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A Novel Backstepping Control for Attitude of Fixed-wing UAVs with Input Disturbance

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Abstract: In this paper, a novel backstepping inverse optimal control algorithm is proposed to realize robust control for attitude of fixed-wing (unmanned aerial vehicles) UAVs with input disturbance. The backstepping controller is used to stabilize all attitude variables by sufficiently applying Lyapunov theory and the input disturbances are cancelled via compensation of an estimator. In flight control, attitude control is consider as inner-loop in common, and it is significant to make sure control of inner-loop is fast and accuracy. The method proposed here determine the control parameters guaranteeing closed-loop stability and improve performance of controller with accurate aircraft dynamic model. Simulation results in MATLAB and X-plane environment show the effectiveness and applicability of the proposed controller.

Key Words: Inverse optimal, Backstepping, Nonlinear control, Attitude, UAVs

1 Introduction

UAVs have a very wide range of applications [1], and that is due to its compact structure and high maneuverability. It can conveniently be adjusted to a specific mission by changing its payload and it mainly be used by military and government organizations. For example, equipping a UAV with infrared and vision sensors, for instance, allows the UAV to monitor/report wildfire in forests or traffic in urban [2]. Fixed-wing UAVs can, be employed for search and rescue missions because they can cover vast distances very quickly and it can take off and landing conveniently.

Meanwhile, fixed-wing UAVs offer a wealth of interesting and challenging control problems [3]. The governing dynamics are nonlinear, the aerodynamics are uncertain, the states are difficulty to measure directly, the movements of body are easily polluted by environment, and the control input is constrained by position and rate limits. Study on UAVs has attracted great attentions of many researchers, and a large number of successful applications can be found in various of areas, which include formation control [4], auto landing [5], and path-following [6].

Since then, most control methods are based on mathematics model of UAVs [7]. In order to improve the performance of controller, precise mathematics model is essential to consider. The structure and order of the model should be based on a priori information on the physical properties of the vehicle on hand. But it is formidable and unnecessary for UAVs to obtain a full state mathematics model in flight control [8]. In addition, physical limitations impose magnitude, rate, and bandwidth constraints on the control surface deflections and all of the aircraft states. However, heading and airspeed commands are first order in nature to simplify the control design, and the most popular approach is to separate the control laws into multiple layers [9]. Further more, attitude, roll and pitch, are consider as inner-loop in common, and it is significant to make sure control of inner-loop is fast and accuracy.

Some recent representative theoretical researches on attitude control of UAVs can be referred. In [10], an integrated, though cascaded Lyapunov-based adaptive backstepping approach is taken and used to design a flight path controller. Longitudinal control of an aircraft that directly accommodates magnitude, rate, and bandwidth constraints on the aircraft states and the actuator signals in [11]. However, this approach is so complicated and conservative that it may excite unmodeled dynamics or saturate the control inputs and, therefore, cause robustness problems. Su [12] considered a nonlinear system subject to input disturbances, and propose to augment the nominal control law, and an upper bound on parameters can be computed such that the augmented controller yields complete disturbance suppression. But when disturbances are introduced elsewhere, this method do not guarantee output regulation.

This article presents a novel backstepping inverse optimal control approach for attitude control of an aircraft with input disturbance. Both theoretical and simulation analyses are included. An inverse optimal control algorithm is extended to the attitude tracking control problem. The most contributions of this paper are: (1) The ability to accommodate disturbance in input and adjust the parameters to optimal; (2) Boundedness of the solution and asymptotic stability can be guaranteed; (3) The approach has been simulated with both MATLAB and X-plane environment, which provides very accurate aircraft models and has very important feature: the possibility to exchange data with external systems. The aircraft models simulated in X-plane are built based on the exact physical model and material. The power characteristics of an engineering tool that can be applied to forecast the fling qualities of fixed and rotary wing aircraft [13].

The paper is outlined as follows. First, the nonlinear dynamics of the aircraft model and several essential assumptions are introduced in Section 2. In Section 3 the control system design is presented decomposed in two cascaded different cases. Section 4 validates the performance of the control laws using numerical simulations performed in MATLAB/Simulink and X-plane environment. A summary of the results and the conclusions are given in Section 5.

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2 Model of Aircraft

The aircraft model employed in this study is that of an conventional distribution aircraft, which structure and aerodynamic characters are reported in many articles [8][9].

Flight control systems can be designed for a large amount of control objectives. The object of proposed controller is to consider general maneuvering in this paper. The work has been divided into two parts: the longitudinal direction and the lateral direction.

In the longitudinal direction, the pitch rate q, in body-fixed coordinate, can be regarded as suitable controlled variable. However, the normal acceleration of aircraft is closely coupled to the angle of attack α, which plays a vital role in the equation of motion in longitude. For nonlinear approach, α is selected as controlled variable here.

In the lateral direction, roll rate p and sideslip β are also considered as command control. For convenient analysis, the body-fixed axis selected as the basic of motion axis. However, the angle of attack α and sideslip β appear in the stable axis in general. For that, the rotation axis can instead be selected as the stable axis, see Fig.1.

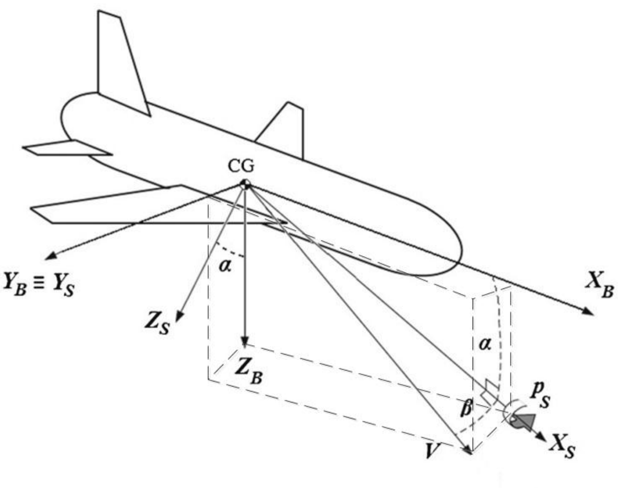

Fig. 1: Definition of body axis and stable axis

The objective of attitude control of aircraft is to design a controller, which guarantee controlled variables (α, β, p_w) tracking a desired trajectory $(\alpha^d, \beta^d, p_w^d)$.

$$\begin{array}{c} \alpha \to \alpha^d \\ \beta \to \beta^d \\ p_w \to p_w^d \end{array} \tag{1}$$

The motion equations of aircraft have been defined in [14], and the results in wind axis can be expressed as following:

$$\begin{array}{c} \dot{V}_T = \frac{1}{m}(-D + F_T \cos\alpha\cos\beta + mg_1) \\ \dot{\alpha} = \frac{1}{\cos\beta}(q_w + \frac{1}{mV_T}(-L - F_T \sin\alpha + mg_2)) \\ \dot{\beta} = -r_w + \frac{1}{mV_T}(Y - F_T\cos\alpha\sin\beta + mg_3) \end{array} \tag{2}$$

where, V_T, m are velocity and mass of aircraft, respectively. p_w, q_w, r_w are roll rate, pitch rate and yaw rate in stable axis, respectively. L, Y, F_T are lift, aerodynamic force and engine thrust, respectively.

$$\begin{array}{c} L = \bar{q}SC_L(\alpha) \\ Y = \bar{q}SC_Y(\beta) \end{array} \tag{3}$$

where, C_L, C_Y are lift coefficient and side force coefficient of aircraft. S is valid area of wing. $\bar{q} = \frac{1}{2}\rho(h)V_T^2$ is aerodynamic pressure. The gravity acceleration components are:

$$\begin{array}{c} g_1 = g(-\cos\alpha\cos\beta\sin\theta + \sin\beta\sin\phi\cos\theta \\ +\sin\alpha\cos\beta\cos\phi\cos\theta) \\ g_2 = g(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta) \\ g_3 = g(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta \\ -\sin\alpha\sin\beta\cos\phi\cos\theta) \end{array} \tag{4}$$

where, g is gravity acceleration, ϕ, θ are roll and pitch.

Define control input

$$\dot{\omega}_w = u \tag{5}$$

where $u = [u_1, u_2, u_3]$, and $\omega_w = [p_w, q_w, r_w]$.

Equations (2) and (5) together with the expressions for aerodynamic forces and moment of aircraft. The equations in this section were developed under the following assumptions:

Assumption 1: Thrust is directed along the x body axis and through the CG (center of gravity) of aircraft.

Assumption 2: The lift coefficient and side force coefficient are only depended on aerodynamic angles and not on airspeed changes or aerodynamic angle rates.

Assumption 3: The aircraft is symmetric about the Oxz plane in body axis and characters of motion in longitudinal direction and lateral direction are not dependence.

3 Controller Design and Strategy

Backstepping is a systematic method for nonlinear control design, which use Lyapunov theory to recur the design procedure. However, it is difficult to find a suitable Lyapunov function for a given system. In linear control, one often seeks for optimal controller in some sense to obtain the best ability of control, and a common method is to minimize a certain cost function in order to consider the control law.

Ezal [15] apply this technique to feedback system and backstepping controllers, and global optimal is obtained. This method is also known as inverse optimal control.

3.1 Zero-disturbance Case

For the zero-disturbance case, a common backstepping method has been developed in [10]:

$$\begin{array}{c} u_1 = k_p(p_w^d - p_w) \\ u_2 = -k_q(q_w + k_\alpha\cos\beta(\alpha - \alpha^d) + f_\alpha(\alpha^d)) \\ u_3 = k_r(-r_w + k_\beta\beta + \frac{1}{V_T}g\cos\theta\sin\phi) \end{array} \tag{6}$$

the parameters defined in (6) should satisfy

$$k_p > 0 \tag{7}$$

$$k_\alpha > max\{0, \kappa_\alpha\} \tag{8}$$

$$k_q > 2k_\alpha \tag{9}$$

$$k_\beta > max\{0, \kappa_\beta\} \tag{10}$$

$$k_r > 2k_\beta > 0 \tag{11}$$

where,

$$\kappa_\alpha = -\frac{\bar{q}S}{mV_T} min\{\frac{\partial C_L(\alpha)}{\partial \alpha}\}$$

$$\kappa_\beta = \frac{\bar{q}S}{mV_T} max\{\frac{C_Y(\beta)}{\beta}\}$$

$$f_\alpha(\alpha) = \frac{1}{mV_T}(-L(\alpha) - F_T \sin\alpha + mg_2) \quad (12)$$

$$f_\beta(\beta) = \frac{1}{mV_T}(y(\beta) - F_T \cos\alpha \sin\beta + mg_3) \quad (13)$$

In this section, the inverse optimal control problem is formulated and some notations are introduced. For a symmetric matrix $R \in \Re^{n\times n}$, $\|x\|_R^2$ represent the quadratic form $x^T Rx$. V_x denote the derivative of the Lyapunov function V, that is $V_x = \frac{\partial V}{\partial x}$.

Assume parameters $\vartheta = \{k_p, k_\alpha, k_q, k_\beta, k_r\}$. The system model introduced in (2) and (5) can be written as

$$\dot{x} = h(x) + ku \quad (14)$$

where, $x = \{\alpha, \beta, p_w, q_w, r_w\}$.

Here, we seek a cost function J that control input u minimizes it and guarantee the system to reach steady as $t \to \infty$. By choosing a proper L, we can obtain that

$$J = \int_0^\infty L(x, u)dt \quad (15)$$

It is just right that the Lyapunov function V desired in backstepping method is positive definite and continuously decrease with time. If we define $J = V$, that means L and $-\dot{V}$ coincide. To simplify analysis, L is chosen as

$$L = Q(x) + \|u\|_R^2 \quad (16)$$

where, $Q(x)$ is positive definite and R is symmetric matrix and positive definite. using Hamilton-Jacobi-Bellman (HJB) equation

$$min[Q(x) + \|u\|_R^2 + V_x\dot{x}] = 0 \quad (17)$$

In order to solve equation(17), two steps are essential and insert (14) into (17) yields

$$\begin{aligned} Q(x) + \|u\|_R^2 + V_x h(x) + V_x ku = Q(x) + V_x h(x) \\ + \|u + \tfrac{1}{2}R^{-1}(V_x k)^T\|_R^2 - \tfrac{1}{4}\|V_x k\|_{R^{-1}}^2 \end{aligned} \quad (18)$$

In (18), control input u only present in the $\|\cdot\|$, positive definite term. Therefore, the minimum only can be obtained when $\|\cdot\| = 0$, then

$$u^* = -\frac{1}{2}R^{-1}(V_x k)^T \quad (19)$$

and also that,

$$Q(x) = -V_x h(x) + \frac{1}{4}\|V_x k\|_{R^{-1}}^2 \quad (20)$$

3.2 Disturbance Case

The case of zero-disturbance has been analysed, and this subsection an unknown, constant disturbance θ is considered in control input. In (19), we have obtained a control law u^* and a corresponding Lyapunov function $V(x)$ such that

$$\dot{V}(x) = V_x(h(x) + ku^*) = -P(x) < 0 \quad (21)$$

If θ is known, the control law

$$u = u^* - \theta \quad (22)$$

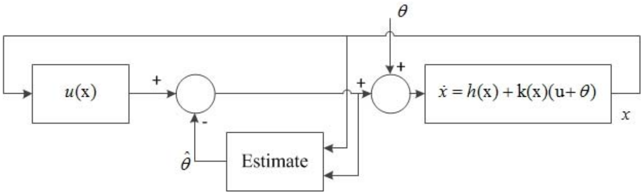

Fig. 2: Estimate the disturbance with closed system

As presented in Fig.2, we use an estimator to achieve $\hat{\theta} = \theta$, then the disturbance can be cancelled by compensation $\hat{\theta}$ in the control input.

Theorem 1: System defined in (14), and control input with unknown, constant disturbance θ. Then, the control law

$$u = u^* - \int_0^t \Xi^{-1}(V_x(x(s))k)^T ds \quad (23)$$

where Ξ is any symmetric, positive definite matrix, makes the the closed loop system to be steady and cost function to minimum.

Proof: Extend the Lyapunov function

$$V(x, \hat{\theta}) = V(x) + \frac{1}{2}\|\tilde{\theta}\|_\Xi^2 \quad (24)$$

where, $\tilde{\theta} = \theta - \hat{\theta}$, Ξ is a positive definite matrix. Define the differential of $\hat{\theta}$

$$\dot{\hat{\theta}} = \Omega(x, \hat{\theta}) \quad (25)$$

Inserting (25) into (24) yields

$$\begin{aligned} \dot{V}(x, \hat{\theta}) &= V_x(h(x) + k(u^* - \hat{\theta} + \theta)) - \Omega(x, \hat{\theta})^T \Xi\tilde{\theta} \\ &= -P(x) + (V_x k - \Omega(x, \hat{\theta})^T \Xi)\tilde{\theta} \end{aligned} \quad (26)$$

selecting

$$\Omega(x, \hat{\theta}) = \Xi^{-1}(V_x k)^T \quad (27)$$

then,

$$\dot{V}(x, \hat{\theta}) = -P(x) < 0 \quad (28)$$

so that

$$u = u^* - \hat{\theta} = u^* - \int \dot{\hat{\theta}} = u^* - \int \Omega(x, \hat{\theta}) \quad (29)$$

4 Simulations

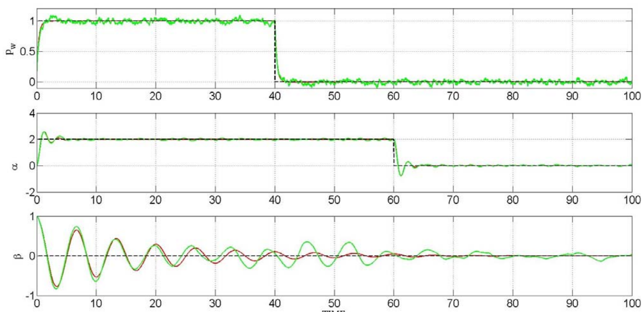

Fig. 3: Tracking results of control variables p_w, α, β. The red line and green line in the figure is response of aircraft with compensation of disturbance and without compensation of disturbance, respectively.

In order to verify the distributed controller proposed in this paper, some numerical simulations are given in this section. The approach has similarities with both MATLAB and X-plane environment.

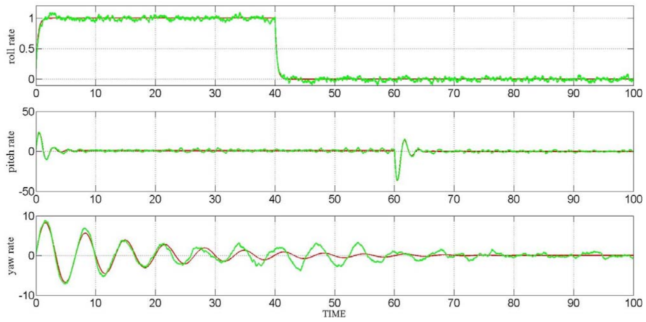

Fig. 4: Angle rate of the aircraft

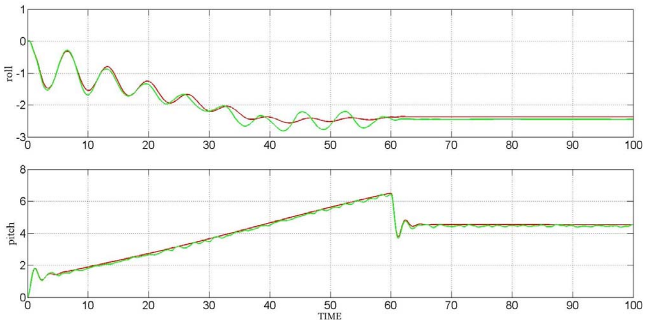

Fig. 5: Roll and pitch of the aircraft

4.1 MATLAB Experiment

Results of simulations are obtained by a Simulink model, which is integrated with a fixed time step of 0.01 seconds. Conditions are chosen as $m = 10.11kg$, $g = 9.81N/m^2$, $V_T = 10m/s$, $L_\alpha = 0.5$, $L_\beta = 0.2$, $F_T = 100N$. The disturbance is introduced as

$$\theta_1 = rand(1,1) * 40 - 20$$
$$\theta_2 = rand(1,1) * 60 - 30$$
$$\theta_3 = rand(1,1) * 30 - 15$$

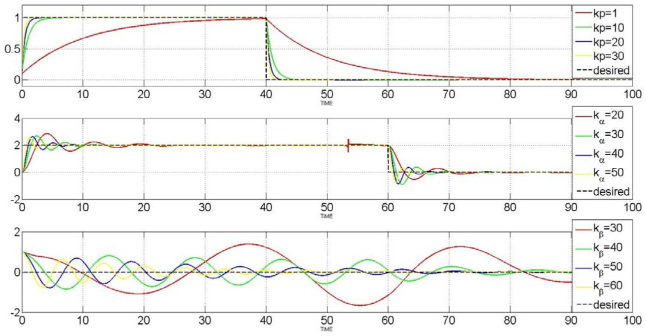

Fig. 6: Results of Four different group of parameters

The controller is designed as (19) and (29), and tracking results of control variables appear in Fig.3. The desired value of variables p_w^d, α^d, β^d are tracking well during the test and steady state value is successfully achieved. Angle rate of the aircraft in stable axis is presented in Fig.4.

Meanwhile, as the aircraft is already trimmed for the conditions, the attitude of the aircraft in the body axis roll and pitch are also presented in Fig.5. The attitude trend to stable and the objective of the controller design is achieved. In addition, the slightly higher oscillatory in the initial part of roll, mostly because the roll rate in stable axis p_w has a pulse in the initial state.

Fig. 7: Flight test in X-plane

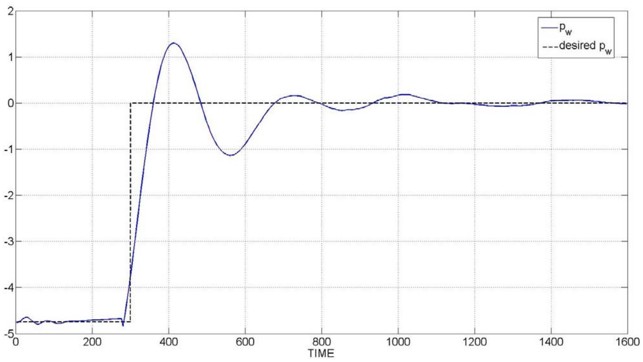

Fig. 8: Test of p_w control in X-plane

Fig.6 shows four groups of different control parameters ϑ of controller proposed in (6) with the same other test conditions without disturbance. It is meaningful that the performance of controller with different parameters can be compared and trend of alteration of parameters can be predicted.

4.2 X-plane Experiment

We also test the proposed controller in the X-plane environment. A Simulink model is also developed and employed to communicate with X-plane by UDP (user datagram protocol) communication. The UDP block is placed in Simulink with the proposed controllers. To initialize the simulation, the X-plane is loaded with aircraft PT60RC. As soon as the test platform runs, the designed controller will take attitude control. The attitude of the aircraft can also be observed dur-

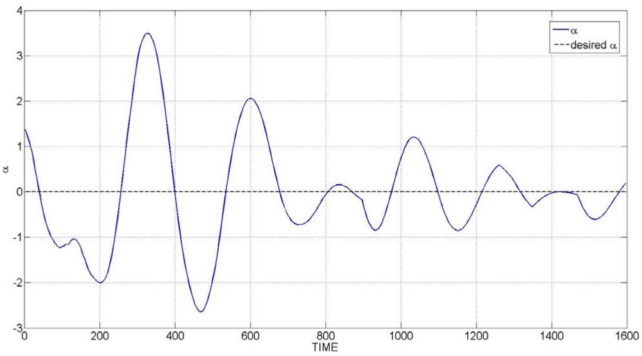

Fig. 9: Test of α control in X-plane

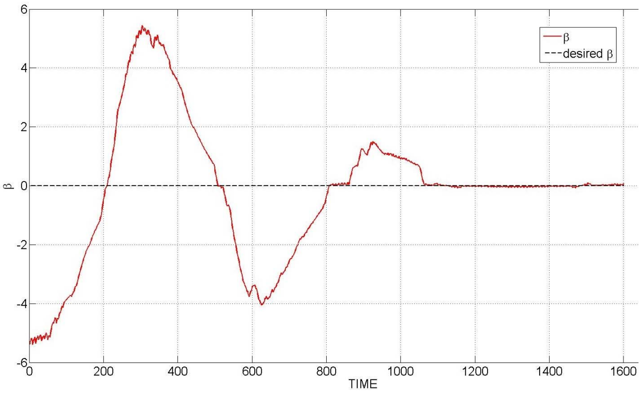

Fig. 10: Test of β control in X-plane

ing the simulation.

Real atmosphere with wind, turbulence are introduced small perturbations to aircraft flight, and the conditions are set as: low-altitude wind speed is 17 kt, wind gust increase is 5 kt, shear direction is 30 deg, turbulence is 1, the temp is 58 deg F.

The results of test is presented in Fig.8, Fig.9 and Fig.10. Desired value of α and β are set to zero, and roll rate set as a step trajectory. We can find that the aircraft executes some oscillatory in the initial state. It is showed that the controller can exactly track the commanded sideslip angle, angle of attack, and roll angle rate, respectively.

5 Conclusion

In this paper, a novel backstepping inverse optimal control algorithm is proposed to realize robust control for attitude of fixed-wing UAVs with input disturbance. Optimal and stable control is achieved with designed controller based on the backstepping technique and the optimal control theory. Meanwhile, an estimator of disturbances has been introduced and valid compensations guarantee the stability of system. Simulation results show that the controller exhibits smooth and continuous effects under arbitrary initial conditions and the tracking stage.

Attitude control of aircraft is inner-loop and it is vital in the flight test. The precision and rate of control in inner-loop is rather high and fast than out-loop. The work of this paper with a view to the control with disturbances, and the robustness and adaptive of controller is enhanced. The future work include all state flight control and auto-landing of aircraft in several different conditions.

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