Week 4 Project

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Problem 1

Calculate and compare the expected value and standard deviation of price at time t (P_t), given each of the 3 types of price returns, assuming $r_t \sim N(0, \sigma^2)$. Simulate each return equation using $r_t \sim N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

Problem 1

1.1 Classical Brownian Motion:

$$P_t = P_{t-1} + r_t$$

Expectation:

$$E[P_t] = E[P_{t-1} + r_t]$$
$$E[P_t] = E[P_{t-1}] + E[r_t]$$
$$E[P_t] = P_{t-1}$$

Standard deviation:

$$Std[P_t] = Std[P_{t-1} + r_t]$$
$$Std[P_t] = Std[P_{t-1}] + Std[r_t]$$
$$Std[P_t] = \sigma$$

Problem 1

1.2 Arithmetic Return System:

$$P_t = P_{t-1}(1 + r_t)$$

Expectation:

$$E[P_t] = E[P_{t-1} + P_{t-1} r_t]$$
$$E[P_t] = E[P_{t-1}] + E[P_{t-1} r_t]$$
$$E[P_t] = P_{t-1}$$

Standard deviation:

$$Std[P_t] = Std[P_{t-1} + P_{t-1} r_t]$$
$$Std[P_t] = Std[P_{t-1} r_t]$$
$$Std[P_t] = P_{t-1}\sigma$$

Problem 1

1.3 Log Return or Geometric Brownian Motion:

$$P_t = P_{t-1}e^{r_t}$$

Expectation:

$$E[ln(P_t)] = E[ln(P_{t-1}e^{r_t})]$$
$$E[ln(P_t)] = E[ln(P_{t-1})] + E[r_t]$$
$$E[ln(P_t)] = ln(P_{t-1})$$

Standard deviation:

$$Std[ln(P_t)] = Std[ln(P_{t-1}e^{r_t})]$$
$$Std[ln(P_t)] = Std[ln(P_{t-1})] + Std[r_t]$$
$$Std[ln(P_t)] = \sigma$$

Problem 1

Assume $P_{t-1} = 50, \sigma = 0.5$, we can get:

1. Classical Brownian Motion:

$$E[P_t] = 50$$
$$Std[P_t] = 0.5$$

2. Arithmetic Return System:

$$E[P_t] = 50$$
$$Std[P_t] = 25$$

3. Log Return or Geometric Brownian Motion:

$$E[ln(P_t)] = ln(50) = 3.91$$
$$Std[ln(P_t)] = 0.5$$

```
49.995860744615534  0.5048931170610959
49.79303723077689  25.24465585305479
3.9078837500436836  0.5048931170610959
```

We can see that the result from simulation matches our calculation.

Problem 2

Implement a function similar to the “return_calculate()” in this week’s code. Allow the user to specify the method of return calculation.

Use INTC.csv. Calculate the arithmetic returns for INTC. Remove the mean from the series so that the mean(INTC)=0

Calculate VaR

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a Historic Simulation.

Compare the 4 values. Look at the empirical distribution of returns, in sample.

Download from Yahoo! Finance the prices since the end of the data in the CSV file (about 3 months). Look the empirical distribution of returns, out of sample.

Problem 2

Normal VaR = 0.0358494343945957

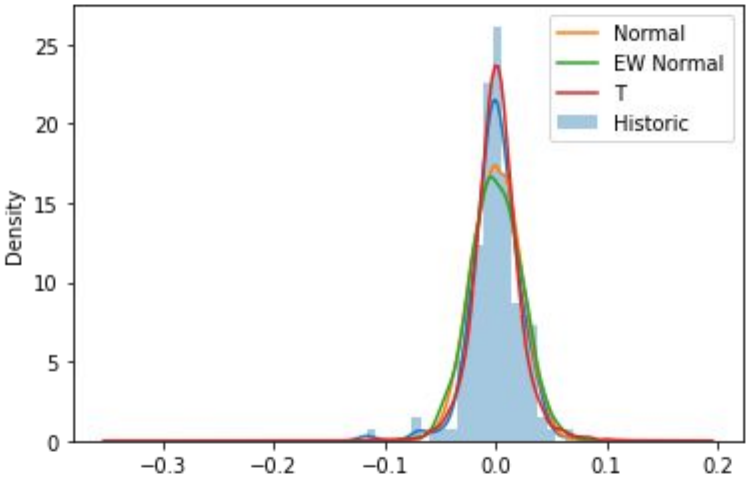
Exponentially Weighted Normal VaR = 0.03855888704529717

T Distribution VaR = 0.0327346066839301

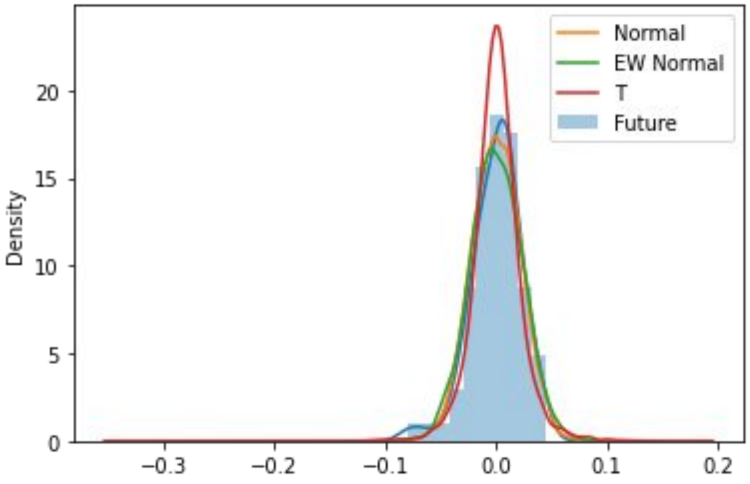
Historic VaR = 0.029574903865632305

Problem 2

On past data



On future data



We can see that the T distribution best describes the historic data, while the normal distribution best describes the future data.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings).

Discuss your methods, why you chose those methods, and your results.

Problem 3

We conducted Shapiro-Wilks test on the data and found the percentages of normally distributed returns for each portfolio are all around 50%. We cannot assume normal distribution for these portfolios. T distribution didn't perform well for prediction in problem 2. We choose to use historic VaR for these portfolios.

Problem 3

```
Portfolio A: 5329.25419921393
Portfolio B: 5579.825670338119
Portfolio C: 3320.2194874334964
Portfolio Total: 12593.869013199
```

Historic VaR.

```
Portfolio A: 6364.898362871958
Portfolio B: 5730.94788590254
Portfolio C: 3986.185355313238 7
Portfolio Total: 15427.41574229567 8
```

Historic VaR smoothed by KDE.