MATH 225 Winter 2017 Section 01---Prof. Dean

MIDTERM EXAM 2

Thurs. January 19, 2017

NAME (please print legibly): _____

Student ID Number: _____

- No calculators or notes are allowed on this exam.
- Please show all your work. You may use the backs of pages if necessary. You may not receive full credit for a correct answer if there is no work shown.
- Don't cheat.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	15	
5	15	
6	10	
7	10	
TOTAL	100	

1) (a) (10 points) Solve the initial value problem

$$2y'' - 9y' - 5y = 0; \quad y(0) = \frac{12}{5}, \quad y'(0) = 1$$

(b) (10 points) Solve the initial value problem

$$y'' - 2y' + 5y = 0; \quad y(0) = 3, \quad y'(0) = 1$$

2) **(20 points) Use the method of undetermined coefficients to find the general solution of the equation**

$$y'' + 4y' + 4y = 4e^{-2t}; \quad y(0) = 5, \; y'(0) = -4$$

3) (20 points) Use variation of parameters to find the general solution of the equation

$$y'' + 9y = \sec 3x$$

4) **(15 points) An 8-lb weight is attached to a frictionless spring, that in turn is suspended from the ceiling. The weight stretches the spring $\frac{8}{9}$ ft and comes to rest in its equilibrium position. The weight is then pushed up 4 inches and released with a downward velocity of $2\sqrt{3}$ ft/sec. Find the initial value problem that describes the motion of the weight, and solve it, writing your solution in the form $u(t) = R\cos(\omega_0 t - \delta)$.**

5) (15 points) A 32-lb weight is attached to a spring suspended from the ceiling. The spring constant is $k = 5$, and the damping constant is $d = 4$. The weight is then pushed down 3 inches, and is released with a downward velocity of 9 in./sec.

(a) Determine the motion of the weight, simplifying your answer into a single term.

(b) Determine the damped amplitude, damped frequency, and damped period of the motion.

6) **(10 points) A mass of 9 slugs is hanging at rest on a frictionless spring whose constant is $k = 16$. Beginning at time $t = 0$, an external force of $F(t) = 4\cos \omega t$ is applied to the system.**

   (a) What is the angular frequency of the forcing function that is in resonance with the system?

   (b) Find the equation of motion of the mass with resonance.

7) (10 points) Use the definition of Laplace transform to find the Laplace transform of the function

$$f(t) = \begin{cases} e^{2t}, & 0 \le t < 2 \\ t, & t \ge 2 \end{cases}$$