STA 32 Winter 2023
Homework 5 - Due 11:59 PM, Friday, Feb 24th on Gradescope

Book Homework

1. In a particular state, a major flood happens once every 3 years on average. If you can assume floods are independent, and that two floods cannot happen simultaneously, find the following:
   (a) The probability that a flood happens within the first 5 years.
   (b) The probability that a flood happens after the first 2 years.
   (c) If there has been no flood for 4 years, what is the probability there is a flood in the next 2 years?
   (d) The expected time until a flood happens, and the standard deviation.
   (e) Find the 50th percentile of the time until a flood.

2. The average amount of time until a car accident on a particular 60 mile stretch of road is 60 minutes. Assume that car accidents are independent, and that two accidents cannot occur at the same time.
   (a) What is the probability of a car accident occurring in the two hours?
   (b) What is the probability of a car accident occurring between 45 and 75 minutes?
   (c) What is the variance of the time until a car accident occurs?
   (d) If a car accident has not happened in 3 hours, what is the probability it will happen in the next two hours?
   (e) What is the probability of three car accidents occurring in the two hours?

3. The wait time at a popular fast food restaurant is uniformly distributed between 1 and 15 minutes on average.
   (a) Find the probability that you wait less than 6 minutes.
   (b) Find the probability that you wait between 5 and 10 minutes.
   (c) What is the expected wait time, and the variance of wait time?
   (d) If you have waited 5 minutes already, what is the probability you wait an additional 5 minutes?

4. Weights of female cats of a certain breed are normally distributed with mean 4.1 kg and standard deviation 0.6 kg.
   (a) What proportion of female cats have weights between 3.7 and 4.4 kg?
   (b) A certain female cat has a weight that is 0.5 standard deviations above the mean. What proportion of female cats are heavier than this one?
   (c) A female cat is chosen at random. What is the probability that she weighs more than 4.5 kg?
   (d) Six female cats are chosen at random. What is the probability that exactly one of them weighs more than 4.5 kg?

5. Assume that IQ is normally distributed, with mean 100 and standard deviation 15.
   (a) What is the probability that a randomly selected persons IQ is over 120?
   (b) Find the value of Q_1, Q_2, and Q_3 for IQ.
   (c) Find the value of the lower $= Q_1 - 1.5(Q_3 - Q_1)$ and upper $= Q_3 + 1.5(Q_3 - Q_1)$ for IQ.
   (d) Find the probability of an outlier for IQ for a single person based on your values for (c).

6. Assume that the selling price of a particular item (call it X_1) is normally distributed, with mean 200 gold coins, and standard deviation 50 coins. Assume the selling price of another item is also normally distributed, with mean 230 gold coins, and standard deviation 10 gold coins (call it X_2). Assume these distributions are independent.
   (a) Find the probability the first item sold for more than the second item.
   (b) Find the probability that both items sold for over 400 gold pieces total.
   (c) Find the probability that the average of the two items sold for between 220 and 250 gold pieces.
   (d) Find the 30th percentile for the average price of the two items.
   (e) Find the 60th percentile for the total of the two items.
   (f) Find the value of the total price for both where only 10% of the time, both items will sell for more than that value.

7. ~~Drums labeled 30 L are filled with a solution from a large vat. The amount of solution put into each drum is random, with true mean 30.01 L and standard deviation 0.1 L.~~
   (a) ~~What is the probability that the total amount of solution contained in 50 drums is more than 1500 L?~~
   (b) ~~If the total amount of solution in the vat is 2401 L, what is the probability that 80 drums can be filled without running out?~~
   (c) ~~How much solution should the vat contain so that the probability is 0.90 that 80 drums can be filled without running out?~~
   (d) ~~Find the 10th percentile for the total amount of solution for 80 drums.~~

8. ~~The heights of men in a certain population follow a normal distribution with mean 69.7 inches and standard deviation 2.8 inches.~~

(a) If 10 men are selected at random, what is the probability their average height is above 72 inches?

(b) If exactly one man is selected at random, what is the probability his average height is above 72 inches?

(c) What is the 80^{th} percentile for the average height of 10 men?

(d) What is the probability the average height of 10 men is between 70 and 71 inches?

9. The population average cholesterol content of a certain brand of egg is 215 milligrams, and the standard deviation is 18 milligrams. (Eggs are randomly selected.)

(a) Find the probability the cholesterol content for a single egg is between 210 and 220.

(b) Find the probability the average cholesterol content for 36 eggs is between 210 and 220.

(c) Find the third quartile for the average cholesterol content for 36 eggs.

(d) If we are told the average for 36 eggs is less than 220 mg, what is the probability the average is greater than 210 mg?