Multiphysics Modelling for Four States of Matter
Practical 3: The original ghost fluid method.

This practical will take the two-material-plus-level-set code you wrote for practical 2, and turn it into something more physically meaningful. The level set value $\phi = 0$ will now represent the material interface between your states $\mathbf{u}_1$ and $\mathbf{u}_2$.

For all these tests, within the computational domain there is always going to be one real material, and one ghost material. Initial data for each material should be specified everywhere, regardless of whether it is real or ghost (though the ghost fluid method will overwrite some of this). Similarly, it is recommended that you apply the ghost fluid boundary conditions to the entire ghost fluid region, don't worry about efficiency until things are working.

Outputting this data, and plotting the data, starts to become a bit more challenging. For plots, you only want to show the real data, but for debugging it is useful to output everything, this can make sure there is no unexpected behaviour happening in your ghost zones. If you are using gnuplot, a few useful commands are below. The assumption here is that all your output is in a single data file, with columns x, ρ_1, v_1, p_1, ρ_2, v_2, p_2, ϕ. Plotting real density can be done by:

```
> plot '<file>' using 1:($8 < 0 ? $2 : $5)
```

Here, we are giving the command to plot from column 2 (ρ_1) if column 8 is negative, i.e. $\phi < 0$, or column 5 (ρ_2) otherwise. This will create a plot in a single colour. You may wish each material to have its own colour, but only be plotted where it is the real material:

```
> plot '<file>' using 1:($8 < 0 ? $2 : 1/0),
    '<file>' using 1:($8 > 0 ? $5 : 1/0)
```

This will now only plot one variable where column 8 is negative, gnuplot just doesn't plot `inf` values (and 1/0 gives `inf`). Similarly, the second command on the same plot only plots the variable where column 8 is positive.

Exercises:

All these tests should be run on a domain $x \in [0, 1]$ and with level set function (either one gives an interface at $x = 0.5$),

$$\phi = x - 0.5 \quad \text{or} \quad \phi = 0.5 - x$$

All initial data is split into left (L) and right (R) states, left state is the real material for $x < 0.5$ and right for $x > 0.5$. It is up to you which sign convention you want to use for ϕ for the left and right states (i.e. which of the two functions in the equation above you are using).

1. **Stationary contact discontinuity**:

   (a) Two identical materials

$$(\rho, v, p)_L^T = (1, 0, 1)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = (0.5, 0, 1)^T, \quad \gamma_R = 1.4$$

   (b) Two different materials

$$(\rho, v, p)_L^T = (1, 0, 1)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = (0.5, 0, 1)^T, \quad \gamma_R = 1.67$$

   For this test, absolutely nothing should happen. Because nothing should happen, the final time your own choice, but $t = 0.2$ should be sufficient to show this

2. **Moving contact discontinuity**:

   (a) Two identical materials

$$(\rho, v, p)_L^T = (1, 0.5, 1)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = (0.5, 0.5, 1)^T, \quad \gamma_R = 1.4$$

   (b) Two different materials

$$(\rho, v, p)_L^T = (1, 0.5, 1)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = (0.5, 0.5, 1)^T, \quad \gamma_R = 1.67$$

   (c) The other direction

$$(\rho, v, p)_L^T = (1, -0.5, 1)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = (0.5, -0.5, 1)^T, \quad \gamma_R = 1.67$$

   For this test, the final time should be $t = 0.25$ and the exact solution should be recovered (the level set function should move to $x = 0.75$, or $x = 0.25$, depending on the sign of the velocity). Only the real density should show any evolution, velocity and pressure should not do anything - check this for yourself though.

3. **Toro's tests**:
Set up these tests for a $\gamma = 1.4$ ideal gas, as before. This time, though, the left material should take the left state everywhere, and the right material should take the right state everywhere, e.g. for test 1 we have

$$(\rho, v, p)_L^T = (1, 0, 1)^T, \quad (\rho, v, p)_R^T = (0.125, 0, 0.1)^T$$

For test 1, the results should look very familiar, but note what happens at the contact discontinuity. For the other tests, do not expect them all to work, which ones fail?

4. **Fedkiw's test A**:
Taken from Fedkiw *et al.* (section 5.2) this is a true two-material version of Toro's test 1,

$$(\rho, v, p)_L^T = \left(1, 0, 10^5\right)^T, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = \left(0.125, 0, 10^4\right)^T, \quad \gamma_R = 1.2$$

The final time is $t = 0.0007$, and results are shown in Figures 7 and 8 of the original paper show results. Compare your results to the plots in the paper for density, pressure and velocity (you don't need to worry about entropy) - or, if you have an exact multimaterial Riemann problem solver, compare to that.

5. **Fedkiw's test B**:
This test demonstrates the effects of a shock wave interacting with an interface in equilibrium, again taken from section 5.2 of Fedkiw *et al.*

$$(\rho, v, p)_L^T = \begin{cases} \left(1.333, 0.3535\sqrt{10^5}, 1.5 \times 10^5\right)^T & x < 0.05 \\ (1, 0, 10^5)^T & x > 0.05 \end{cases}, \quad \gamma_L = 1.4$$

$$(\rho, v, p)_R^T = \left(0.1379, 0, 10^5\right)^T, \quad \gamma_R = 1.67$$

This test should be run until $t = 0.0012$, and again density, pressure and velocity should be compared to the paper (or an exact solver, which can reproduce this test if you start from when the shock wave hits the interface). Does the behaviour of the density at the contact discontinuity change in any way for this test compared to the previous Riemann problem tests?