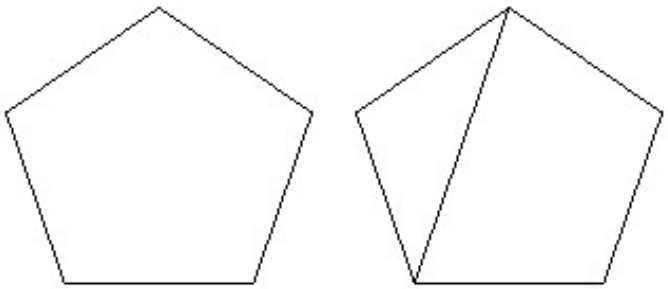
0.1 Transforming the Network

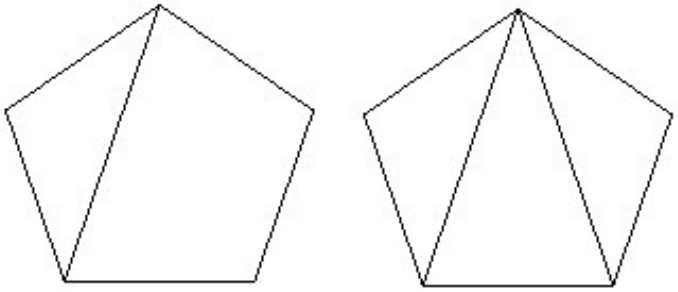
There are three types of operation which we can perform upon our network. We'll introduce three steps involving these.

Step 1 We start by looking at the polygonal faces of the network and ask: is there a face with more than three sides? If there is, we draw a diagonal as shown in the diagram below, splitting the face into two smaller faces.



Obr. 1: Dividing faces

We repeat this with our chosen face until the face has been broken up into triangles.

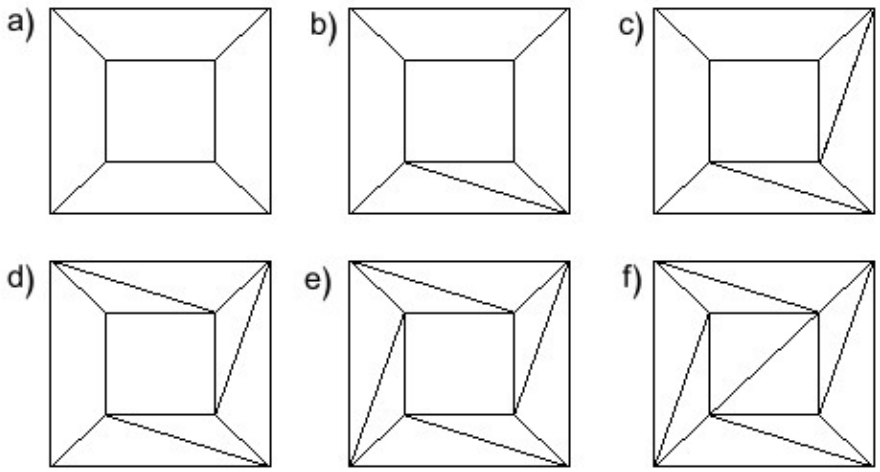


Obr. 2: In the end we are left with triangular faces.

If there is a further face with more than three sides, we use Step 1 on that face until it too has been broken up into triangular faces. In this way, we can break every face up into triangular faces, and we get a new network, all of whose faces are triangular. We illustrate this process by showing how we would transform the network we made from a cube.

We go back to **Step 1**, and look at the network we get after performing **Step 1** just once. Now, by drawing a diagonal we added one edge. Our original face has become two faces, so we have added one to the number of faces. We haven't changed the number of vertices. The network now has V vertices, $E+1$

edges and $F+1$ faces. So how has $V-E+F$ changed after we performed **Step 1** once? Using what we know about the changes in V , E and F we can see that



Obr. 3: This is what happens to the cube's network as we repeatedly perform **Step 1.**

V - E + F has become V - (E+1) + (F+1). Now we have:

$$V-(E+1)+(F+1)=V-E-1+F+1=V-E+F \tag{1}$$

So V - E + F has not changed after **Step 1**! Because each use of **Step 1** leaves V - E + F unchanged, it is still unchanged when we reach our new network made up entirely of triangles! The effect on V - E + F as we transform the network made from the cube is shown in the Table 1.

Tabuľka 1: Steps of the Euler law derivation.

Round	V	E	F	$V-E+F$
(a)	8	12	6	2
(b)	8	13	7	2
(c)	8	14	8	2
(d)	8	15	9	2
(e)	8	16	10	2
(f)	8	17	11	2