HW3 Assignment

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Probelm set 1

(1)

```
library(matrixcalc)
A <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), 4, byrow=T)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
matrix.rank(A)
```

```
## [1] 4
```

(2) If m >n, the maximum rank can be n, and the minimum rank will be 1 since we assume that the matrix is non-zero.

(3)

```
B <- matrix(c(1,2,1,3,6,3,2,4,2), 3, byrow=T)
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
matrix.rank(B)
```

```
## [1] 1
```

Problem set 2

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

$$x = \begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}$$

$$A - x = \begin{pmatrix} 1-x & 2 & 3 \\ 0 & 4-x & 5 \\ 0 & 0 & 6-x \end{pmatrix}$$

$det(A - x) = (1 - x)(24 - 10x + x^2)$

$(1 - x)(24 - 10x + x^2) = 0$

$(1 - x)(x - 4)(x - 6) = 0$

$x = 1, x = 4, x = 6$

$a) eigenvalue = 1$

$$B = \begin{pmatrix} 0 & 2 & 3 & |0 \\ 0 & 3 & 5 & |0 \\ 0 & 0 & 5 & |0 \end{pmatrix}$$

$$reduced\ rechelon\ form\ B = \begin{pmatrix} 0 & 1 & 0 & |0 \\ 0 & 0 & 1 & |0 \\ 0 & 0 & 0 & |0 \end{pmatrix}$$

$x_2 = 0, x_3 = 0, x_1 = 1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} * \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$b) eigenvalue = 4$

$$C = \begin{pmatrix} -3 & 2 & 3 & |0 \\ 0 & 0 & 5 & |0 \\ 0 & 0 & 2 & |0 \end{pmatrix}$$

$$reduced\ rechelon\ form\ C = \begin{pmatrix} 1 & -2/3 & 0 & |0 \\ 0 & 0 & 1 & |0 \\ 0 & 0 & 0 & |0 \end{pmatrix}$$

$x_1 = 2, x_2 = 3, x_3 = 0$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} * \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 0 \end{pmatrix} = 4 * \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$c) eigenvalue = 6$

$$D = \begin{pmatrix} -5 & 2 & 3 & |0 \\ 0 & -2 & 5 & |0 \\ 0 & 0 & 0 & |0 \end{pmatrix}$$

$$reduced\ rechelon\ form\ D = \begin{pmatrix} 1 & 0 & -8/5 & |0 \\ 0 & 1 & -5/2 & |0 \\ 0 & 0 & 0 & |0 \end{pmatrix}$$

$x_1 = 8/5, x_2 = 5/2, x_3 = 1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} * \begin{pmatrix} 8/5 \\ 5/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9.6 \\ 15 \\ 6 \end{pmatrix} = 6 * \begin{pmatrix} 8/5 \\ 5/2 \\ 1 \end{pmatrix}$$