STA32 Homework 4 Solution

Book Homework

1. (a) Let X be the number of 18-20 year-old students who consumed alcoholic beverage out of 10. Then $X \sim Binomial(10, 0.697)$. 1) Individuals are independent (random sample) 2) There are two possible outcomes - consumed alcoholic beverage or not. 3) Fixed n 4) Fixed p.Thus, the X is a binomial random variable.

   (b) $P(X = 6) = \binom{10}{6}(0.697)^6(0.303)^4 = 0.2029$.

   (c) Let Y be the number of 18-20 year-old students who **not** consumed alcoholic beverage out of 10. Then $X \sim Binomial(10, 0.303)$. Then $P(Y = 4) = \binom{10}{4}(0.303)^6(0.697)^4 = 0.2029$.

   (d) Let X be the number of 18-20 year-old students who consumed alcoholic beverage out of 5. Then $X \sim Binomial(5, 0.697)$. $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1671$.

   (e) $X \sim Binomial(5, 0.697)$. $P(X \geq 1) = 1 - P(X = 0) = 0.9974$.

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2. Let X be the number of children up to and including the first blue-eyed child they have. Then $X \sim Geo(0.125)$

   (a) $P(X = 3) = (1 - 0.125)^2(0.125) = 0.0957$.

   (b) $\mu_X = 1/p = 1/0.125 = 8$.

   (c) $\sigma_X = \sqrt{(1-p)/p^2} = \sqrt{(1-0.125)/0.125^2} = 7.4833$.

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3. (a) Let X_i be the ith child's eyes' color. $P(X_1 = G \cap X_2 = G^C) = P(X_1 = G)P(X_2 = G^C) = 0.125 * 0.875 = 0.1094$.

   (b) Let X be the number of children having green eyes out of 2. $X \sim Binomial(2, 0.125)$ $P(X = 1) = \binom{2}{1}(0.125)(1 - 0.125) = 0.2187$.

   (c) Let X be the number of children having green eyes out of 6. $X \sim Binomial(6, 0.125)$ $P(X = 2) = \binom{6}{2}(0.125)^2(1 - 0.125)^4 = 0.1374$.

   (d) $P(X \leq 1) = 1 - P(X = 0) = 1 - \binom{6}{0}(0.125)^0(1 - 0.125)^6 = 0.5512$.

   (e) Let Y be the number of children up to and including the first green eyed child. $P(Y = 4) = (1 - 0.125)^4(0.125) = 0.0733$.

   (f) Let X be the number of children having brown eyes out of 6, then $X \sim Bino(6, 0.75)$, $P(X = 2) = 0.033$. It would be considered unusual since the probability is less than 0.05.

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4. (a) Geometric $X \sim Geo(1/6)$, the probability the first 6 on the fifth roll is $(1 - 1/6)^4(1/6) = 0.0804$.

   (b) Binomial $X \sim Bino(6, 1/6)$, $P(X = 3) = \binom{6}{3}(1/6)^3(1 - 1/6)^3 = 0.0536$.

   (c) Negative binomial $X \sim NB(r = 3, p = 1/6)$. $P(X = 5) = \binom{5-1}{3-1}(1/6)^3(1 - 1/6)^2 = 0.0129$.

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5. (a) Negative Binomial Model because each survey is a Bernoulli trial, and we are interested in the number of trials up to and including 2nd success.

   (b) Negative binomial $X \sim NB(r = 2, p = 0.55)$. $P(X = 4) = \binom{4-1}{2-1}(0.55)^2(1 - 0.55)^2 = 0.1011$.

   (c) $\binom{4-1}{2-1} = \binom{3}{1=3}$.

   (d) We know the last trial is the success, so we only need to consider combination of $x - 1$ successes in the rest $(n - 1)$ trials.

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6. Let N be the number of defective components produced. Then $N \sim Poisson(20)$.
   (a) $P(N = 15) = e^{-20}\frac{20^{15}}{15!} = 0.0516$.
   (b) Let X be the number that are repairable. Then $X \sim Bin(15, 0.6)$.
   $P(X = 10) = {}_{15}C_{10}(0.6)^{10}(1 - 0.6)^{15-10} = 0.1859$.
   (c) Given $N, X \sim Bin(N, 0.6)$, or $X|N \sim Bin(N, 0.6)$
   (d) Let N be the number of defective components, and let X be the number that are repairable.
   $P(N = 15 \cap X = 10) = P(N = 15)P(X = 10|N = 15) = (e^{-20}\frac{20^{15}}{15!})({}_{15}C_{10}(0.6)^{10}(1 - 0.6)^{15-10}) = 0.00960$.

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7. Let X be the number of typo per hour, then $X \sim Poisson(1)$ because the mean is 1 typo per hour.
   (a) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.0153$
   (b) $\mu = 1, \sigma = 1$
   (c) $P(X = 4) = \frac{e^{1}1^{4}}{4!} = 0.0153$ which is less than 5%, so we can think it's unusual.

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8. (a) TRUE, there are gaps between the values that Y takes on.
   (b) FALSE, if there is more than two outcomes, the outcomes do not follow the Bernoulli distribution.
   (c) FALSE, Y can also take 0.
   (d) FALSE, let n is number of successes and k is the success probability. Then, the minimum number of trial is the number of successes not 0.
