STA 32 Winter 2023
Homework 3 - Due by 11:59 PM Wednesday, Feb 1^{st}, onto Gradescope

Book Homework

1. Answer the following questions as True or False and explain explain your answers.
   (a) For two events A and B where $P(A) > 0$ and$P(B) > 0, P(A \cap B^C) = 1 - P(A \cap B)$.
   (b) For two events A and B are independent, where $P(A) > 0$ and$P(B) > 0$, then $P(A \cup B) = P(A) + P(B)(1 - P(A))$.
   (c) For a discrete random variable X, where a is a constant,$P(X > a) = P(X \geq a + 1)$ if X can only take on integer values.
   (d) For a discrete random variable X, where a is a constant,$F(a) = 1 - P(X > a)$.

2. Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

   (a) Find $P(X \leq 2)$.
   (b) Find $P(X > 1)$.
   (c) Find μ_X.
   (d) Find σ_X^2

3. A chemical supply company ships a certain solvent in 10-gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume X has the following probability mass function:

x	1	2	3	4	5
$p(x)$	0.4	0.2	0.2	0.1	0.1

   (a) Find the mean number of drums ordered.
   (b) Find the standard deviation of the number of drums orderd.
   (c) Let Y be the number of gallons ordered. Find the probability mass function of Y.
   (d) How many gallons are expected to be ordered?

4. The CDF for a discrete random variable representing the winnings (in dollars) from a particular game is

x	-2	-1	0	1	2
$F(x)$	0.1	0.15	0.65	0.85	1.0

   Where X is the amount of winnings (or losses). I.e, $X = -2$ means they lost 2 dollars, $X = 2$ means they won two dollars. Assume the game costs nothing to play.
   (a) Write down the pmf for this random variable as a table with row x and $P(X = x)$
   (b) Find the expected winnings for this random variable, $E(X)$.
   (c) Find the probability that if someone won money, they won 1 dollar.
   (d) Find the probability that someone loses money.

5. The main bearing clearance (in mm) in a certain type of engine is a random variable with probability density function

$$f(x) = \begin{cases} 625x & 0 < x \leq 0.04 \\ 50 - 625x & 0.04 < x \leq 0.08 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the probability that the clearance is less than 0.02 mm?
   (b) Find the mean clearance.
   (c) Find the standard deviation of the clearances.
   (d) Find the cumulative distribution function of the clearance.
   (e) Find the median clearance.

6. The thickness X of a wooden shim (in mm) has probability density function

$$f(x) = \begin{cases} \frac{3}{4} - \frac{3(x-5)^2}{4} & 4 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find μ_X.
   (b) Find σ_X^2
   (c) Let Y denote the thickness of a shim in inches (1 mm = 0.0394 inches). Find μ_Y and σ_Y^2.
   (d) If three shims are selected independently and stacked one atop another, find the mean and variance of the total thickness.

7. A continuous random variable is defned with the following probability density function:

$$f(x) = \begin{cases} c(x^2 + 2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value c such that $\int f(x)dx = 1$.
   (b) Find the cumulative distribution function, $F(x)$.
   (c) Find the probability X lies between zero and 0.50.
   (d) Find the mean and standard deviation of X.

8. If X and Y are independent random variables with means $\mu_X = 9.5, \mu_Y = 6.8$, and standard deviations $\sigma_X = 0.4, \sigma_Y = 0.1$, find the means and standard deviations of the following. Show your work.
   (a) $Z_1 = 3X$
   (b) $Z_2 = Y - X$
   (c) $Z_3 = X + 4Y$

9. The cumulative distribution function for a continuous random variable is:

$$F(x) = \begin{cases} 1 - \exp(-2x) & 0 \le x \le \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the median of X.

(b) Find the 10th percentile of X.

(c) Find the probability that X is between 1 and 3.

(d) Find the probability that X is larger than 3.