$