STA 32 Winter 2023
Homework 4 - Due Friday, Feb 17th at 11:59 PM,
Gradescope

Book Homework

1. Data collected by the Substance Abuse and Mental Health Services Administration (SAMSHA) suggests that 69.7% of 18-20 year-old consumed alcoholic beverages in any given year.

   (a) Suppose a random sample of ten 18-20 year-old is taken. Is the use of the binomial distribution appropriate for calculating the probability that exactly six consumed alcoholic beverages? Explain.

   (b) Calculate the probability that exactly 6 out of 10 randomly sampled 18-20 year-old consumed an alcoholic drink.

   (c) What is the probability that exactly four out of ten 18-20 year-old have not consumed an alcoholic beverage?

   (d) What is the probability that at most 2 out of 5 randomly sampled 18-20 year-old have consumed alcoholic beverages?

   (e) What is the probability that at least 1 out of 5 randomly sampled 18-20 year-old have consumed alcoholic beverages?

2. A husband and wife both have brown eyes but carry genes that make it possible for their children to have brown eyes (probability 0.75), blue eyes (0.125), or green eyes (0.125).

   (a) What is the probability the first blue-eyed child they have in their third child? Assume that the eye colors of the children are independent of each other.

   (b) On average, how many children would such a pair of parents have before having a blue-eyed child?

   (c) What is the standard deviation of the number of children they would expect to have until the first blue-eyed child?

3. Problem 2 introduces a couple with brown eyes who have 0.75 probability of having children with brown eyes, 0.125 probability of having with blue eyes, and 0.125 probability of having children with green eyes.

   (a) What is the probability that their first child will have green eyes and the second will not?

   (b) What is the probability that exactly one of their two children will have green eyes?

   (c) If they have sis children, what is the probability that exactly two will have green eyes?

   (d) If they have six children, what is the probability that at least one will have green eyes?

   (e) What is the probability that the first green eyed child will be the 4th child?

   (f) Would it be considered unusual if only 2 out of their 6 children had brown eyes?

4. Calculate the following probabilities and indicate which probability distribution model is appropriate in each case. You roll a fair die 5 times. What is the probability of rolling

   (a) the first 6 on the fifth roll?

   (b) exactly three 6s?

   (c) the third 6 on the fifth roll?

5. For a sociology class project you are asked to conduct a survey on 20 students at your school. You decide to stand outside of your dorm's cafeteria and conduct the survey on a random sample of 20 students leaving the cafeteria after dinner one evening. Your dorm is comprised of 45% males and 55% females.

   (a) Which probability model is the most appropriate for calculating the probability that the 4th person you survey is the 2nd female? Explain.

   (b) Compute the probability from (a).

   (c) The three possible scenarios that lead to 4th person you survey being the 2nd female are

   $$\{M, M, F, F\}, \{M, F, M, F\}, \{F, M, M, F\}$$

   One common feature among these scenarios is that the last trial is always female. In the first three trials there are 2 males and 1 female. Use the binomial coefficient to confirm that there are 3 ways of ordering 2 males and 1 female.

   (d) Use the findings presented in part (c) to explain why the formula for the coefficient for the negative binomial is $\binom{n-1}{x-1}$ while the formula for the binomial coefficient is $\binom{n}{x}$.

6. The number of defective components produced by a certain process in one day has a Poisson distribution with mean 20. Each defective component has probability 0.60 of being repairable.

   (a) Find the probability that exactly 15 defective components are produced.

   (b) Given that exactly 15 defective components are produced, find the probability that exactly 10 of them are repairable.

   (c) Let N be the number of defective components produced, and let X be the number of them that are repairable. Given the value of N, what is the distribution of X?

   (d) Find the probability that exactly 15 defective components are produced, with exactly 10 of them being repairable.

7. A very skilled court stenographer makes one typographical error (typo) per hour on average.

   (a) What probability distribution is the most appropriate for calculating the probability of a given number of typos this stenographer makes in an hour?

(b) Calculate the probability that this stenographer makes at most 2 typos in a given hour.

(c) What are the mean and the standard deviation of the number of typos this stenographer makes?

(d) Would it be consider unusual if this stenographer made 4 typos this stenographer makes?

8. Answer the questions with TRUE or FALSE and explain your answers as well.

(a) If a random variable Y only takes on values $\{0, 0.1, 0.2, 0.3, 0.4\}$, it is a discrete random variable.

(b) Outcome of each roll of a die follows Bernoulli distribution.

(c) If we have 5 independent identical Bernoulli trials, and Y =# of successes in the 5 trials, Y can take on only values $\{1, 2, 3, 4, 5\}$.

(d) Let X be a negative binomial with n and k, then X can take on values from 0.