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Backstepping Control Design on the Dynamics of the Omni-directional Mobile Robot

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Abstract. The dynamical model of an omni-directional mobile robot is bulit based on the Newtonian mechanics. Correspondingly, a backstepping-based controller is then proposed with proven global stability by selecting a Lyapunov function and introducing a virtual control input for the built dynamical model. Simulation results show the effectiveness of the proposed controller.

Introduction

The RoboCup is a high technology activity in recent years, which has attracted wide concerns in the worldwide. The motion control, which is conventionally divided into kinematics-based control and dynamics-based control, is a very important issue in matchs. Currently almost all the teams are using kinematics-based control [1, 2]， with the requirements of instantaneity and accuracy, some people begin to pay attention on the dynamics-based control [3].

The establishment of dynamic model is the foundation of dynamics-based control. To correctly establish the dynamic model relies on the complete and thorough dynamical analysis of the driving motors and the mechanical couplings in the robot.

Backstepping method is a nonlinear system stability design theory which is developed by Petar V.Kokotovic and his cooperators in past decade. For the uncertain characters of the matching and non-matching parameters existing in nonlinear systems, we can systemically design a stable controller with backstepping method by correcting the algorithm, and finally globally stable regulator or tracker of the system can be realized. In each step, backstepping method connects change of state coordinates and self-adaptive function of uncertain parameters with the stabilization function of virtual control system of a known Lyapunov function [4, 5, 6]. The introduced virtual input is a static compensation in essence. The stabilization of the fore subsystems cannot be possible without the virtual control of the subsystem. In recent years, to design the self-adaptive controllers of robot with backstepping method has drawn grate concerns of relevant scholars [7, 8].

In this paper, according to the classical force analysis of NuBot middle-size robot, the dynamic model was built. A waypoint stabilization control for the established dynamical model is studied based on time-varying feedback control of backstepping method. Global stable character of the system is also proved by using Lyapunov theory.

Dynamic Modeling

Coordinates descriptions. Fig.1 is a photo of the NuBot robot, which is built by the NuBot Team of National University of Defense Technology from scratch. Four DC motor drivers are orthogonally arrayed on the robot chassis (shown as Fig.2), which provides the omni-directional mobile ability for the robot.

Three right-hand coordinates are set-up on NuBot robot, shown in Fig.3. To be more specifical, they are:

Global coordinate system xOy (inertial coordinate): the origin of this system is located at the center of the filed with x axis directing to the opponent goal;

Robot body coordinate system $x_B O_B y_B$: the origin of this system is located at the mass center of the robot with x axis being ahead to the head of the robot;

Robot view coordinate system $x_T O_T y_T$: the origin of this system is located at the mass center of the robot, and the x_T axis is coincides with the robot target view.

We employ θ and ϕ to denote the angle between robot body coordinate system $x_B O_B y_B$ and the x axis in global coordinate system xOy , and the angle between robot view coordinate system $x_B O_B y_B$ and the x axis in global coordinate system xOy , respectively.


Fig.1 a photo of a NuBot robot


Fig.2 four wheels in NuBot robot

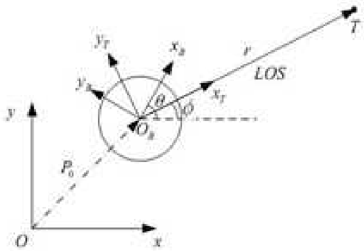


Fig.3 the coordinate systems built on NuBot robot

The dynamical model. According to Fig.2, the wheels' configuration of the robot can be abstracted as that shown in Fig.4.

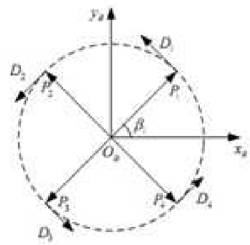


Fig.4 Abstraction of the wheels' configuration

With the similar tricks in the literature [3], the dynamics of the omni-directional robot can be described by

$$\begin{bmatrix} (M+m)\ddot{x} \\ (M+m)\ddot{y} \\ (J+2mL^2)\ddot{\gamma} \end{bmatrix} + k_2 \begin{bmatrix} 2\dot{x} \\ 2\dot{y} \\ 4L^2\dot{\gamma} \end{bmatrix} + m\dot{\gamma} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \end{bmatrix} = k_1 \mathbf{J}^T(\gamma) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{1}$$

Where
$\gamma = \theta + \pi/4$, quantities M , m , J and L are the mass of the whole robot, the mass of a single wheel, the inertia moment of the robot and the length from O_B to the wheels, respectively, k_1 and k_2 are the cofactors depended on the character, $u_i (i = 1,2,3,4)$ are the voltages acting on the armature of the i-th motor, which are the actual control inputs and meet $-24V \le u_i \le 24V$.

$$J(\gamma)=\begin{bmatrix}-\sin(\gamma) & \cos(\gamma) & L\\ -\cos(\gamma) & -\sin(\gamma) & L\\ \sin(\gamma) & -\cos(\gamma) & L\\ \cos(\gamma) & \sin(\gamma) & L\end{bmatrix} \tag{2}$$

Backstepping Control design

The key to solve the stabilization problem is to design a control law to ensure the robot can reach any given target and keep still at the target. In this paper, a asymptotically stable controller based on backstepping method is designed to solve the waypoint stabilization problem.

We assume that the posture of the robot is $q_r=[x_r,y_r,\theta_r]^{\mathrm{T}}$. Then we define the posture error of the robot in global coordinate system as follow:

$$\mathbf{e}=\mathbf{q}-\mathbf{q}_{\mathbf{r}}=\begin{pmatrix}x-x_r\\ y-y_r\\ \theta-\theta_r\end{pmatrix}=\begin{pmatrix}x_e\\ y_e\\ \theta_e\end{pmatrix} \tag{3}$$

The objective of the point stabilization control is to design control laws $\mathbf{U}=[u_1,u_2,u_3,u_4]$ such that $\lim_{t\to\infty}e(t)=0$ and $\lim_{t\to\infty}u(t)=0$.

The following section describes the design process of the controller.

Model transformation. According to the dynamical model (1), the studied omni-directional mobile robot is a time-varying system with control redundancy because it has four inputs as oppose to three outputs. We introduce three temporary variables F_x,F_y and T to divide (1) into two parts as (4) and (5).

$$\begin{bmatrix}(M+m)\ddot{x}\\ (M+m)\ddot{y}\\ (J+2mL^2)\ddot{\gamma}\end{bmatrix}+k_2\begin{bmatrix}2\dot{x}\\ 2\dot{y}\\ 4L^2\dot{\gamma}\end{bmatrix}+m\dot{\gamma}\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\gamma}\end{bmatrix}=\begin{bmatrix}F_x\\ F_y\\ T\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}F_x\\ F_y\\ T\end{bmatrix}=k_1\mathbf{A}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix} \tag{5}$$

Thus, we have:

$$\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix}=\frac{1}{k_1}\mathbf{A}^{\dagger}\begin{bmatrix}F_x\\ F_y\\ T\end{bmatrix}=\frac{1}{k_1}\begin{bmatrix}-\sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} & \frac{1}{4}L\\ -\cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} & \frac{1}{4}L\\ \sin\frac{\gamma}{2} & -\cos\frac{\gamma}{2} & \frac{1}{4}L\\ \cos\frac{\gamma}{2} & \sin\frac{\gamma}{2} & \frac{1}{4}L\end{bmatrix}\begin{bmatrix}F_x\\ F_y\\ T\end{bmatrix} \tag{6}$$

Design of the controller. The controller is designed by combining PD and backstepping control methods, whose structure is shown in Fig.5.

According to backstepping method, we choose Lyapunov function as follow:

$$V_I=\frac{1}{2}\mathbf{e}^{\mathbf{T}}\mathbf{e} \tag{7}$$

By differentiating V_1, we obtain

$$\dot{V}_I=\mathbf{e}^{\mathbf{T}}\dot{\mathbf{e}}=\mathbf{e}^{\mathbf{T}}(\dot{\mathbf{q}}-\dot{\mathbf{q}}_{\mathbf{r}})=\mathbf{e}^{\mathbf{T}}\dot{\mathbf{q}}=\mathbf{e}^{\mathbf{T}}\mathbf{v} \tag{8}$$

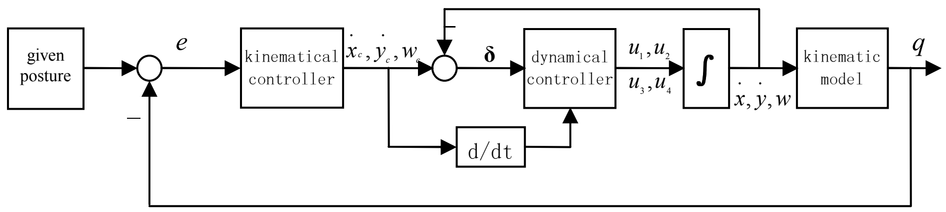


Fig.5 The structure of the controller

According to the design of PD controller, we have:

$$\mathbf{v_c} = -\mathbf{c_1 e} - \mathbf{c_2}\dot{\mathbf{e}} = -\mathbf{c_1 e} - \mathbf{c_2 v} \tag{9}$$

where $\mathbf{v_c} = [x_c, y_c, w_c]^{\mathrm{T}}$ is the velocity vector with x_c and y_c representing the expected velocity in direction of x axis and y axis in global coordinate system, and w_c representing the expected angle velocity of robot, and $\mathbf{c_1} = diag[c_{11}, c_{12}, c_{13}], \mathbf{c_2} = diag[c_{21}, c_{22}, c_{23}]$ (c_{11}, c_{12}, c_{13}, c_{21}, c_{22}, c_{23} are all positive) are the P and D parameter matrices of the PD controller.
Substitute (9) to(8), we obtain:

$$\begin{aligned}
\dot{V}_1 &= -\frac{c_{21}}{c_{11}}v_x^2 - \frac{c_{22}}{c_{12}}v_y^2 - \frac{c_{23}}{c_{13}}w^2 - \frac{v_x v_{xc}}{c_{11}} - \frac{v_y v_{yc}}{c_{12}} - \frac{w w_c}{c_{13}} \\
&= -\frac{c_{21}}{c_{11}}v_x^2 - \frac{c_{22}}{c_{12}}v_y^2 - \frac{c_{23}}{c_{13}}w^2 + \frac{1}{2}\frac{1}{c_{11}}[(v_x - v_{xc})^2 - v_x^2 - v_{xc}^2] + \frac{1}{2}\frac{1}{c_{12}}[(v_y - v_{yc})^2 - v_y^2 - v_{yc}^2] + \frac{1}{2}\frac{1}{c_{13}}[(w - w_c)^2 - w^2 - w_c^2] \\
&= -(\frac{c_{21}}{c_{11}} + \frac{1}{2c_{11}})v_x^2 - (\frac{c_{22}}{c_{12}} + \frac{1}{2c_{12}})v_y^2 - (\frac{c_{23}}{c_{13}} + \frac{1}{2c_{13}})w^2 + \frac{1}{2}\frac{1}{c_{11}}(v_x - v_{xc})^2 + \frac{1}{2}\frac{1}{c_{12}}(v_y - v_{yc})^2 + \frac{1}{2}\frac{1}{c_{13}}(w - w_c)^2
\end{aligned} \tag{10}$$

In the following, we will introduce the design of control law of $\mathbf{F} = \left[F_x, F_y, T\right]^{\mathrm{T}}$. Choose a Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\bar{\mathbf{M}}\boldsymbol{\delta} \tag{11}$$

where

$$\boldsymbol{\delta} = \mathbf{v} - \mathbf{v_c} = [\tilde{v}_x, \tilde{v}_y, \tilde{w}], \bar{\mathbf{M}} = \begin{bmatrix} M+m & 0 & 0 \\ 0 & M+m & 0 \\ 0 & 0 & J+2mL^2 \end{bmatrix}.$$

Differentiating both sides of (11), we get:

$$\begin{aligned}
\dot{V}_2 &= -(\frac{c_{21}}{c_{11}} + \frac{1}{2c_{11}})v_x^2 - (\frac{c_{22}}{c_{12}} + \frac{1}{2c_{12}})v_y^2 - (\frac{c_{23}}{c_{13}} + \frac{1}{2c_{13}})w^2 + \frac{1}{2}\frac{1}{c_{11}}(v_x - v_{xc})^2 + \frac{1}{2}\frac{1}{c_{12}}(v_y - v_{yc})^2 + \frac{1}{2}\frac{1}{c_{13}}(w - w_c)^2 + \boldsymbol{\delta}^{\mathrm{T}}\bar{\mathbf{M}}\dot{\boldsymbol{\delta}} \\
&= -(\frac{c_{21}}{c_{11}} + \frac{1}{2c_{11}})v_x^2 - (\frac{c_{22}}{c_{12}} + \frac{1}{2c_{12}})v_y^2 - (\frac{c_{23}}{c_{13}} + \frac{1}{2c_{13}})w^2 + \frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\begin{bmatrix} 1/c_{11} & 0 & 0 \\ 0 & 1/c_{12} & 0 \\ 0 & 0 & 1/c_{13} \end{bmatrix}\boldsymbol{\delta} + \boldsymbol{\delta}^{\mathrm{T}}(\mathbf{F} - k_2\begin{bmatrix} 2\dot{x} \\ 2\dot{y} \\ 4L^2\dot{\gamma} \end{bmatrix} - m\dot{\gamma}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \end{bmatrix} - \bar{\mathbf{M}}\dot{\mathbf{v}}_{\mathbf{c}}) \\
&= -\frac{c_{21}}{c_{11}}v_x^2 - \frac{c_{22}}{c_{12}}v_y^2 - \frac{c_{23}}{c_{13}}w^2 + \boldsymbol{\delta}^{\mathrm{T}}(\begin{bmatrix} 1/2c_{11} & 0 & 0 \\ 0 & 1/2c_{12} & 0 \\ 0 & 0 & 1/2c_{13} \end{bmatrix}\boldsymbol{\delta} + \mathbf{F} - k_2\begin{bmatrix} 2\dot{x} \\ 2\dot{y} \\ 4L^2\dot{\gamma} \end{bmatrix} - m\dot{\gamma}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \end{bmatrix} - \bar{\mathbf{M}}\dot{\mathbf{v}}_{\mathbf{c}})
\end{aligned} \tag{12}$$

We choose:

$$\mathbf{F} = -\left(\mathbf{k_d} + \begin{bmatrix} 1/2c_{11} & 0 & 0 \\ 0 & 1/2c_{12} & 0 \\ 0 & 0 & 1/2c_{13} \end{bmatrix}\right)\boldsymbol{\delta} + k_2 \begin{bmatrix} 2\dot{x} \\ 2\dot{y} \\ 4L^2\dot{\gamma} \end{bmatrix} + m\dot{\gamma}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \end{bmatrix} + \bar{M}\dot{\mathbf{v}}_{\mathbf{c}}$$

$$= -\tilde{\mathbf{k}}_{\mathbf{d}}\boldsymbol{\delta} + k_2 \begin{bmatrix} 2\dot{x} \\ 2\dot{y} \\ 4L^2\dot{\gamma} \end{bmatrix} + m\dot{\gamma}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\gamma} \end{bmatrix} + \bar{M}\dot{\mathbf{v}}_{\mathbf{c}} \quad (13)$$

which implies

$$\begin{cases} F_x = -k_{d1}\tilde{v}_x + 2k_2\dot{x} + m\dot{\gamma}\dot{y} + (M+m)\ddot{x}_c \\ F_x = -k_{d2}\tilde{v}_y + 2k_2\dot{x} - m\dot{\gamma}\dot{x} + (M+m)\ddot{y}_c \\ T = -k_{d3}\tilde{w} + 4k_2L^2\dot{\gamma} + (M+m)\ddot{\gamma}_c \end{cases} \quad (14)$$

Therefore we have:

$$\dot{V}_2 = -\frac{c_{21}}{c_{11}}v_x^{\ 2} - \frac{c_{22}}{c_{12}}v_y^{\ 2} - \frac{c_{23}}{c_{13}}w^2 - \tilde{\mathbf{k}}_{\mathbf{d}}\boldsymbol{\delta}^{\mathbf{T}}\boldsymbol{\delta} = -\mathbf{e}\begin{bmatrix} c_{21}/c_{11} & 0 & 0 \\ 0 & c_{22}/c_{12} & 0 \\ 0 & 0 & c_{23}/c_{13} \end{bmatrix}\mathbf{e}^{\mathbf{T}} - \tilde{\mathbf{k}}_{\mathbf{d}}\boldsymbol{\delta}^{\mathbf{T}}\boldsymbol{\delta} \quad (15)$$

where $\tilde{\mathbf{k}}_{\mathbf{d}} = diag(k_{d1}, k_{d2}, k_{d3})$, $(k_{d1}, k_{d2}, k_{d3} > 0)$.

Proof of the global stability. According to (15), we have:

$$\dot{V}_2 \le -\mathbf{e}\begin{bmatrix} c_{21}/c_{11} & 0 & 0 \\ 0 & c_{22}/c_{12} & 0 \\ 0 & 0 & c_{23}/c_{13} \end{bmatrix}\mathbf{e}^{\mathbf{T}} \quad (16)$$

thus, we obain：

$$V_2(0) - V_2(\infty) \ge \int_0^\infty -\mathbf{e}\begin{bmatrix} c_{21}/c_{11} & 0 & 0 \\ 0 & c_{22}/c_{12} & 0 \\ 0 & 0 & c_{23}/c_{13} \end{bmatrix}\mathbf{e}^{\mathbf{T}}dt \quad (17)$$

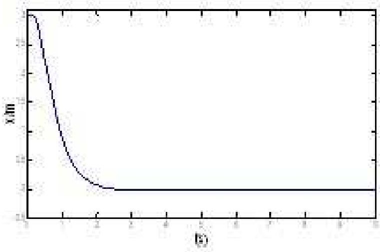
So V_2 is bounded, according to Barbalat lemma [9], we have:
$\lim_{t\to\infty}\mathbf{e} = 0$. In the same way, $\lim_{t\to\infty}\boldsymbol{\delta} = 0$.

According to the above proof, we can draw a conclusion that the backstepping controller has the ability to ensure the global uniformly stabilization of the control system, and the tracking error asymptotically converges to zero. Thus, we can choose (14) as the controller.

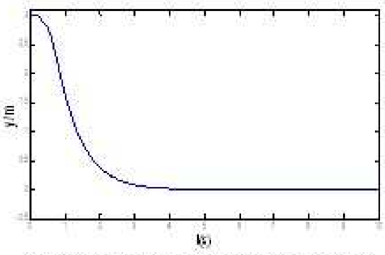
Simulation Results

Take NuBot as the application platform, and the parameters in the dynamical model are J=2.8kg·㎡, M=31kg, m=0.65kg, L=0.195m, k_1=18, k_2=150. The starting and ending points are set to be $[x, y, \theta]^{\mathrm{T}} = [3, 3, \pi]$ and $[x, y, \theta]^{\mathrm{T}} = [0, 0, 0]$, respectively. The simulation results are shown in Fig.6.

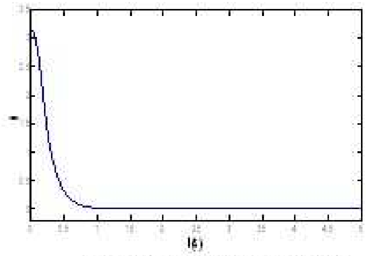
The simulation results show that the posture error is nearly to be zero and it takes only 2s to achieve the target point. The proposed control algorithm is asymptotically stable.



(a) the evolutions of x direction



(b) the evolutions of y direction



(c) the evolutions of θ

Fig.6 the evolutions of NuBot

Conclusion

In this paper, a globally stable backstepping controller is designed for the omni-directional mobile robot. Simulation results indicate that the controller has a good performance on rapidity, stability and accuracy.

References

[1] Zhang,H., Wang, X., Lu, H., et al. NuBot Team Description Paper 2009, Robocup 2009, Graz, CD-ROM(2009).

[2] Stuttgart T. Buchheim et al. Team Description Paper 2005 CoPS.

[3] T.Kalmar-Nagy et al. Near-optimal dynamic trajectory generation and control of an ommidirectional vehicle. Robotics and Autonomous Systems. Vol. 46 (2004): p.47-64.

[4] Kokotovic P. The joy of feedback: nonlinear and adaptive: 1991 Bode Prize lecture, IEEE control systems magazine, Vol. 12(3) (1992): p. 7-17.

[5] M.Kratic, L.Konellokopuouloo, P.V.Kokotovic, Adaptive nonlinear control without over-parameterization, System & Control letters, Vol. 19(3) (1992): p. 177-185.

[6] Pourboghrat, Farzad, Karlsson, Mattias P., Adaptive control of dynamic mobile robots with nonholonomic constrains, Computers and Engineering, Vol. 28(4) (2002): p.241-253.

[7] JiangZ.P., Henk Nijmeijer, Tracking control of mobile robots: a case study of backstepping, Automatica, Vol, 33(7) (1997): p. 1397-1399.

[8] R.Fierro, F.L.Lewis, Control of a nonholonomic mobile robot: backstepping kinematics into dynamics, Proceedings of the 34th Conference on Decision and Control, New Orleans, LA, December, 1995: p. 3805-3810.

[9] Narendra K, Annaswamy A. Stable adaptive systems [M]. USA: Prentice Hall. (1989).

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DOI References

[4] Kokotovic P. The joy of feedback: nonlinear and adaptive: 1991 Bode Prize lecture, IEEE control systems magazine, Vol. 12(3) (1992): pp.7-17.
doi:10.1109/37.165507
[5] M. Kratic, L. Konellokopuouloo, P.V. Kokotovic, Adaptive nonlinear control without over-parameterization, System & Control letters, Vol. 19(3) (1992): pp.177-185.
doi:10.1109/37.165507
[6] Pourboghrat, Farzad, Karlsson, Mattias P., Adaptive control of dynamic mobile robots with nonholonomic constrains, Computers and Engineering, Vol. 28(4) (2002): pp.241-253.
doi:10.1016/S0045-7906(00)00053-7
[8] R. Fierro, F.L. Lewis, Control of a nonholonomic mobile robot: backstepping kinematics into dynamics, Proceedings of the 34th Conference on Decision and Control, New Orleans, LA, December, 1995: pp.3805-3810.
doi:10.1109/CDC.1995.479190
[9] Narendra K, Annaswamy A. Stable adaptive systems [M]. USA: Prentice Hall. (1989).
doi:10.1109/CDC.1989.70277