MATH 225 Winter 2017 Section 01---Prof. Dean

MIDTERM EXAM 1

Thurs. January 12, 2017

NAME (please print legibly): _____

Student ID Number: _____

- No calculators or notes are allowed on this exam.
- Please show all your work. You may use the backs of pages if necessary. You may not receive full credit for a correct answer if there is no work shown.
- Don't cheat.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	10	
6	20	
TOTAL	100	

1) (a) (10 points) Find the general solution of the equation

$$x^2 \frac{dy}{dx} - 4xy = x^6 \cos 2x$$

(b) (10 points) Find the explicit solution of the initial value problem

$$\frac{dy}{dx} = \frac{\sqrt{2x+1}}{y^2}, \quad y(4) = 2$$

2) **(a) (10 points) A 360 g sample of a certain radioactive substance decays to 240 g in 5 months. How long would it take for the sample to decay to 40 g?**

(b) (10 points) A drink whose temperature is $44^{\circ}\mathrm{F}$ is placed in a $68^{\circ}\mathrm{F}$ room. After 8 minutes, the drink is $50^{\circ}\mathrm{F}$. How long will it take for the drink to warm to $60^{\circ}\mathrm{F}$?

3) **(20 points) Initially, a full 80-gallon tank contains a salt solution with a concentration of 0.10 lb/gal. A salt solution with a concentration of 1.5 lb/gal flows into the tank at a rate of 2 gal/min. The mixture is kept well-stirred and flows out of the tank at a rate of 4 gal/min. Find $Q(t)$, the amount of salt in the tank until the tank empties.**

4) (a) (10 points) Find the explicit solution of the initial value problem

$$\left(2y^3 - \frac{5}{2\sqrt{x}}\right) dx + 6xy^2\, dy = 0, \;\; y(4) = -1$$

(The equation is exact. You do not need to verify that first.)

(b) (10 points) A object weighing $mg = 12.8$ lb falls from rest, with coefficient of air resistance $k = 0.2$. Find its velocity when $t = 5$ seconds, and the total distance fallen by the object in the first 5 seconds.

5) **(10 points) Use Euler's method with step size $h = 0.1$ to approximate the solution to the following initial value problem at points $x = 3.1$ and $x = 3.2$.**

$$y' = y^2 + xy, \quad y(3) = 1$$

6) **(a) (10 points) Given that the functions $y_1(x) = e^{-4x}$ and $y_2(x) = xe^{-4x}$ are solutions of the differential equation**

$$y'' + 8y' + 16y = 0$$

(you do not need to verify that), use the Wronskian to show that y_1 and y_2 are linearly independent, and then find the solution of the initial value problem

$$y'' + 8y' + 16y = 0; \quad y(0) = -2, \; y'(0) = \frac{19}{2}$$

(b) (10 points) Given that $y_1(x) = x^4$ is a solution of

$$x^2y'' + 2xy' - 20y = 0$$

use reduction of order to find a second linearly independent solution for $x > 0$.