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Discrete Active Disturbance Rejection Iterative Learning Control Based on Dynamic Linearization

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Abstract: A discrete active disturbance rejection iterative learning control method based on dynamic linearization is proposed for a class of discrete-time, nonlinear and non-affine system that run repeatedly within a finite time. The controlled system is dynamically linearized into an affine form related to the control input within the iteration domain. The control gain is initialized through the pseudo partial derivative of dynamic linearization model when needed and then fixed. The estimated errors of parameter, system uncertainty and external disturbance are compacted into a nonlinear term as the total disturbance of the system. Via iterative sliding mode scheme, the iterative extended state observer is designed to estimate the total disturbance and a discrete active disturbance rejection iterative learning control law is proposed. The convergence of the iterative extended state observer and tracking errors of the system are analyzed. The proposed method is a new intuitive and concise data-driven control method which does not need the system model information. The effectiveness of the proposed method is verified by simulations.

Key Words: dynamic linearization, active disturbance rejection control, extended state observer, sliding mode control, iterative learning control, data driven control

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1 Introduction

Iterative learning control (ILC) is an effective control method for repeated operations in a limited interval. It improves the control performance by updating the control input based on the previous control input and tracking error iteration by iteration. However, the actual control system is often a nonlinear time-varying system with disturbance or uncertain initial conditions, traditional ILC may lead to undesirable large transient behavior)[1, 2].

To improve ILC performance and apply it to non-strict pe- oriodic situations, some scholars introduced the idea of model free adaptive control (MFAC)[3–6] into iterative learning process. MFAC is an effective method to deal with uncertainties through establishing dynamic linearization model. By extending dynamic linearization method in time domain to the iterative domain, data-driven optimal ILC(DDOILC)[7–10] is proposed. It mainly relies on the system I/O data, while does not require the system to meet the same initial conditions and global Lipschitz conditions. However, the external disturbances of the original nonlinear system is not additionally considered. Moreover, though uncertainty of system has been compacted into a new term called pseudo partial derivative (PPD) of the linearized model, if the uncertainty and nonlinearity of the controlled system are too strong, the dynamic characteristics of the PPD will become so complex that it is difficult to effectively be estimated.

Active Disturbance Rejection Control (ADRC)[11], is also used to deal with system uncertainties and disturbances. One of the key components in ADRC is the extended state observer (ESO), through which the uncertainties and disturbances are online estimated and then rejected by compensating them into the control input. Inspired by ADRC, [12] proposed a data-driven optimal ILC based on iterative ESO (IESO-DDOILC). This method proposes improved iterative dynamic linearization, which compacts system uncertainty and periodic disturbances into a nonlinear term, and designs a new IESO to estimate the nonlinear term. As for pursuing the perfection and accuracy of a dynamic linearization model is not necessary in data- driven control, a simplified IESO-DDOILC is proposed in [13]. This method proposes a simplified update law of PPD and shows the estimated error of parameter can be estimated by IESO and compensated in the control law.

Also, there are some other jobs introducing the idea of ADRC into iterative learning process. [14] extends the time-domain uncertainty compensation in ADRC to the periodic iterative domain, and then proposes IESO for the first time. IESO is designed as a sliding mode surface of tracking error function. The convergence is proved in details using the Lyapunov-like method. [15] proposed an ADRC based ILC (ADRILC) based on IESO. This method has better convergence rate than traditional ILC, but also makes up for the limitation of ADRC for periodic process. However, unlike DDOILC proposed for discrete nonlinear systems, this ADRILC is designed for continuous systems. As for time-varying discrete systems with strong nonlinearities, there are still more job to do for ADRILC.

In order to develop ADRILC in discrete-time nonlinear systems directly, this paper proposes a discrete active disturbance rejection iterative learning control method based on dynamic linearization. First, the controlled system is dynamically linearized along the iterative axis. Secondly, inspired by ADRC, a gain constant is used to roughly estimate the PPD in the dynamic linearization model. The estimated error of PPD, uncertainty, and external disturbance are compacted into the so-called total disturbance and estimated by the designed IESO, thereby reducing the dynamic complexity of the parameter. Finally, the idea of sliding mode control is extended to the periodic iteration domain, and the discrete ADRILC law is derived by the iterative sliding mode surface and the reaching law. Theoretical analysis and simulation results prove the effectiveness of the proposed method. Compared with DDOILC, the proposed method is simpler

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and more intuitive. It also makes up for the deficiency of ADRILC in the control of fast time-varying nonlinear discrete systems. The proposed method does not require system model information, so it is a kind of data-driven control.

This paper is organized as follows. Section 2 is the basic problem description, giving the paradigm of the active disturbance rejection model based on iterative dynamic linearization. Section 3 presents the design of IESO and the DL-ADRILC. In Section 4, the mathematical proof of the convergence of the IESO estimated error and the tracking error of system are given. In Section 5, simulations are given to verify the effectiveness of the proposed method. Finally, section 6 summarize the work of this paper.

2 Problem Formulation

Consider a class of discrete-time, nonlinear and non-affine SISO system that run repeatedly within a finite time:

$$\begin{array}{l} y(k,t+1) = \\ f(y(k,t), y(k,t-1), \cdots, y(k,t-n_y), u(k,t), \\ u(k,t-1), \cdots, u(k,t-n_u)) + d(k,t+1), \end{array} \tag{1}$$

where $u(k,t)$ and $y(k,t)$ represent the input and output data of the system respectively, $k = 1,2,3,\cdots$ repretents the k-th iteration, t represents discrete time,$f(\cdot)$ represents an unknown nonlinear mapping about system input and output, $f(0,0) = 0$; n_y and n_u represent unknown system order; $d(k,t+1)$ is a bounded disturbance changed along time axis and iteration axis, $|d(k,t+1)| \leqslant b_{\mathrm{d}}$, where b_{d} is a positive constant.

Assume that the partial derivative of the nonlinear function with respect to the control input $u(k,t)$ is continuous and bounded.In addition, the sign of the partial derivative is known and unchanged. According to [12], differential (1) along the iteration axis, one can obtain,

$$\begin{array}{r} \Delta y(k,t+1) = \dfrac{\partial f^*}{\partial u(k,t)}(u(k,t) - u(k-1,t)) \\ + F(k,t) + \Delta d(k,t) \end{array} \tag{2}$$

where $\Delta y(k,t) = y(k,t) - y(k-1,t)$, $\dfrac{\partial f^*}{\partial u(k,t)}$ represents the partial derivative of $f(\cdot)$ with respect to the input $u(k,t)$; $\Delta d(k,t) = d(k,t) - d(k-1,t)$ and

$$\begin{array}{l} F(k,t) = f(y(k,t), \cdots, y(k,t-n_y), \\ u(k-1,t), u(k,t-1), \cdots, u(k,t-n_u)) \\ -f(y(k-1,t), \cdots, y(k-1,t-n_y), \\ u(k-1,t), u(k-1,t-1), \cdots, u(k-1,t-n_u)) \end{array} \tag{3}$$

Define $\phi(k,t) = \dfrac{\partial f^*}{\partial u(k,t)}$, $\xi(k,t) = F(k,t) + \Delta d(k,t)$, then according to (2), (1) can be dynamically linearized into the following form:

$$\Delta y(k,t+1) = \phi(k,t)\Delta u(k,t) + \xi(k,t), \tag{4}$$

where $\Delta u(k,t) = u(k,t) - u(k-1,t)$. $\phi(k,t)$ is bounded, $|\phi_k(t)| \leqslant b_{\text{pu}}$; $\xi(k,t)$ is bounded, $|\xi_k(t)| \leqslant b_{\xi}$, b_{pu} and b_{ξ} are positive constant. The proof is detailed in [12].

3 Discrete ADRILC based on Dynamic Linearization

The proposed method consists of two parts: First, the model paradigm of discrete active disturbance rejection in the iterative domain is established based on the dynamic linearization model (4), and a new IESO is designed to estimate the total disturbance of the system; then according to the idea of sliding mode equivalent control, an iterative control law is designed to compensate the total disturbance of the system, so that the controlled system can achieve bounded tracking.

3.1 IESO Design

Based on the idea of ADRC, let b , a gain constant, be a rough estimate value of the partial derivative $\phi(k,t)$. Then the estimated error of the parameter is compacted into the total disturbance. (4) can be rewritten into the following form:

$$\Delta y(k,t+1) = b\Delta u(k,t) + w(k,t), \tag{5}$$

where $w(k,t) = (\phi(k,t) - b)\Delta u(k,t) + \xi(k,t)$ is so-called total disturbance.$\phi(k,t)$, $\xi(k,t)$ and $\Delta u(k,t)$ are bounded, so $w(k,t)$ is bounded. $|w_k(t)| \leqslant b_{\mathrm{w}}, \forall t$ and $\forall k$, where b_{w} is a positive constant.

Compared with the integral series model as the model paradigm of continuous active disturbance rejection, (5) can be regarded as a model paradigm of discrete active disturbance rejection in the iterative domain.

Let $x_1(k,t) = y(k,t)$; let $x_2(k,t) = w(k,t)$ as extended state; $\Delta w(k-1,t) = x_2(k,t) - x_2(k-1,t)$, then (5) can be rewritten as follows:

$$\begin{cases} x_1(k,t+1) = x_1(k-1,t+1) + x_2(k-1,t) + \\ \qquad b\Delta u(k,t), \\ x_2(k,t) = x_2(k-1,t) + \Delta w(k-1,t). \end{cases} \tag{6}$$

The discrete model based on iterative dynamic linearization shown in (6) can be written as the following state equation:

$$\begin{cases} \begin{pmatrix} x_1(k,t+1) \\ x_2(k,t) \end{pmatrix} = A \begin{pmatrix} x_1(k-1,t+1) \\ x_2(k-1,t) \end{pmatrix} + \\ \qquad B\Delta u(k,t) + D\Delta w(k-1,t) \\ y(k,t+1) = C \begin{pmatrix} x_1(k,t+1) \\ x_2(k,t) \end{pmatrix} \end{cases} \tag{7}$$

where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} b \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Referring to the design of discrete ESO in [16], according to (7), design IESO as follows:

$$\begin{cases} \begin{pmatrix} \hat{x}_1(k,t+1) \\ \hat{x}_2(k,t) \end{pmatrix} = \\ A \begin{pmatrix} \hat{x}_1(k-1,t+1) \\ \hat{x}_2(k-1,t) \end{pmatrix} + B\Delta u(k,t) + \\ L(y(k-1,t+1) - \hat{y}(k-1,t+1)) \\ \hat{y}(k,t+1) = C \begin{pmatrix} \hat{x}_1(k,t+1) \\ \hat{x}_2(k,t) \end{pmatrix} \end{cases} \tag{8}$$

where $L = \begin{bmatrix} l_1 & l_2 \end{bmatrix}^{\mathrm{T}}$ is the adjustable gain matrix of the state observer. $\hat{x}_1(k,t+1)$, $x_2(k,t)$, $\hat{y}(k,t+1)$ represent the estimate value of $x_1(k,t+1)$, $x_2(k,t)$, $y(k,t+1)$

respectively. $\hat{x}_2(k,t)$ is obtained by (8), and it should be within the bounds of $x_2(k,t)$. Taking into account the numerical stability of the calculation process, the limiting iterative learning law is adopted.

$$\hat{x}_2(k,t) = \begin{cases} b_{\mathrm{f}}, & x_2(k,t) > b_{\mathrm{f}}; \\ x_2(k,t), & |x_2(k,t)| \leqslant b_{\mathrm{f}}; \\ -b_{\mathrm{f}}, & x_2(k,t) < -b_{\mathrm{f}}; \end{cases} \tag{9}$$

It is worth noting that compared with the discrete-time ESO proposed in [16], the IESO proposed in this paper runs along the iterative axis. It makes up for the shortcomings of discrete-time ESO in estimating the state of a finite-time repeated operation. Unlike the IESO in [12], the IESO derived in this paper can not only estimate the expansion state, but also the actual output state of the system, and the convergence is proved below. Moreover, it is proposed for nonlinear non-affine discrete systems, and it makes up for the shortcomings of the IESO proposed in [14] for the state estimation of fast time-varying nonlinear discrete systems.

3.2 Controller Design

Define the desired trajectory as $y_{\mathrm{d}}(t), t \in \{0, \cdots, T\}$. The tracking error is:

$$e(k,t) = y(k,t) - y_{\mathrm{d}}(t). \tag{10}$$

The goal is that when the number of iterations tends to infinity, the system outputs $y(k,t)$ to completely track the desired trajectory $y_{\mathrm{d}}(t)$. That is, $\lim\limits_{k\to\infty} e_k(t) = 0$.

Denote sliding surface as:

$$s(k,t+1) = e(k,t+1). \tag{11}$$

For continuous systems, the exponential reaching law is widely used, and its expression is as follows:

$$\dot{s}(t) = -qs(t) - \varepsilon \mathrm{sign}(s(t)). \tag{12}$$

Discretize the above formula to obtain the exponential reaching law of the discrete system[17]:

$$s(t+1) = (1 - qT_{\mathrm{s}})s(t) - \varepsilon T_{\mathrm{s}} \mathrm{sign}(s(t)). \tag{13}$$

where T_s is sample time, $(1 - qT_{\mathrm{s}}) \in (0,1), \varepsilon > 0$.

The independent variable of the system along the iteration axis is the number of iterations instead of time, so there is no sampling time parameter. The iterative sliding mode reaching law of the discrete system is proposed as follows:

$$\begin{aligned} s(k,t+1) = {} & qs(k-1,t+1) - \\ & \varepsilon \mathrm{sign}(s(k-1,t+1)). \end{aligned} \tag{14}$$

where $q \in (0,1), \varepsilon > 0$.

According to (5) and (10), (11) can be rewritten as follows:

$$\begin{aligned} s(k,t+1) = {} & y(k-1,t+1) + b\Delta u(k,t) + \\ & w(k,t) - y_{\mathrm{d}}(t+1). \end{aligned} \tag{15}$$

Substituting (14) into (15), the control law is obtained as follows:

$$\begin{aligned} \Delta u(k,t) = {} & b^{-1}(y_{\mathrm{d}}(t+1) - y(k-1,t+1) - \\ & w(k,t) + qs(k-1,t+1) - \\ & \varepsilon \mathrm{sign}(s(k-1,t+1))). \end{aligned} \tag{16}$$

where the total system disturbance $w(k,t)$ is unknown, and the IESO shown in (8) is used for estimating $w(k,t)$. Thus the final control law is:

$$\begin{aligned} u(k,t) = {} & u(k-1,t) + b^{-1}(y_{\mathrm{d}}(t+1) - \\ & y(k-1,t+1) - \alpha \hat{x}_2(k,t) + \\ & qs(k-1,t+1) - \varepsilon \mathrm{sign}(s(k-1,t+1))). \end{aligned} \tag{17}$$

where $\alpha > 0$ is the disturbance compensation factor. Since the estimated error of IESO is bounded rather than fully estimated, the disturbance compensation term in the algorithm introduces the disturbance compensation factor α to reduce the disadvantages caused by the estimation error.

Then (8), (9) and (17) constitute the algorithm proposed in this paper.

4 Convergence Analysis

The proof of the convergence is divided into two parts. The first part proves the bounded convergence of the IESO, and the second one proves the bounded convergence of the system tracking error.

4.1 Convergence Analysis of IESO

Define $\delta(k,t) = x(k,t) - \hat{x}(k,t)$ as the estimation error of IESO, and make the difference between (5) and (6) to obtain the observation error state equation:

$$\begin{aligned} \begin{pmatrix} \delta_1(k,t+1) \\ \delta_2(k,t) \end{pmatrix} = (A - LC) \begin{pmatrix} \delta_1(k-1,t+1) \\ \delta_2(k-1,t) \end{pmatrix} + \\ D\Delta w(k-1,t+1). \end{aligned} \tag{18}$$

Because the total perturbation $w(k,t)$ is bounded, $\Delta w(k-1,t)$ is bounded. By choosing a suitable observer gain, the spectral radius $s(A - LC)$ is less than 1, that is, it satisfies:

$$\max\left(\left|\frac{2-l_1+\sqrt{l_1^2-4l_2}}{2}\right|, \left|\frac{2-l_1-\sqrt{l_1^2-4l_2}}{2}\right|\right) < 1. \tag{19}$$

Then there is a constant ς that makes the following formula true:

$$\|A - LC\|_p \leqslant s(A - LC) + \varsigma \leqslant \gamma < 1. \tag{20}$$

where $0 < \gamma < 1$.

Because $\Delta w(k-1,t)$ is bounded, let

$$\|D\Delta w(k-1,t)\|_p \leqslant \sigma_1. \tag{21}$$

where σ_1 is a positive constant, $\sigma_1 = b_{\mathrm{w}}$, then

$$\begin{aligned} \left\| \begin{matrix} \delta_1(k,t+1) \\ \delta_2(k,t) \end{matrix} \right\|_p & \leqslant \gamma \left\| \begin{matrix} \delta_1(k-1,t+1) \\ \delta_2(k-1,t) \end{matrix} \right\|_p + \sigma_1 \\ & \leqslant \gamma^2 \left\| \begin{matrix} \delta_1(k-2,t+1) \\ \delta_2(k-2,t) \end{matrix} \right\|_p + \gamma\sigma_1 + \sigma_1 \\ & \leqslant \cdots \leqslant \gamma^k \left\| \begin{matrix} \delta_1(0,t+1) \\ \delta_2(0,t) \end{matrix} \right\|_p + \frac{\sigma_1}{1-\gamma}. \end{aligned} \tag{22}$$

$\gamma \in (0,1)$, and the initial error $\delta(0,t)$ is bounded, so when the number of iterations tends to infinity, the error converges to a bounded range,

$$\lim_{k\to\infty} \|\delta(k,t)\|_p \leqslant \frac{\sigma_1}{1-\gamma}. \tag{23}$$

4.2 Convergence Analysis of Tracking Error

Substituting the control law shown in (15) into (5), the error equation is obtained as:

$$\begin{aligned} e(k,t+1) = w(k,t) - \alpha\hat{x}_2(k,t) + \\ qs(k-1,t+1) - \\ \varepsilon sign(s(k-1,t+1)). \end{aligned} \tag{24}$$

so,

$$\begin{aligned} |e(k,t+1)| \leqslant |w(k,t) - \alpha\hat{x}_2(k,t) - \\ \varepsilon \mathrm{sign}(s(k-1,t+1))| + \\ qs(k-1,t+1). \end{aligned} \tag{25}$$

Because $w(k,t)$, $\hat{x}_2(k,t)$ and $\mathrm{sign}(s(k-1,t+1))$ are bounded, let

$$\begin{aligned} |w(k,t) - \alpha\hat{x}_2(k,t) - \\ \varepsilon \mathrm{sign}(e(k-1,t+1))| < \sigma_2. \end{aligned} \tag{26}$$

where σ_2 is a positive constant, $\sigma_2 = b_{\mathrm{w}} + \alpha b_{\mathrm{f}} + \varepsilon$, then

$$\begin{aligned} |e(k,t+1)| &\leqslant q\,|e(k-1,t+1)| + \sigma_2 \\ &\leqslant q^2\,|e(k-2,t+1)| + q\sigma_2 + \sigma_2 \\ &\leqslant \cdots \leqslant q^k\,|e(0,t+1)| + \frac{\sigma_2}{1-q}. \end{aligned} \tag{27}$$

$q \in (0,1)$, and the initial error $e(0,t)$ is bounded, so when the number of iterations tends to infinity, the error converges to a bounded range,

$$\lim_{k\to\infty} e(k,t) \leqslant \frac{\sigma_2}{1-q}. \tag{28}$$

Proof finished.

5 Simulation Study

The method proposed in this paper is mainly compared with IESO-DDOILC algorithm proposed in [12], which estimates the nonlinear term $\xi(k,t)$ by the IESO shown below ,

$$\begin{cases} \hat{\xi}(k,t) = \hat{\xi}(k-1,t) - \alpha(y_{\mathrm{d}}(t+1) - \\ \qquad y(k-1,t+1)), \\ \hat{\xi}(k,t) = \begin{cases} b_\xi, & \hat{\xi}(k,t) > b_\xi; \\ \hat{\xi}(k,t), & |\hat{\xi}(k,t)| \leqslant b_\xi; \\ -b_\xi, & \hat{\xi}(k,t) < -b_\xi; \end{cases} \end{cases} \tag{29}$$

Then the pseudo partial derivative update law and the optimal control law are derived by the optimization technique:

$$\begin{aligned} \hat{\phi}(k,t) = \hat{\phi}(k-1,t) + \frac{\eta\Delta u(k-1,t)}{\mu + \Delta u^2(k-1,t)} \times \\ (\Delta y(k-1,t+1) - \hat{\xi}(k-1,t) - \\ \hat{\phi}(k-1,t)\Delta u(k-1,t)), \end{aligned} \tag{30}$$

$$\begin{aligned} u(k,t) = u(k-1,t) + \\ \frac{\rho\hat{\phi}(k,t)(e(k-1,t+1) - \hat{\xi}(k,t))}{\lambda + \hat{\phi}^2(k,t)}, \end{aligned} \tag{31}$$

To ensure the stability of the algorithm, if $\hat{\phi}(k,t) \leqslant \varepsilon$ or $|\Delta u(k-1,t)| < \varepsilon$, ε is any small positive number, reset the algorithm as follows:

$$\hat{\phi}(k,t) = \hat{\phi}(0,t). \tag{32}$$

Then (29) to (32) constitute the IESO-DDOILC algorithm.

Think over a nonlinear non-affine discrete system that runs repeatedly over a finite time interval given in [12],

$$\begin{aligned} &y(k,t+1) = \\ &\begin{cases} \dfrac{y(k,t)}{1+y^2(k,t)} + u^3(k,t) + d(k,t+1), \\ \qquad\qquad 0 \le t \le 50; \\ \dfrac{y(k,t)y(k,t-1)y(k,t-2)u(k,t-1)}{1+y^2(k,t-1)+y^2(k,t-2)} \times \\ ((y(k,t-2)-1) + u(k,t)) + d(k,t+1), \\ \qquad\qquad 50 < t \le 100; \end{cases} \end{aligned} \tag{33}$$

The desired trajectory is

$$\begin{aligned} &y_{\mathrm{d}}(t+1) = \\ &\begin{cases} 0.5 \times (-1)^{\mathrm{round}(t/10)}, 0 \le t \le 30, \\ 0.5\sin\dfrac{t\pi}{10} + 0.3\cos\dfrac{t\pi}{10}, 30 < t \le 70, \\ 0.5 \times (-1)^{\mathrm{round}(t/10)}, 70 < t \le 100. \end{cases} \end{aligned} \tag{34}$$

Assume the initial value of the system changes iteratively. Set to $y(k,0) = 0.1\sin\dfrac{\pi k}{40}$. The non-repetitive external disturbance is $d(k,t+1) = 0.2\sin(t + \dfrac{k}{25})$.

The parameters of IESO are $l_1 = 0.8$, $l_2 = 0.2$, the value of l_1 and l_2 satisfy (23); $b_{\mathrm{f}} = 0.05$; The parameters of controller are $b = 2$, $\alpha = 1.2$, $q = 0.1$, $\varepsilon = 0.04$; The initial control input $u(0,t) = 0$, and $\hat{x}_1(0,t) = 0, \hat{x}_2(0,t) = 0$.

Fig.1, Fig.2 shows the results of the 100th iteration of the observer, which shows that the proposed IESO can effectively estimate the system state as well as the extended state. The dotted line in Fig.1 indicates the actual output of the system, and the red line indicates the output of the state observer, from which it can be seen that the error between the estimation result and the actual output is small, and the bounded estimation performance can be achieved.

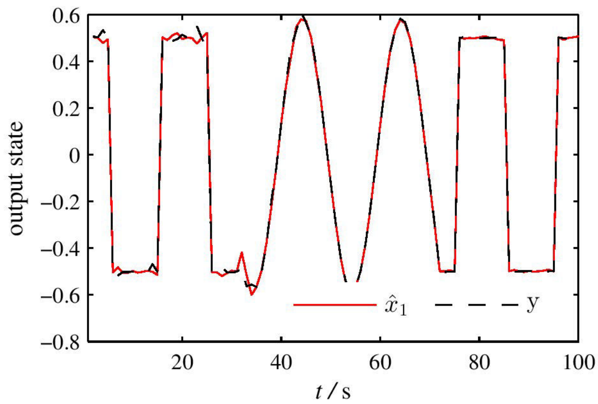

Fig. 1: Estimation of x_1

The black dashed line in Fig.2 indicates the actual so-called total disturbance, the red line indicates the output results of the proposed IESO for the extended state, and the blue dashed line indicates the output results of the IESO proposed in the [12]. We can see that the observations can roughly estimate the actual total disturbance. Define the maximum absolute error value for the kth iteration as $e_{\max}(k) = \max\limits_{t=1,2,\dots,T}\{|e(t,k)|\}$, Define the average estimation error for kth iteration as $e_{\text{avg}}(k) = \frac{1}{T}\sum\limits_{t\in 1,2,\dots,T}|e(t,k)|$. The $e_{\max}$ for 100th iteration of the proposed IESO is 0.0579, e_{avg} is 0.0075. The $e_{\max}$ for 100 iterations of the IESO in [12] is 0.0841, e_{avg} is 0.014. It shows that the proposed IESO has better performance.

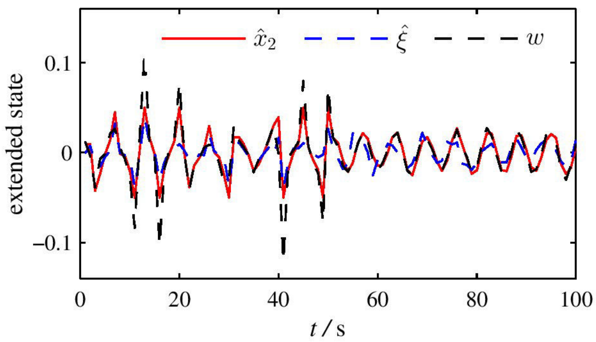

Fig. 2: Estimation of extended state

For comparison, in addition to the previously mentioned IESO-DDOILC, the proposed method is also compared with the anticipatory type ILC (A-type-ILC)[18], DDOILC[7]. The A-type-ILC algorithm is as follows:

$$u(k,t) = u(k-1,t) + ke(k-1,t+\Delta). \tag{35}$$

where k is the learning gain, $\Delta > 0$ indicates the lead time to compensate the delay. It is comparable to PD-type ILC control at low frequencies, but does not require a differential signal of error and is suitable for discrete systems. IESO-DDOILC without IESO compensation is the DDOILC algorithm. These methods do not require model information of the controlled plant and are able to control nonlinear non-affine discrete systems.

Fig.3 shows the tracking effect of the system in the 100th iteration, the blue line indicates the desired trajectory of the system. The black dashed line, the blue dashed line, the black line and the red line indicate the A-type-ILC, DDOILC, IESO-DDOILC and the proposed method, respectively. The above methods are able to achieve bounded tracking performance when there are iteratively varying initial values and external disturbances of the system. Their $e_{\max}$ are 0.1349, 0.1103, 0.0611 and 0.0484, e_{avg} are 0.0262, 0.0206, 0.009 and 0.0063. the proposed method has the smallest tracking error, which indicates that the proposed method has better tracking performance.

Fig.4 and Fig.5 visualizes the convergence of $e_{\max}$ and e_{avg} for the above four methods as the number of iterations k increases. The controlled system is a nonlinear non affine system with changing structure, and there are iteratively changing initial values and external disturbances of the system, which verifies the robustness of the control methods. A-type-ILC learns to correct the control input by using the error of the next moment of the previous cycle, which is a simple

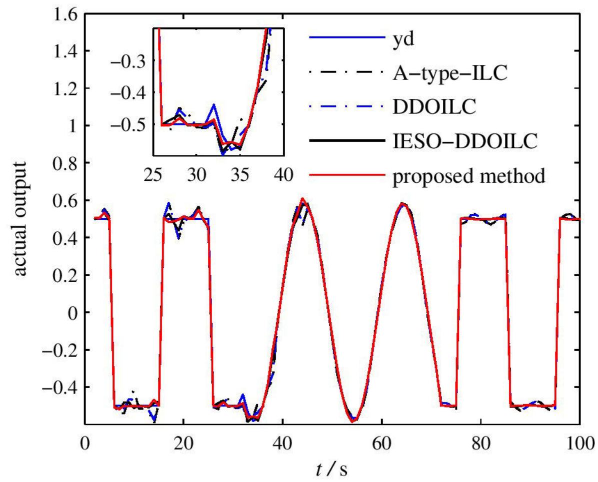

Fig. 3: The system output tracking performance

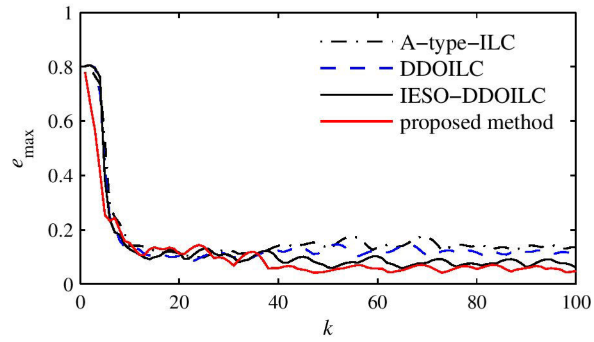

Fig. 4: Convergence of the maximum tracking error

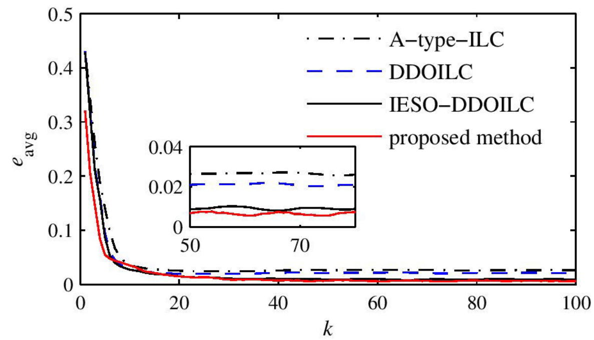

Fig. 5: Convergence of the average tracking error

method but has no adaptive mechanism to cope with the system uncertainty, so its tracking error is the largest. DDOILC reflects and estimates the system uncertainty to some extent by updating the parameter $\hat{\phi}(k,t)$. The IESO-DDOILC method additionally uses IESO to estimate and compensate for the system uncertainty, thus achieving better control performance than the traditional DDOILC method. The proposed method derives the control law based on the idea of sliding mode control, and the proposed IESO has a better estimation performance than the IESO of IESO-DDOILC. Therefore, the proposed method has the same or even better control performance than IESO-DDOILC.

6 Conclusions

In this paper, a discrete active disturbance rejection iterative learning control method based on dynamic linearization is proposed for a class of discrete-time nonlinear non-affine uncertain systems with external disturbance running repeat edly in finite time. The controlled system is linearized dynamically along the iterative axis. Then the control gain is estimated and fixed as a gain constant. The parameter estimation error, system uncertainties and external disturbances are compacted into a nonlinear term as the total disturbance which are estimated by the designed IESO and then compensated into control law. The proposed method presents a simple scheme both for discrete ADRC and ADRILC, in which the control gain b can be initially determined by the PPD update law if needed, and then fixed in most of the time. It can effectively solve the problem of estimating b in ADRC. Using different dynamic linearization forms, discrete ESO, different iterative sliding mode surfaces, sliding mode convergence law are supposed to exhibit better control performance results, which are next promising research directions.

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