A Proof for the bound of $P_{m=1}^{detect,diff}$

For simplicity, in this appendix we denote the redundancy overhead $r_{l,i}$ as r, and the number of devices N_l as N. $r \in (0,1)$, $N \in [2,+\infty)$, $N \in Z$

$$\begin{aligned} P_{m=1}^{detect,diff} &= P_{m=1}^{detect,cen} - P_{m=1}^{detect,ind} \\ &= r - 1 + (1 - \frac{r}{N-1})^{(N-1)} \end{aligned} \tag{1}$$

Assume $x = N - 1$, then $x \in [1,+\infty)$, $x \in Z$, then:

$$P_{m=1}^{detect,diff} = r - 1 + (1 - \frac{r}{x})^x \tag{2}$$

Assume $g(x,r) = r - 1 + (1 - \frac{r}{x})^x$, $f(x,r) = (1 - \frac{r}{x})^x$. Because $\frac{r}{x} \in (0,1)$, then $1 - \frac{r}{x} \in (0,1)$, therefore $f(x,r) = (1 - \frac{r}{x})^x \in (0,1)$.

$\ln f(x,r) = x \ \ln(1 - \frac{r}{x})$

Derive both sides of equations with x:

$$\begin{aligned} \frac{\frac{df(x,r)}{dx}}{f(x,r)} &= \ln(1 - \frac{r}{x}) + x\,\frac{r}{x^2}\frac{1}{1-\frac{r}{x}} \\ &= \ln(1 - \frac{r}{x}) + \frac{r}{x-r} \end{aligned} \tag{3}$$

$$\frac{df(x,r)}{dx} = f(x,r)\left(\ln(1 - \frac{r}{x}) + \frac{r}{x-r}\right) \tag{4}$$

Assume $h(x,r) = \ln(1 - \frac{r}{x}) + \frac{r}{x-r}$

$$\begin{aligned} \frac{dh(x,r)}{dx} &= \frac{1}{1-\frac{r}{x}}(-1)(-\frac{r}{x^2}) - \frac{r}{(x-r)^2} \\ &= \frac{-r^2}{x(x-r)^2} \end{aligned} \tag{5}$$

Clearly, $\frac{dh(x,r)}{dx} < 0$, this indicates $h(x,r)$ monotonically decreas with the increase of x, when $x \in [1,+\infty)$, $x \in Z$. Therefore, we know:

$$\begin{aligned} h(x,r)_{min} &= h(x,r)_{x\to+\infty} \\ &= \ln(1^-) + 0^+ \\ &= 0 \end{aligned} \tag{6}$$

Thus $h(x,r) > 0$ is always true when $x \in [1,+\infty)$, $x \in Z$, and $r \in (0,1)$.

From Eq. 4:

$$\frac{df(x,r)}{dx} = f(x,r) \cdot h(x,r) \tag{7}$$

As it is proven that $h(x,r) > 0$, $f(x,r) = (1 - \frac{r}{x})^x \in (0,1) > 0$ in the given range of x and r, then we have $\frac{df(x,r)}{dx} > 0$. This proves that $f(x,r)$ monotonically increases with the increase of x. The upper bound of $f(x,r)$ should be:

$$\begin{aligned} f(x,r)_{max} &= f(x,r)_{x\to+\infty} \\ &= (1 - \frac{r}{x})^x_{x\to+\infty} \end{aligned} \tag{8}$$

According to the definition of e, $e^k = \lim_{n\to+\infty}(1 + \frac{k}{n})^n$, then $f(x,r)_{max} = e^{-r}$. From here, we can conclude that:

$$\begin{aligned} P_{m=1}^{detect,diff} = g(x,r) &= r - 1 + (1 - \frac{r}{x})^x \\ &= r - 1 + f(x,r) \\ &< r - 1 + e^{-r} \end{aligned} \tag{9}$$

To find out the trend of $g(x,r)_{max}$ according to the change of r, we derive $g(x,r)$ with r:

$$\begin{aligned} g(x,r)_{max} &= r - 1 + e^{-r} \\ \frac{dg(x,r)_{max}}{dr} &= 1 - e^{-r} \in (0, 1 - \frac{1}{e}) > 0 \end{aligned} \tag{10}$$

This shows that $g(x,r)_{max}$ increases monotonically with the increase of r. The largest possible value of $P_{m=1}^{detect,diff}$ is $g(x,r)_{max}(r=1^{-}) = \frac{1}{e} \approx 0.368$. When $r = 10\%$, the upper bound of $P_{m=1}^{detect,diff}$ is around 0.005, and when $r = 20\%$, the upper bound of $P_{m=1}^{detect,diff}$ is around 0.019.