BỘ GIÁO DỤC VÀ ĐÀO TẠO

TRƯỜNG ĐẠI HỌC BÁCH KHOA HÀ NỘI

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Tên công trình:

TRACKING CONTROL FOR A RANGE OF MECHANICAL SYSTEMS UNDER INPUT CONSTRAINTS

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SUMMARY

A tracking control scheme is proposed for a class of multiple-input and multiple-output mechanical systems under actuator saturation. The actuator saturation compensator is adopted to attenuate the adverse effects of actuator saturation. Then, the backstepping method is applied to design the tracking control law. The designed control law is proved to guarantee the tracking trajectories and the global ultimate stability of the closed-loop control system. Simulations on a two-link robotic manipulator shows the effectiveness of the proposed control scheme. Furthermore, part of a solution to mechanical systems with constraint force is suggested for feature research.

Keywords: actuator saturation, backstepping method, mechanical systems,

I. INTRODUCTION

The motion of a physical system such as robotic manipulators can be illustrated by MIMO mechanical systems. The control signals of that systems might be restricted because of the physical constraint of actuators. Then, the perfomance and even stability of the control systems can be negatively affected. Hence, the control regulations of the MIMO mechanical systems should be able to deal with the adverse effects of actuator saturation.

The actuator saturation problem seems not to be considered widely by the previous works.

Therefore, considering actuator saturation, we propose a tracking control scheme for the MIMO nonlinear mechanical systems via an actuator saturation compensator with the backstepping method. The actuator saturation compensator is used to eliminate the effects of actuator saturation.

The main contribution of this paper is that an actuator saturation compensator with the input control error between the actual and the command control signals, is introduced into the backstepping design to reduce the saturation effects.

II. RESEARCH RESULTS

2.1 Problem formulation

Consider the nonlinear mechanical systems subject to actuator saturation (without unknown disturbances):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mathrm{g}(q) = \tau \qquad (1)$$

where $q \in R^n, \dot{q} \in R^n, and\ \ddot{q} \in R^n$ are the generalized position, velocity, and acceleration vectors, respectively. Vector q is taken as the system output vector, $M(q) \in R^{nxn}$ is the generalized inertia matrix, $C(q,\dot{q}) \in R^{nxn}$ is the generalized centripetal-Coriolis matrix, $\mathrm{g}(q) \in R^{nxn}$ is the generalized gravity vector, $\tau \in R^{nxn}$ is the actual control input vector.

Property 1. The inertia matrix $M(q)$ is positive definite and symmetric.

Property 2. The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric.

In practice, the saturation can be described as:

$$\tau_i = sat(\tau_{ci}) = \begin{cases} sign(\tau_{ci})\tau_{Mi}, & |\tau_{ci}| \geq \tau_{Mi} \\ \tau_{ci}, & |\tau_{ci}| < \tau_{Mi} \end{cases} \qquad (2)$$

where $\tau_{Mi} > 0$ (i = 1, 2, …, n) denotes the saturation limit, $\tau_c = [\tau_{c1}, \ldots, \tau_{cn}]^T$ denotes the command control input vector calculated by the control law to be designed later.

Control objective: a tracking control law τ_c is designed for the MIMO nonlinear mechanical system (1) with actuator saturation (2), such that all signals of the closed-loop control system are bounded and the system output vector q can track the desired trajectory $q_d(t) \in R^n$, where $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are all bounded.

2.2 Tracking control design

A tracking control law will be designed for the MIMO nonlinear mechanical system (1) by integrating an actuator saturation compensator into the backstepping method. The schematic of the closed-loop control system is illustrated in Figure 1.

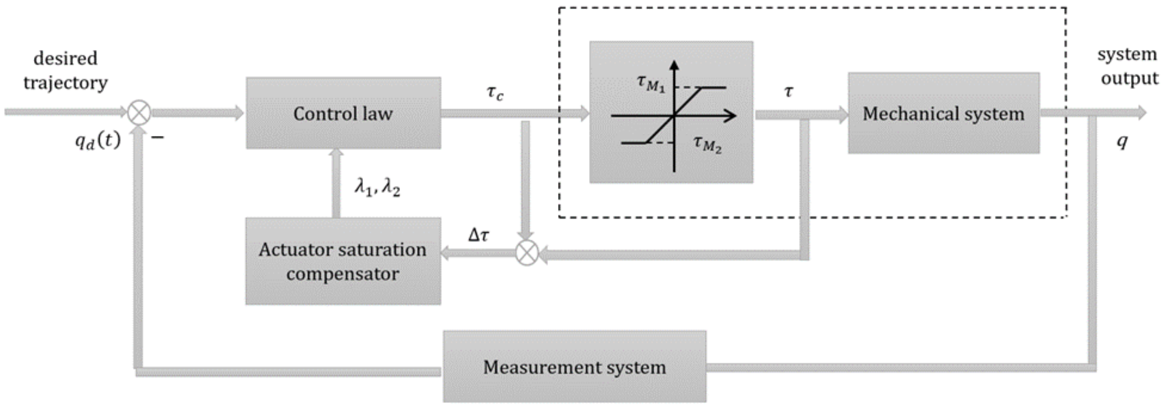

Figure 1 Schematic of closed-loop control system of mechanical system

To facilitate the control design, define the coordinate changes

$$\zeta_1 = q - q_d(t) - \lambda_1 \qquad (3)$$

$$\zeta_2 = \dot{q} - \alpha - \lambda_2 \qquad (4)$$

where $\alpha \in R^n$ is the intermediate control function vector to be designed later, and $\lambda_1 \in R^n$ and $\lambda_2 \in R^n$ are generated by the following actuator saturation compensator:

$$\begin{cases} \dot{\lambda}_1 = -K_1\lambda_1 + \lambda_2 \\ \dot{\lambda}_2 = M^{-1}(q)(-K_2\lambda_2 + \Delta\tau) \end{cases} \qquad (5)$$

where $K_1 = K_1^T \in R^2$ and $K_2 = K_2^T \in R^2$ are $n\ x\ n$ positive-definite design matrices and $\Delta\tau = \tau - \tau_c$ denotes the control deviation between actual and command control vectors.

It is indicated from (5) that the signals λ_1 and λ_2 are the filtered versions of the saturation effects on the variables being controlled. When the actuator saturation occurs, $\Delta\tau \neq 0$, the constructed actuator saturation compensator responses such that the signals λ_1 and λ_2 are used to modify the position errors ζ_1 and ζ_2 for attenuating the adverse effects of actuator saturation.

The control design procedure is presented as follows.

Step 1. Along with the trajectory of (5), it can be verified that

$$\dot{\zeta}_1 = \dot{q} - \dot{q}_d(t) + K_1\lambda_1 - \lambda_2 \qquad (6)$$

For (6), select the Lyapunov function candidate

$$V_1 = \frac{1}{2}\zeta_1^T\zeta_1 \qquad (7)$$

According to (4) and (6), the time derivative of (7) is

$$\dot{V}_1 = \zeta_1^T[\zeta_2 + \alpha - \dot{q}_d(t) + K_1\lambda_1] \qquad (8)$$

Let the intermediate control function vector α be

$$\alpha = -K_1\zeta_1 - K_1\lambda_1 + \dot{q}_d(t) \qquad (9)$$

Substituting (9) into (8) yields

$$\dot{V}_1 = -\zeta_1^T K_1 \zeta_1 + \zeta_1^T \zeta_2 \qquad (10)$$

Step 2. In the light of (1), (5) and (8), the time derivative of (4) is

$$\dot{\zeta}_2 = M^{-1}(q)[-C(q,\dot{q})\dot{q} - g(q) + \tau_c + \Delta\tau - M(q)\dot{\alpha} + K_2\lambda_2 - \Delta\tau]$$
$$= M^{-1}(q)[-C(q,\dot{q})\dot{q} - g(q) + \tau_c - M(q)\dot{\alpha} + K_2\lambda_2] \qquad (11)$$

For (6) and (11), select the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\zeta_2^T M(q)\zeta_2 \qquad (12)$$

In the light of (10), (11) and Property 2, the time derivative of (12) is

$$\dot{V}_2 = \dot{V}_1 + \zeta_2^T M(q)\dot{\zeta}_2 + \frac{1}{2}\zeta_2^T \dot{M}(q)\zeta_2$$
$$= -\zeta_1^T K_1\zeta_1 + \zeta_1^T\zeta_2 + \zeta_2^T\left[-C(q,\dot{q}) + \frac{1}{2}\dot{M}(q)\right]\zeta_2 + \zeta_2^T[-C(q,\dot{q})(\alpha+\lambda_2) - g(q) + \tau_c - M(q)\dot{\alpha} + K_2\lambda_2]$$
$$= -\zeta_1^T K_1\zeta_1 + \zeta_1^T\zeta_2 + \zeta_2^T[-C(q,\dot{q})(\alpha+\lambda_2) - g(q) + \tau_c - M(q)\dot{\alpha} + K_2\lambda_2] \qquad (13)$$

Design the control law as follows:

$$\tau_c = -\zeta_1 - K_2\zeta_2 + C(q,\dot{q})(\alpha+\lambda_2) + g(q) + M(q)\dot{\alpha} - K_2\lambda_2 \qquad (14)$$

2.3 Stability analysis

We will analyze the stability of the closed-loop control system under the control law in the presence of actuator saturation.

The main results are summarized in the following theorem.

Theorem. For the MIMO nonlinear mechanical system (1) with actuator saturation (2), by designing the actuator saturation compensator (5), and the control law (14), the system output vector q tracks the desired trajectory $q_d(t)$ and the closed-loop control system is guaranteed to be globally uniformly ultimately stable.

Proof. Select the Lyapunov function candidate $V_2 = V_1 + \frac{1}{2}\zeta_2^T M(q)\zeta_2$ (15)

From (13) and (14), we have:

$$\dot{V}_2 = \dot{V}_1 + \zeta_2^T M(q)\dot{\zeta}_2 + \frac{1}{2}\zeta_2^T\dot{M}(q)\zeta_2 = -\zeta_1^T K_1\zeta_1 - \zeta_2^T K_2\zeta_2$$
$$\leq -k.(\|\zeta_1\|^2 + \|\zeta_2\|^2) \qquad (16)$$

Where $k = min\{\lambda_{min}(K_1), \lambda_{min}(K_2)\}$ with $\lambda_{min}(.)$ being the minimum eigenvalue of a matrix.

It can be seen that V_2 is positive definite and $\dot{V}_2$ is negative semidefinite. So that based on Lyapunov stability theory, we have

$$\lim_{t\to\infty}\|\zeta_1\| = \lim_{t\to\infty}\|q - q_d(t) - \lambda_1\| = 0 \qquad (17)$$

Next, we consider another Lyapunov function candidate:

$$V_\lambda = \frac{1}{2}{\lambda_1}^T\lambda_1 + \frac{1}{2}{\lambda_2}^T M(q)\lambda_2 \qquad (18)$$

In the light of (5) and Young's inequality, the time derivative of (18) is

$$\dot{V_\lambda} = -\lambda_1{}^T K_1 \lambda_1 + \lambda_1{}^T \lambda_2 - \lambda_2{}^T K_2 \lambda_2 + \lambda_2{}^T \Delta\tau + \frac{1}{2}\lambda_2{}^T \dot{M}(q)\lambda_2$$

$$\leq -\lambda_1{}^T\left(K_1 - \frac{1}{2}I\right)\lambda_1 - \lambda_2{}^T\left(K_2 - I - \frac{1}{2}m.I\right)\lambda_2 + \frac{1}{2}\|\Delta\tau\|^2$$

$$\leq -\alpha_\lambda(\|\lambda_1\|^2 + \|\lambda_2\|^2) + \frac{1}{2}C^2 \qquad (19)$$

Where $\alpha_\lambda = min\left\{\lambda_{min}\left(K_1 - \frac{1}{2}I\right), \lambda_{min}\left(K_2 - I - \frac{1}{2}m.I\right)\right\}$ and $C \geq 0$ is the upper bound of $\|\Delta\tau\|$. So we should choose $K_1$and K_2 to satisfy:

$$\lambda_{min}(K_1) > \frac{1}{2}\ , \lambda_{min}(K_2) > 1 + \frac{1}{2}m \qquad (20)$$

We aslo obtain:

$$0 \leq V_\lambda \leq \frac{C^2}{2\alpha_\lambda} + \left[V_\lambda(0) - \frac{C^2}{2\alpha_\lambda}\right]e^{-\alpha_\lambda t} \qquad (21)$$

So that:

$$\lim_{t\to\infty}\|\lambda_1\| = \frac{C}{\sqrt{\alpha_\lambda}} \qquad (22)$$

And, we get:

$$\lim_{t\to\infty}\|q - q_d(t)\| = \frac{C}{\sqrt{\alpha_\lambda}} \qquad (23)$$

As we can see, we can have $\|q - q_d(t)\|$ small as possible as we want by adjusting the matrices $K_1$and K_2. Theorem is thus proved.

2.4 Simulation

To verify the effectiveness of the proposed tracking control scheme, the simulations are given on a two-link robotic manipulator (Figure 2) with time-varying disturbances under actuator saturation.

The motion mathematical model of the two-link robotic manipulator is

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \text{g}(q) = \tau \qquad (15)$$

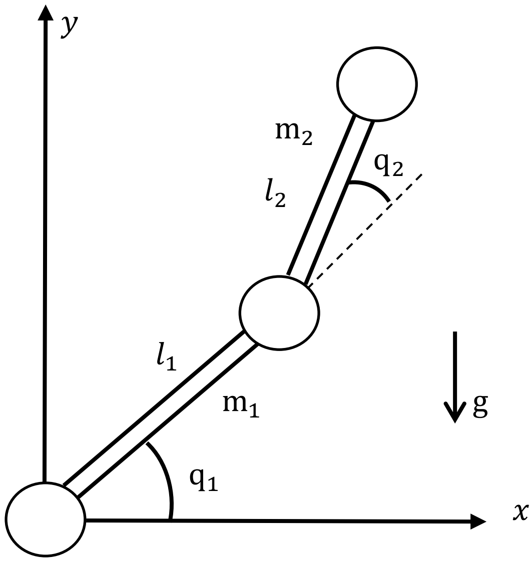

Figure 2 Coordinate of a two-link robotic manipulator

expressed in detail as follows:

$$M(q) = \begin{bmatrix} r_1 + r_2 + 2r_3\cos(q_2) & r_2 + r_3\cos(q_2) \\ r_2 + r_3\cos(q_2) & r_2 \end{bmatrix} \tag{16}$$

$$C(q, \dot{q}) = \begin{bmatrix} -r_3\,\dot{q_2}\sin(q_2) & -r_3(\dot{q_1} + \dot{q_2})\sin(q_2) \\ r_3\dot{q}_1\sin(q_2) & 0 \end{bmatrix} \tag{17}$$

$$\mathrm{g}(q) = \begin{bmatrix} r_4 \mathrm{g} cos(q_1) + r_5 \mathrm{g} cos(q_1 + q_2) \\ r_5 \mathrm{g} cos(q_1 + q_2) \end{bmatrix} \tag{18}$$

where $r_1 = m_1 l_1^2 + m_2 l_1^2 + I_1, r_2 = m_2 l_2^2 + I_2, r_3 = m_2 l_1 l_2, r_4 = m_1 l_2 + m_2 l_1$ and $r_5 = m_2 l_2$. The model parameters are $m_1 = 2.0\text{kg}, m_2 = 0.85\text{kg}, \mathrm{g} = 9.8,\ l_1 = 0.35m, l_2 = 0.31\text{m}, I_1 = 61.25 \times 10^{-3}\ \text{kgm}^2$ and $I_2 = 20.42 \times 10^{-3}\ \text{kgm}^2$. The actuator saturation limits are τ_{M_1}= 15 and τ_{M_2} = 4. The control objective is to make the manipulator position track the desired trajectory

$$q_d(t) = \begin{bmatrix} q_{1d} \\ q_{2d} \end{bmatrix} = \begin{bmatrix} 0.2 - 0.2\cos(\pi t) \\ 0.2 + 0.2\sin(\pi t) \end{bmatrix} \tag{19}$$

According to the proposed control scheme, we define the following coordinate changes:

$$\zeta_1 = q - q_d(t) - \lambda_1 \tag{20}$$

$$\zeta_2 = \dot{q} - \alpha - \lambda_2 \tag{21}$$

Let the intermediate control function vector $\alpha \in R^2$ be

$$\alpha = -K_1\zeta_1 - K_1\lambda_1 + \dot{q_d}(t) \tag{22}$$

The signals $\lambda_1 \in R^2$ and $\lambda_2 \in R^2$ are generated by the actuator saturation compensator

$$\begin{cases} \dot{\lambda_1} = -K_1\lambda_1 + \lambda_2 \\ \dot{\lambda_2} = M^{-1}(q)(-K_2\lambda_2 + \Delta\tau) \end{cases} \tag{23}$$

Where $K_1 = K_1^T \in R^2$ and $K_2 = K_2^T \in R^2$ are design matrices and $\Delta\tau = \tau - \tau_c$. Then, we design the control law as follows:

$$\tau_c = -\zeta_1 - K_2\zeta_2 + C(q,\dot{q})(\alpha + \lambda_2) + g(q) + M(q)\dot{\alpha} - K_2\lambda_2 \tag{24}$$

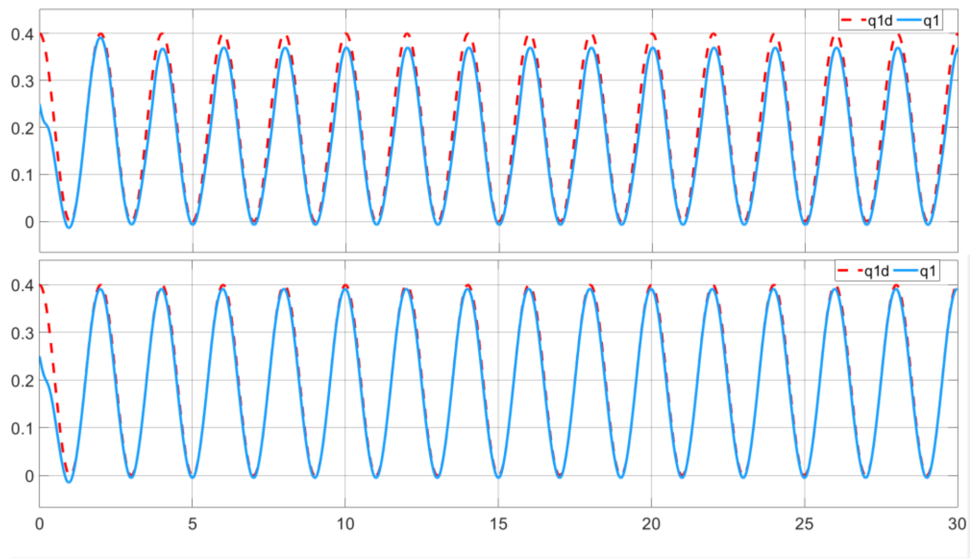

Figure 3 Position q1 tracking performance comparison

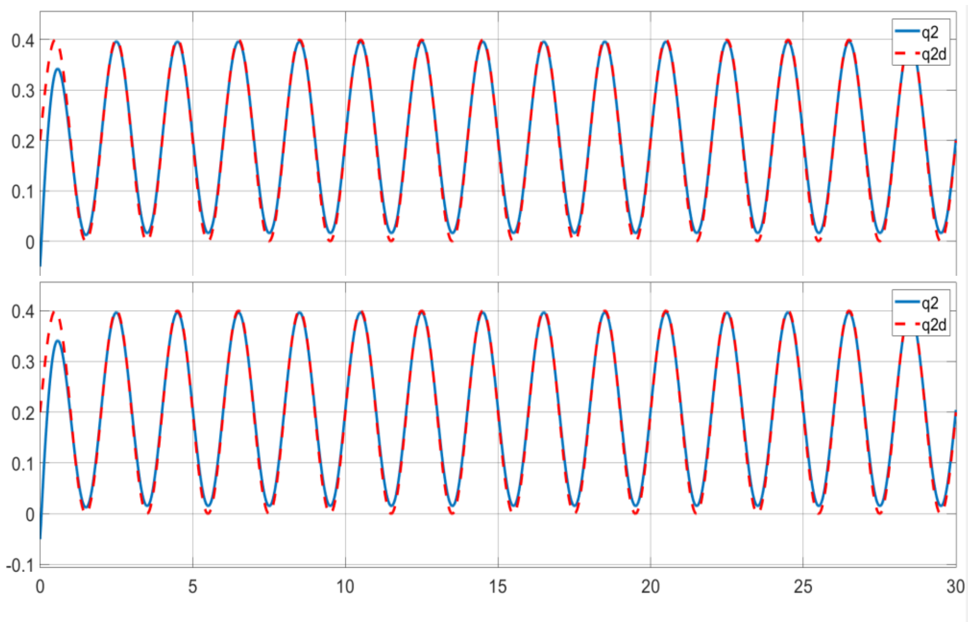

Figure 4 Position q2 tracking performance comparison

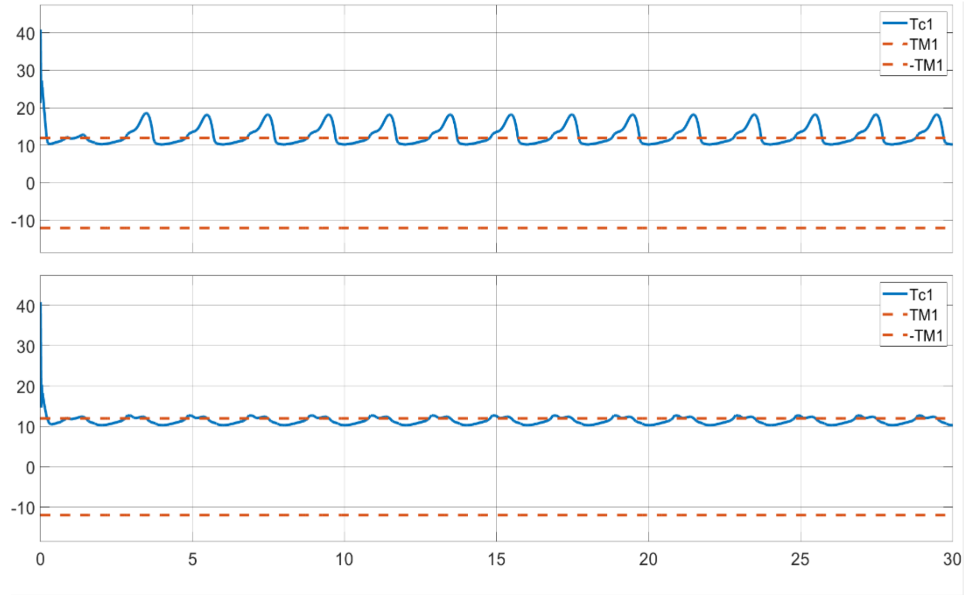

Figure 6 Control input signals τ_{c1} comparison

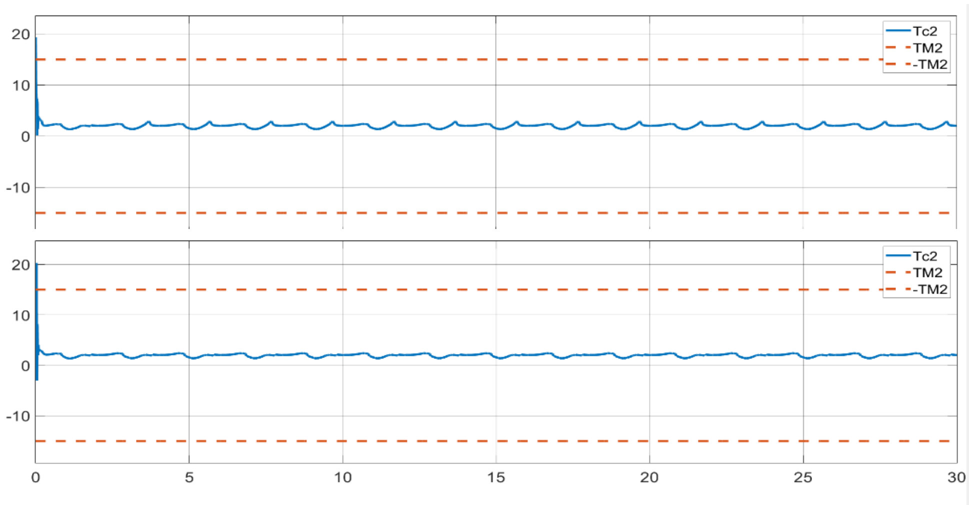

Figure 5 Control input signals τ_{c2} comparison

The simulation results are plotted in Figures 3 to 6. It can be seen from Figure 3 and 4 that the positions of the robotic manipulator track the desired trajectories with the satisfactory control performance. It can be seen from Figure 5 and 6 that the control input signals are bounded and reasonable.

2.5 Expand to mechanical systems with force constraints

The model system with force constraints:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J^T.\lambda_h \qquad (25)$$

There exists a matrix $L(q)$ that satisfies:

$$L^T.J^T = 0 \qquad (26)$$

Multiplying L^T on both sides of (1), we can obtain:

$$M_L\ddot{q} + C_L\dot{q} + G_L = L^T\tau \qquad (27)$$

Where

$$M_L = L^T.M, C_L = L^T.C, G_L = L^T.g \qquad (28)$$

This is the model simplified. The advantage it brings is that we can ignore the force constraints to control our system as before. But when it comes to the task of motion/force control, we have to recover λ_h. And it is done by the following steps:

From (28) we have:

$$J^T.\lambda_h = -(M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mathrm{g}(q)) + \tau \qquad (29)$$

The idea is that we multiply on both side (25) by matrix Z which satisfies:

$$\begin{cases} Z.J^T = I & (30) \\ Z.M(q).L(q) = 0 & (31) \end{cases}$$

And we have

$$L^T.J^T = 0 \qquad (32)$$

To guarantee (30), we choose Z in the form of

$$Z = (K.J^T)^{-1}.K \qquad (33)$$

To guarantee (31), we must have

$$(K.J^T)^{-1}.K.M.L = 0 \qquad (34)$$

So we can easily choose

$$K = J.M^{-1} \qquad (35)$$

In conclusion, we can recover the force constraint by

$$\lambda_h = Z.[-(M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mathrm{g}(q)) + \tau] \qquad (36)$$

Where

$$Z = (J.M^{-1}.J^T)^{-1}.J.M^{-1} \qquad (37)$$

III. CONCLUSION

Under actuator saturation, a tracking control scheme for the MIMO nonlinear mechanical systems has been proposed. By designing an compensator with the backstepping method, the influences of actuator saturation are minimized. The proposed control theme makes the system outputs stably follow the desired trajectories and guarantees the global uniform ultimate stability of the closed-loop control system.

The future research will develop a tracking motion/force control scheme for the MIMO nonlinear mechanical systems simultaneously with constraints (force and actuator).

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