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Determine whether or not V is a vector space with these operations.

Proof: Let $A = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} z \\ w \end{bmatrix}, C = \begin{bmatrix} a \\ b \end{bmatrix}$

AA.

(A+B)+C = $\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} z \\ w \end{bmatrix}\right) + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y+w \\ x+z \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x+z+b \\ y+w+a \end{bmatrix}$

A+ (B+C)= $\begin{bmatrix} x \\ y \end{bmatrix} + (\begin{bmatrix} z \\ w \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}) = \begin{bmatrix} x \\ y \end{bmatrix} + (\begin{bmatrix} b+w \\ a+z \end{bmatrix}) = \begin{bmatrix} y+z+a \\ x+w+b \end{bmatrix}$

(A+B) +C $\neq A + (B + C)$

Since it doesn't apply the property of Additive Associativity, it is not a vector space.