

# HW4 Assignment

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## Problem set 1

```
A <- matrix(c(1,2,3,-1,0,4),2, byrow = T)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]   -1    0    4
```

```
# Find  $X=AA^T$  and  $Y=A^TA$ 
```

```
x <- A %*% t(A)
x
```

```
##      [,1] [,2]
## [1,]   14   11
## [2,]   11   17
```

```
y <- t(A) %*% A
y
```

```
##      [,1] [,2] [,3]
## [1,]    2    2   -1
## [2,]    2    4    6
## [3,]   -1    6   25
```

```
# Use the built in function to compute the eigenvalues and eigenvectors of X and Y
```

```
eigen_x <- eigen(x)
eigen_x
```

```
## $values
## [1] 26.601802  4.398198
```

```
##
## $vectors
##      [,1]      [,2]
## [1,] 0.6576043 -0.7533635
## [2,] 0.7533635  0.6576043
```

```
eigen_y <- eigen(y)
eigen_y
```

```
## $values
## [1] 2.660180e+01 4.398198e+00 1.058982e-16
```

```
##
## $vectors
##      [,1]      [,2]      [,3]
## [1,] -0.01856629 -0.6727903  0.7396003
## [2,]  0.25499937 -0.7184510 -0.6471502
```

```
## [3,] 0.96676296 0.1765824 0.1849001
```

$$A = U \Sigma V^T$$

$$A^T A = V \Sigma^T \Sigma V^T = Y$$

$$A A^T = U \Sigma \Sigma^T U^T = X$$

```
# Compute the left-singular, singular values, and right-singular vectors
```

```
svd_A <- svd(A)
```

```
svd_A
```

```
## $d
```

```
## [1] 5.157693 2.097188
```

```
##
```

```
## $u
```

```
##          [,1]          [,2]
```

```
## [1,] -0.6576043 -0.7533635
```

```
## [2,] -0.7533635 0.6576043
```

```
##
```

```
## $v
```

```
##          [,1]          [,2]
```

```
## [1,] 0.01856629 -0.6727903
```

```
## [2,] -0.25499937 -0.7184510
```

```
## [3,] -0.96676296 0.1765824
```

```
# v is right-singular u is left-singular
```

*Sigma*

```
ssvd <- matrix(c(svd(A)$d[1],0,0,svd(A)$d[2],0,0),nrow=2)
```

```
ssu_eig <- matrix(c(sqrt(eigen(x)$values[1]),0,0,sqrt(eigen(x)$values[2]),0,0),nrow=2)
```

```
ssv_eig <- matrix(c(sqrt(eigen(y)$values[1]),0,0,sqrt(eigen(y)$values[2]),0,0),nrow=2)
```

*Decomposition*

```
usvd <- svd_A$u
```

```
vsvd <- cbind(svd_A$v, c(0,0,0))
```

```
usvd %*% ssvd %*% t(vsvd)
```

```
##          [,1]          [,2] [,3]
```

```
## [1,]      1 2.000000e+00      3
```

```
## [2,]     -1 1.110223e-16      4
```

```
ueig <- eigen(x)$vectors
```

```
veig <- cbind(eigen(y)$vectors[,1:2], c(0,0,0))
```

```
ueig %*% ssu_eig %*% t(veig)
```

```
##          [,1]          [,2] [,3]
```

```
## [1,]      1 2.000000e+00      3
```

```
## [2,]     -1 -9.992007e-16      4
```

## Problem Set 2

```
myinverse <- function(M){  
  if (det(M)==0){  
    stop('Try another one!')  
  }  
  cofactor <- diag(ncol <-nrow(M))  
  for (i in 1:nrow(M)){  
    for(j in 1:nrow(M)){  
      cofactor[i,j] <- (-1)^(i+j)*det(M[-i,-j])  
    }  
  }  
  return( (t(cofactor)/ det(M)))  
}
```

```
A <- matrix(c(1,2,3,0,4,5,1,0,6), 3, byrow=T)  
A
```

```
##      [,1] [,2] [,3]  
## [1,]    1    2    3  
## [2,]    0    4    5  
## [3,]    1    0    6
```

```
B <-myinverse(A)  
B
```

```
##      [,1]      [,2]      [,3]  
## [1,] 1.0909091 -0.54545455 -0.09090909  
## [2,] 0.2272727 0.13636364 -0.22727273  
## [3,] -0.1818182 0.09090909 0.18181818
```