Event-Triggered Consensus of Homogeneous and Heterogeneous Multiagent Systems With Jointly Connected Switching Topologies

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***Abstract*—This paper investigates the distributed event-based consensus problem of switching networks satisfying the jointly connected condition. Both the state consensus of homogeneous linear networks and the output consensus of heterogeneous networks are studied. Two kinds of event-based protocols based on local sampled information are designed, without the need to solve any matrix equation or inequality. Theoretical analysis indicates that the proposed event-based protocols guarantee the achievement of consensus and the exclusion of Zeno behaviors for jointly connected undirected switching graphs. These protocols, relying on no global knowledge of the network topology and independent of switching rules, can be devised and utilized in a completely distributed manner. They are able to avoid continuous information exchanges for either controllers' updating or triggering functions' monitoring, which ensures the feasibility of the presented protocols.**

***Index Terms*—Consensus, event-triggered control, heterogeneous network, homogeneous network, jointly connected switching topologies.**

I. Introduction

EVENT-DRIVEN coordination has been widely studied and started maturing to soon stand alone in the control area in the last decade [1]–[9]. Compared to classic continuous control approaches, event-based control has numerous advantages especially in enhancing control efficiency, such as avoiding continuously updating controllers and continuous communications among neighboring agents. The latter advantage is particularly evident when we focus on Internet of Things and other large-scale networks where the cyber operations, including processing, storage, and communication, must be viewed as a scare, globally shared resource [10]. Due to these practical considerations, it is not surprising that so many researchers are interested in event-triggered control and present plenty of results. Applying event-driven control in networked systems poses some new challenges that do not exist in either area alone [10]. As pointed out in [10], Nowzari *et al.* must considered how to deal with the natural asynchronism introduced into the systems and how to rule out the Zeno behavior. Another challenge is that the separation principle cannot be used for event-triggered control systems anymore [11].

Existing works have presented a large number of insights into general coordination of networked systems with event-triggered mechanisms. As a specific case study, event-triggered consensus is a longstanding area of research in multiagent systems (see [12]–[19]). Many survey papers about event-driven control were published, such as [10] and [20]–[22]. Generally speaking, existing consensus protocols are designed for either the state consensus of homogeneous networks or the output consensus of heterogeneous networks. Noting that for heterogeneous networks, where even the dimensions of states may be different, output consensus is a more meaningful topic than state consensus.

In the field of state consensus of homogeneous networks, Garcia *et al.* [23], Meng and Chen [24], and Dimarogonas *et al.* [25] presented event-based protocols for single-integrator agents under undirected graphs. To remove the limitation that continuous information was still required in triggering functions of early works, Seyboth *et al.* [26] proposed triggering functions only based on discrete sampled information. Yang *et al.* [27], Zhu *et al.* [28], and Guo *et al.* [29] presented event-driven consensus algorithms for general linear networks. Zhang *et al.* [30] studied the event-driven consensus using output feedback control. The event-based consensus control problem with external disturbances was studied in [31]–[33]. Event-driven output consensus of heterogeneous networks was studied in [34] and [35]. Hu and Liu [36] studied event-based cooperative output regulation problem of heterogeneous networks.

It should be noted that the proposed protocols in the above works were only designed for fixed and connected topologies. However, in many practical cases, the topologies may be switching [37]–[40] and do not satisfy the connected condition. Adaldo *et al.* [41] proposed an event-driven protocol for networks with switching communication graphs.

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One limitation of the protocol in [41], that the triggering functions were designed based on continuous information, may limit its practical applicability. To avoid continuous interagent communication, Cheng *et al.* [42] proposed decentralized event-based controllers for leader–follower networks under fixed or switching graphs. The results of [42] relied on an assumption that the (switching) topology is connected at every moment, which was not always satisfied for general switching topologies. In particular, there were even no any connections among agents at some special instants. This assumption was removed by Wu *et al.* [43] and Xu *et al.* [44], in which similar problems were considered. The designs of the protocols proposed in [43] and [44], nevertheless, required to solve two coupled inequalities, while the existence of the solution is unclear in general cases. The switching nature of topologies coupled with event-triggered communications makes it troublesome to propose distributed consensus algorithms, and the existence of heterogeneity renders the task for heterogeneous networks more challenging. How to devise event-triggered consensus algorithms for linear homogeneous (or heterogeneous) networks with general switching topologies needs further investigation.

In this paper, we study the event-driven consensus control problems with switching graphs, including the state consensus of homogeneous linear networks and the output consensus of heterogeneous linear networks. For the homogeneous case, we present an event-based protocol, composed of controllers and triggering rules. Under this protocol, communications will not take place until the topology switches or the designed measurement error exceeds an appropriate threshold. It is shown that state consensus is achieved and Zeno behaviors are ruled out. The protocol can be explicitly constructed and do not need to solve any matrix equation or inequality. We also consider an event-based output consensus of heterogeneous networks with switching topologies and an exogenous signal that can be viewed as a reference input or an external disturbance. For this problem, we first devise distributed observers to estimate the exogenous signal and then propose local control inputs.

The main contributions of this paper are listed as follows. We have solved both the event-based state consensus control problem of homogeneous networks and the event-based output consensus control problem of heterogeneous networks. Different from existing related papers, the proposed event-triggered protocols of this paper can be used for any switching graphs satisfying the jointly connected condition, including fixed graphs as a special case. The proposed protocols, requiring no global information associated with the whole network and independent of the switching rules, can be devised and utilized in a completely distributed manner. The Zeno behavior can be excluded at any finite time by showing that the interval between any different triggering instants is not less than a strictly positive value. This feature ensures the feasibility of the above protocols when they are implemented on practical systems.

Here is the outline of this paper. In Section II, we consider the event-driven state consensus of homogeneous networks. We then study event-based output consensus of heterogeneous networks in Section III. Numerical simulations and conclusions are presented in Sections IV and V, respectively.

II. Event-Based State Consensus of Homogeneous Multiagent Systems

A. Problem Formulation

In this section, we consider N homogeneous linear agents, whose dynamics satisfy

$$\dot{x}_i = Ax_i + Bu_i,\ \ i = 1, \ldots, N \tag{1}$$

where $x_i \in \mathbf{R}^n$ denotes the state, $u_i \in \mathbf{R}^p$ represents the control input, and $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times p}$ are constant matrices.

Assumption 1: The pair (A, B) is stabilizable and A is neutrally stable.[1]

Denote $\theta : [0, +\infty) \to \Theta$ as a switching signal with a positive dwelling time τ. Let $\mathcal{G}_{\theta(t)} \triangleq (\mathcal{V}, \mathcal{E}_{\theta(t)})$ represent an undirected graph among the N agents, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}_{\theta(t)} \subseteq \mathcal{V}\times\mathcal{V}$ denote the sets of nodes and edges, respectively. Consider an infinite time sequence composed of nonempty, bounded, and contiguous intervals $[\bar{t}_0, \bar{t}_1), \ldots, [\bar{t}_k, \bar{t}_{k+1}), \ldots$, with $\bar{t}_0 = 0$. Suppose $\bar{t}_{k+1} - \bar{t}_k \le T$ with T being some positive constant and during each interval $[\bar{t}_k, \bar{t}_{k+1})$, there are finite nonoverlapping subintervals

$$\left[\bar{t}_k^0, \bar{t}_k^1\right), \left[\bar{t}_k^1, \bar{t}_k^2\right), \ldots, \left[\bar{t}_k^{m_k-1}, \bar{t}_k^{m_k}\right),\ \bar{t}_k = \bar{t}_k^0,\ \bar{t}_{k+1} = \bar{t}_k^m$$

satisfying $\bar{t}_k^{j+1} - \bar{t}_k^j \ge \tau$, $j = 0, 1, \ldots, m_k - 1$. And $\mathcal{G}_{\theta(t)}$ is fixed during each subinterval. An edge of $\mathcal{E}_{\theta(t)}$ is composed of two distinct nodes of $\mathcal{V}$. If $(i, j) \in \mathcal{E}_{\theta(t)}$, i and j are neighbors under graph $\mathcal{G}_{\theta(t)}$. An undirected path between nodes i and j is denoted as $(i_1, i_2), (i_2, i_3), \ldots, (i_q, j)$. Denote the adjacency matrix of graph $\mathcal{G}_{\theta(t)}$ by $\mathcal{A}(t) = [a_{ij}(t)] \in \mathbf{R}^{N\times N}$, where $a_{ii}(t) = 0$, $a_{ij}(t) = 1$ if $(j, i) \in \mathcal{E}_{\theta(t)}$ and $a_{ij}(t) = 0$ otherwise. Denote the Laplacian matrix $\mathcal{L}_{\theta(t)} = [l_{ij}(t)] \in \mathbf{R}^{N\times N}$ of $\mathcal{G}_\theta$ by $l_{ii}(t) = \sum_{j=1}^N a_{ij}(t)$ and $l_{ij}(t) = -a_{ij}(t)$, $i \ne j$. Define the degree as $d_i(t) = l_{ii}(t)$, $i \in \mathcal{V}$. Then, define $\bigcup_{t\in[\bar{t}_k,\bar{t}_{k+1})} \mathcal{G}_{\theta(t)}$ as a union graph in the collection for time t from $\bar{t}_k$ to $\bar{t}_{k+1}$.

Assumption 2: The undirected graph $\mathcal{G}_{\theta(t)}$ of the N agents is jointly connected, i.e., $\bigcup_{t\in[\bar{t}_k,\bar{t}_{k+1})} \mathcal{G}_{\theta(t)}$ is connected.

The objective here is to present distributed event-based algorithms under which all subsystems described by (1) converge to a common state trajectory and Zeno behaviors can be eliminated.

Instead of using agents' actual states, define the state estimate as $\tilde{x}_i(t) \triangleq e^{A(t-t_k^i)}x_i(t_k^i)$, $\forall t \in [t_k^i, t_{k+1}^i)$, $i = 1, \ldots, N$, where t_k^i denotes the kth event instant of agent i. The event instants $t_0^i, t_1^i, \ldots$, are determined by the triggering function to be designed later. Using the relative state estimates of neighboring agents, we present a distributed event-based controller as

$$u_i(t) = cG\sum_{j=1}^N a_{ij}(t)\left(\tilde{x}_i - \tilde{x}_j\right),\ \ i \in \mathcal{V} \tag{2}$$

where $c \in \mathbf{R}_{>0}$ and $G \in \mathbf{R}^{p\times n}$ are design parameters.

[1] A matrix $A \in \mathbf{C}^{n\times n}$ is neutrally stable in the continuous-time sense if it has no eigenvalue with positive real part and the Jordan block corresponding to any eigenvalue on the imaginary axis is of size 1, while is Hurwitz if all of its eigenvalues have strictly negative real parts [4].

Define $\xi = [\xi_1^T, \ldots, \xi_N^T]^T$ and $\tilde{\xi} = [\tilde{\xi}_1^T, \ldots, \tilde{\xi}_N^T]^T$ with $\xi_i \triangleq x_i - (1/N)\sum_{j=1}^N x_j$ and $\tilde{\xi}_i \triangleq \tilde{x}_i - (1/N)\sum_{j=1}^N \tilde{x}_j$, $i = 1, \ldots, N$. Letting $x \triangleq [z_1^T, \ldots, x_N^T]^T$ gives $\xi = (M \otimes I_n)x$ and $\tilde{\xi} = (M \otimes I_n)\tilde{x}$, where $M = I_N - (1/N)\mathbf{1}_N\mathbf{1}_N^T$. Noting that $\xi = 0$ if and only if $x_1 = \cdots = x_N$, we call ξ the consensus error, whose dynamics is given by

$$\dot{\xi} = (I_N \otimes A)\xi + (c\mathcal{L}_\theta \otimes \mathrm{BG})\tilde{\xi}. \tag{3}$$

Note that the control law (2) is only updated according to the information received at the latest event time instant, defined by

$$t_{k+1}^i \triangleq \inf\{t > t_k^i \mid f_i(t) \geq 0 \text{ or } a_{ij}(t) \neq a_{ij}(t_k^i) \text{ for some } j \in \mathcal{V}\} \tag{4}$$

where $t_0^i \triangleq 0$ and $f_i(t)$ is the triggering function defined as follows:

$$f_i(t) = 4d_i(t)\|G\|^2\|e_i\|^2 - \delta\sum_{j=1}^N a_{ij}(t)\left\|G(\tilde{x}_i - \tilde{x}_j)\right\|^2 - \mu e^{-\nu t}, \ i = 1, \ldots, N \tag{5}$$

with δ, μ, and ν being positive constants, and $e_i \triangleq \tilde{x}_i - x_i$ being the measurement error. Once f_i triggers, agent i broadcasts its current state to neighbors. The controllers (2) of i and its neighbors update immediately, and $e_i(t)$ resets at the same time.

B. Event-Based Consensus Conditions

Since A is neutrally stable, in light of [4, Lemmas 22 and 23], we can choose $E \in \mathbf{R}^{n_1 \times n}$ and $F \in \mathbf{R}^{(n-n_1)\times n}$ satisfying

$$\begin{bmatrix} E \\ F \end{bmatrix} A \begin{bmatrix} E \\ F \end{bmatrix}^{-1} = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}$$

where $X \in \mathbf{R}^{n_1 \times n_1}$ is skew-symmetric and $Y \in \mathbf{R}^{(n-n_1)\times(n-n_1)}$ is Hurwitz.

Remark 1: It should be pointed out that the matrices E and F can be derived by rendering the matrix A into the real Jordan canonical form [45].

Choose $z = \left(I_N \otimes \begin{bmatrix} E \\ F \end{bmatrix}\right)\xi$ and $\tilde{z} = \left(I_N \otimes \begin{bmatrix} E \\ F \end{bmatrix}\right)\tilde{\xi}$. The derivative of z is given by

$$\dot{z} = \left(I_N \otimes \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}\right)z + \left(c\mathcal{L} \otimes \begin{bmatrix} E \\ F \end{bmatrix}\mathrm{BG}\begin{bmatrix} E \\ F \end{bmatrix}^{-1}\right)\tilde{z}. \tag{6}$$

Let $H = EB$. According to Assumption 1, (X, H) is controllable. Choose $E^+ \in \mathbf{R}^{n \times n_1}$ and $F^+ \in \mathbf{R}^{n \times (n-n_1)}$ satisfying $[E^+ \ F^+] = \begin{bmatrix} E \\ F \end{bmatrix}^{-1}$, with $EE^+ = I$, $FF^+ = I$, $FE^+ = 0$, and $EF^+ = 0$. Since $G = -B^TE^TE$, then we have

$$\dot{z} = \left(I_N \otimes \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}\right)z - \left(c\mathcal{L}_\theta \otimes \begin{bmatrix} EBB^TE^T & 0 \\ FBB^TE^T & 0 \end{bmatrix}\right)\tilde{z}. \tag{7}$$

Define $z_I = (I_N \otimes E)\xi$, $\tilde{z}_I = (I_N \otimes E)\tilde{\xi}$, $z_{II} = (I_N \otimes F)\xi$, and $\tilde{z}_{II} = (I_N \otimes F)\tilde{\xi}$. Rewrite (7) as

$$\dot{z}_I = (I_N \otimes X)z_I - (c\mathcal{L}_\theta \otimes HH^T)\tilde{z}_I \tag{8a}$$

$$\dot{z}_{II} = (I_N \otimes Y)z_{II} - (c\mathcal{L}_\theta \otimes FBB^TE^T)\tilde{z}_I. \tag{8b}$$

Lemma 1 (Cauchy's Convergence Criterion [43]): The sequence $V(\bar{t}_k)$, $k = 0, 1, 2, \ldots$ converges if and only if for $\forall\varepsilon > 0$, $\exists M_\varepsilon \in \mathbf{Z}_+$ satisfying $\forall k > M_\varepsilon$, $|V(\bar{t}_{k+1}) - V(\bar{t}_k)| < \varepsilon$.

Lemma 2 (Barbalat's Lemma [46]): If $\lim_{t\to\infty} g(t) = a$ (a is bounded) and $g''(t)$ is also bounded, then $\lim_{t\to\infty} g'(t) = 0$.

Next, we introduce the main results of this section.

Theorem 1: State consensus of the homogeneous subsystems (1) is achieved under the event-driven algorithm composed of (2) and (5) with $c > 0$, $0 < \delta < 1$, $\mu > 0$, $\nu > 0$, and $G = -B^TE^TE$.[2]

Proof: Let

$$V_1 = \frac{1}{2}z_I^Tz_I. \tag{9}$$

In light of (8a), differentiating V_1 with respect to t gives

$$\dot{V}_1 = \frac{1}{2}z_I^T\left[I_N \otimes \left(X + X^T\right)\right]z_I - z_I^T\left(c\mathcal{L}_\theta \otimes HH^T\right)\tilde{z}_I. \tag{10}$$

Since X is skew-symmetric, $z_I^T[I_N \otimes (X + X^T)]z_I = 0$. Then, we have

$$\dot{V}_1 = -\frac{1}{2}z_I^T\left(c\mathcal{L}_\theta \otimes HH^T\right)z_I - \frac{1}{2}\tilde{z}_I^T\left(c\mathcal{L}_\theta \otimes HH^T\right)\tilde{z}_I + \frac{1}{2}e^T\left(c\mathcal{L}_\theta \otimes G^TG\right)e. \tag{11}$$

Let

$$V_2 = \frac{1}{2}z_{II}^T(I_N \otimes P)z_{II} \tag{12}$$

where P satisfies

$$PY + Y^TP + 2I = 0. \tag{13}$$

In light of (8b), differentiating V_2 with respect to t gives

$$\dot{V}_2 = \frac{1}{2}z_{II}^T\left[I_N \otimes \left(PY + Y^TP\right)\right]z_{II} - z_{II}^T\left(c\mathcal{L}_\theta \otimes PFBB^TE^T\right)\tilde{z}_I. \tag{14}$$

Using the Young's inequality [10] gives

$$\begin{aligned} &-z_{II}^T\left(c\mathcal{L}_\theta \otimes PFBB^TE^T\right)\tilde{z}_I \\ &\leq \frac{1}{2}z_{II}^Tz_{II} + \frac{c^2\lambda_N(\mathcal{L}_\theta)}{2}\tilde{z}_I^T\left(\mathcal{L}_\theta \otimes EBB^TF^TPPFBB^TE^T\right)\tilde{z}_I \\ &\leq \frac{1}{2}z_{II}^Tz_{II} + \frac{c\alpha_1}{2}\tilde{x}^T\left(\mathcal{L}_\theta \otimes G^TG\right)\tilde{x} \end{aligned} \tag{15}$$

where $\alpha_1 = c\lambda_N(\mathcal{L})\|PFB\|^2$ and $\lambda_N(\mathcal{L})$ denotes the largest eigenvalue of $\mathcal{L}_{\theta(t)}$ for all $t > 0$.

Construct the Lyapunov function candidate as

$$V_3 = \frac{\alpha_1}{1-\delta}V_1 + V_2. \tag{16}$$

Evidently, V_3 is positive definite, whose derivative is given by

$$\begin{aligned} \dot{V}_3 \leq\ & \frac{c\alpha_1}{2(1-\delta)}\left[-z_I^T\left(\mathcal{L}_\theta \otimes HH^T\right)z_I + e^T\left(\mathcal{L}_\theta \otimes G^TG\right)e - \tilde{x}^T\left(\mathcal{L}_\theta \otimes G^TG\right)\tilde{x}\right] \\ &+ \frac{1}{2}z_{II}^T\left[I_N \otimes \left(PY + Y^TP + I\right)\right]z_{II} + \frac{c\alpha_1}{2}\tilde{x}^T\left(\mathcal{L}_\theta \otimes G^TG\right)\tilde{x} \\ \leq\ & -\alpha_2 z_I^T\left(\mathcal{L}_\theta \otimes HH^T\right)z_I - \frac{1}{2}z_{II}^Tz_{II} \\ &+ \alpha_2\left[e^T\left(\mathcal{L}_\theta \otimes G^TG\right)e - \delta\tilde{x}^T\left(\mathcal{L}_\theta \otimes G^TG\right)\tilde{x}\right] \end{aligned} \tag{17}$$

[2]The matrix E can be obtained according to Remark 1.

where $\alpha_2 = (c\alpha_1/[2(1-\delta)])$. Because $a_{ij}(t) = a_{ji}(t)$, we have

$$\begin{aligned} e^T\big(\mathcal{L}_\theta \otimes G^T G\big)e &= \sum_{i=1}^N \sum_{j=1}^N a_{ij}(t) e_i^T G^T G\big(e_i - e_j\big) \\ &\le 2\sum_{i=1}^N \sum_{j=1}^N a_{ij}(t) e_i^T G^T G e_i \\ &\le 2\sum_{i=1}^N d_i(t)\|G\|^2\|e_i\|^2 \end{aligned} \tag{18}$$

and

$$\begin{aligned} \tilde{x}^T\big(\mathcal{L}_\theta \otimes G^T G\big)\tilde{x} &= \sum_{i=1}^N \sum_{j=1}^N a_{ij}(t)\tilde{x}_i^T G^T G\big(\tilde{x}_i - \tilde{x}_j\big) \\ &= \frac{1}{2}\sum_{i=1}^N \sum_{j=1}^N a_{ij}(t)\big(\tilde{x}_i - \tilde{x}_j\big)^T G^T G\big(\tilde{x}_i - \tilde{x}_j\big) \\ &= \frac{1}{2}\sum_{i=1}^N \sum_{j=1}^N a_{ij}(t)\big\|G\big(\tilde{x}_i - \tilde{x}_j\big)\big\|^2. \end{aligned} \tag{19}$$

By substituting (5), (18), and (19) into (17), we have

$$\begin{aligned} \dot{V}_3 &\le -\alpha_2 z_I^T\big(\mathcal{L}_\theta \otimes HH^T\big)z_I - \frac{1}{2}z_{II}^T z_{II} \\ &\quad + \frac{\alpha_2}{2}\sum_{i=1}^N \left\{4d_i(t)\|G\|^2\|e_i\|^2 - \delta\sum_{j=1}^N a_{ij}(t)\big\|G\big(\tilde{x}_i - \tilde{x}_j\big)\big\|^2\right\} \\ &\le -\varsigma_1 z_I^T(\mathcal{L}_\theta \otimes I)z_I - \frac{1}{2}z_{II}^T z_{II} + \frac{\mu\alpha_2 N}{2}e^{-\nu t} \end{aligned} \tag{20}$$

where $\varsigma_1 = \alpha_2\|HH^T\|$.

Define $\tilde{V}_3(t) = V_3(t) + (\mu\alpha_2 N/2\nu)e^{-\nu t}$. Then, we have

$$\dot{\tilde{V}}_3 \le -\varsigma_1 z_I^T(\mathcal{L}_\theta \otimes I)z_I - \frac{1}{2}z_{II}^T z_{II}. \tag{21}$$

Combining with $\dot{\tilde{V}}_3(t) \le 0$ and $\tilde{V}_3(t) \ge 0$, we have that $\tilde{V}_3$ is bounded and $\lim_{t\to+\infty}\tilde{V}_3(t)$ exists. Based on Lemma 1, for $\forall\varepsilon > 0$, $\exists M_\varepsilon \in \mathbf{Z}_+$ satisfying $\forall k \ge M_\varepsilon$

$$\big|\tilde{V}_3\big(\bar{t}_{k+1}\big) - \tilde{V}_3\big(\bar{t}_k\big)\big| < \varepsilon$$

or

$$\left|\int_{\bar{t}_k}^{\bar{t}_{k+1}} \dot{\tilde{V}}_3(t)dt\right| < \varepsilon.$$

It follows that:

$$\left|\int_{\bar{t}_k^0}^{\bar{t}_k^1} \dot{\tilde{V}}_3(t)dt\right| + \cdots + \left|\int_{\bar{t}_k^{m_k-1}}^{\bar{t}_k^{m_k}} \dot{\tilde{V}}_3(t)dt\right| < \varepsilon. \tag{22}$$

In light of (21), for each subinterval $[\bar{t}_k^j, \bar{t}_k^{j+1})$, $j = 0, 1, \ldots, m_k - 1$, we have that

$$\begin{aligned} \left|\int_{\bar{t}_k^j}^{\bar{t}_k^{j+1}} \dot{\tilde{V}}_3(t)dt\right| &\ge \varsigma_1\int_{\bar{t}_k^j}^{\bar{t}_k^{j+1}} z_I^T(t)\Big(\mathcal{L}_{\theta(\bar{t}_k^j)} \otimes I\Big)z_I(t)dt \\ &\quad + \frac{1}{2}\int_{\bar{t}_k^j}^{\bar{t}_k^{j+1}} z_{II}^T(t)z_{II}(t)dt \\ &\ge \varsigma_1\int_{\bar{t}_k^j}^{\bar{t}_k^j+\tau} z_I^T(t)\Big(\mathcal{L}_{\theta(\bar{t}_k^j)} \otimes I\Big)z_I(t)dt \\ &\quad + \frac{1}{2}\int_{\bar{t}_k^j}^{\bar{t}_k^j+\tau} z_{II}^T(t)z_{II}(t)dt. \end{aligned} \tag{23}$$

Combining (22) with (23) gives

$$\begin{aligned} \varepsilon > \varsigma_1\Bigg\{&\int_{\bar{t}_k^0}^{\bar{t}_k^0+\tau} z_I^T(t)\Big(\mathcal{L}_{\theta(\bar{t}_k^0)} \otimes I\Big)z_I(t)dt \\ &+ \cdots + \int_{\bar{t}_k^{m_k-1}}^{\bar{t}_k^{m_k-1}+\tau} z_I^T(t)\Big(\mathcal{L}_{\theta(\bar{t}_k^{m_k-1})} \otimes I\Big)z_I(t)dt\Bigg\} \end{aligned}$$

which implies that for $\forall k > M_\varepsilon$

$$\int_{\bar{t}_k^j}^{\bar{t}_k^j+\tau} z_I^T(t)\Big(\mathcal{L}_{\theta(\bar{t}_k^j)} \otimes I\Big)z_I(t)dt < \frac{\varepsilon}{\varsigma_1} j = 0, 1, \ldots, m_k - 1. \tag{24}$$

From (24), we have

$$\lim_{t\to\infty}\int_t^{t+\tau} z_I^T(s)\Big(\mathcal{L}_{\theta(\bar{t}_k^j)} \otimes I\Big)z_I(s)ds = 0 \quad j = 0, 1, \ldots, m_k - 1.$$

Since only finite switches take place during $[\bar{t}_k, \bar{t}_{k+1})$, we obtain that

$$\begin{aligned} \lim_{t\to\infty}\int_t^{t+\tau}\Big\{&z_I^T(s)\Big(\mathcal{L}_{\theta(\bar{t}_k^0)} \otimes I\Big)z_I(s) + \cdots \\ &+ z_I^T(s)\Big(\mathcal{L}_{\theta(\bar{t}_k^{m_k-1})} \otimes I\Big)z_I(s)\Big\}ds = 0 \end{aligned}$$

which can be rewritten as

$$\lim_{t\to\infty}\int_t^{t+\tau}\big\{z_I^T(s)(\mathcal{L}_\Sigma \otimes I)z_I(s)\big\}ds = 0 \tag{25}$$

where $\mathcal{L}_\Sigma = \mathcal{L}_{\theta(\bar{t}_k^0)} + \cdots + \mathcal{L}_{\theta(\bar{t}_k^{m_k-1})}$. According to Assumption 2, $\mathcal{L}_\Sigma$ is connected. We can find an orthogonal matrix T_Σ such that $T_\Sigma\mathcal{L}_\Sigma T_\Sigma^T = \Lambda_\Sigma \triangleq \mathrm{diag}(0, \lambda_\Sigma^2, \ldots, \lambda_\Sigma^N)$, where $\lambda_\Sigma^i > 0$, $i = 2, \ldots, N$, are the eigenvalues of $\mathcal{L}_\Sigma$. Define $\rho = [\rho_1^T, \ldots, \rho_N^T]^T = (T_\Sigma \otimes q)z_I$. It is not difficult to verify that $\rho_1 \equiv 0$. Then, (25) implies that

$$\lim_{t\to\infty}\int_t^{t+\tau}\left\{\sum_{i=2}^N \lambda_\Sigma^i\rho_i^T(s)\rho_i(s)\right\}ds = 0.$$

Because $\tilde{V}_3 \ge 0$ is bounded and $0 \le V_3 \le \tilde{V}_3$, we conclude that V_3 is bounded. In light of (16), $\rho(t)$ is bounded. Noting Assumption 1, we get that $\dot{\rho}(t)$ is bounded according to (8a). Furthermore,

$$\frac{d^2}{dt^2}\int_t^{t+\tau}\left\{\sum_{i=2}^N \lambda_\Sigma^i\rho_i^T(s)\rho_i(s)\right\}ds = 2\sum_{i=2}^N \lambda_\Sigma^i\rho_i^T(t)\dot{\rho}_i(t)$$

which is also bounded. According to Lemma 2, we have that $\lim_{t\to\infty}\{\sum_{i=2}^N \lambda_\Sigma^i\rho_i^T(t)\rho_i(t)\} = 0$, which further indicates that $\lim_{t\to\infty}\rho_i = 0$, $\forall i \in \mathcal{V}$, i.e., $\lim_{t\to\infty} z_I(t) = 0$. Similarly, we can show that $\lim_{t\to\infty} z_{II} = 0$. Consequently, state consensus is achieved. ■

Remark 2: In light of Remark 1, the feedback matrix G is easy to determine such that the event-based protocol (2) and (5) satisfies Theorem 1. Contrary to [43] and [44], where the designs of the event-based protocols rely on a solution to two coupled matrix inequalities, the existence of which is unclear in general cases, the protocol proposed in this paper can be explicitly constructed, without the need to solve any matrix equality or inequality. Besides, our protocol, requiring neither the switching rule of topologies nor nonzero eigenvalues of the Laplacian matrix, can be devised and utilized in a completely distributed manner.

Theorem 2: The closed-loop system (3) exhibits no Zeno behaviors and the interval between two consecutive triggering instants for any agent is strictly positive in finite time.

Proof: To exclude Zeno behaviors, we consider the following four cases.

1) In the first case, both t_k^i and t_{k+1}^i are determined by the triggering function (5). Under Assumption 2, we only need to exclude Zeno behaviors for the network (3) when $d_i(t) > 0$. Combining with (1) and (2) gives

$$\dot{e}_i = Ae_i - c\sum_{j=1}^{N} a_{ij}(t)\mathrm{BG}\big(\tilde{x}_i - \tilde{x}_j\big)$$

which implies that

$$\frac{d\|e_i\|}{dt} \leq \|A\|\|e_i\| + c\sum_{j=1}^{N} a_{ij}(t)\|\mathrm{BG}\|\big\|\tilde{x}_i - \tilde{x}_j\big\|. \quad (26)$$

Theorem 1 shows that ξ is bounded. Since A is neutrally stable (by Assumption 1), it is easy to see that $\tilde{\xi}$ is also bounded. Combining (1) and (2) gives $\dot{x} = (I_N \otimes A)x + (c\mathcal{L}_\theta \otimes \mathrm{BG})\tilde{\xi}$. Thus, x is bounded, which further indicates the boundedness of $\tilde{x}$. Then, it follows from (26) that:

$$\frac{d\|e_i\|}{dt} \leq \|A\|\|e_i\| + c\sigma_i \quad (27)$$

where σ_i denotes the upper bound of $\sum_{j=1}^{N} a_{ij}(t)\|\mathrm{BG}\|\|\tilde{x}_i - \tilde{x}_j\|$ for t from t_k^i to t_{k+1}^i. Define a function $\psi : [0, \infty) \to \mathbf{R}_{\geq 0}$, satisfying

$$\dot{\psi} = \|A\|\psi + c\sigma_i,\ \ \psi(0) = \big\|e_i\big(t_k^i\big)\big\| = 0. \quad (28)$$

Then, we obtain that $\|e_i(t)\| \leq \psi(t - t_k^i)$, where $\psi(t)$ is the analytical solution to (28), given by $\psi(t) = (c\sigma_i/\|A\|)(e^{\|A\|t} - 1)$. On the other hand, the triggering function (5) satisfies $f_i(t) \leq 0$, if we have the following condition:

$$\|e_i\|^2 \leq \frac{\mu e^{-\nu t}}{d_i(t)\|G\|^2}. \quad (29)$$

Then, the interval between two triggering instants t_k^i and t_{k+1}^i for agent v_i can be lower bounded by the time for $\psi^2(t - t_k^i)$ evolving from 0 to the right hand of (29). Thus, a lower bound of $t_{k+1}^i - t_k^i$, denoted as τ_k^i, can be obtained by solving the following inequality:

$$\frac{c^2\sigma_i^2}{\|A\|^2}\Big(e^{\|A\|t} - 1\Big)^2 \geq \frac{\mu e^{-\nu t}}{d_i(t)\|G\|^2}$$

from which, we have that

$$\tau_k^i \geq \frac{1}{\|A\|}\ln\left(1 + \frac{\|A\|}{c\sigma_i\|G\|}\sqrt{\frac{\mu e^{-\nu(t_k^i+\tau_k^i)}}{d_i(t)}}\right). \quad (30)$$

2) In the second case, t_k^i is determined by the switch of the topology, while t_{k+1}^i is determined by the triggering function (5). Since the measurement error e_i is reset to zero at t_k^i, this case is similar to the first case and the details are omitted here for brevity.
3) In the third case, both t_k^i and t_{k+1}^i are determined by the switches of the topology. It is obvious that the interval is not less than the dwelling time τ.
4) In the last case, t_k^i is determined by the triggering function (5), while t_{k+1}^i is determined by the switch of the topology. Note that in finite time, there is only a finite number of switches. Therefore, the minimum of the finite interval $\tau_k^i = t_{k+1}^i - t_k^i$ is nonzero, and there exists a minimum interevent time, while its value is not available in this case.

In conclusion, Zeno behaviors are excluded and the interval between two consecutive triggering instants is strictly positive in finite time. ■

Remark 3: Generally speaking, the Zeno behavior is excluded if there does not exist infinite triggers within a finite period of time. However, as pointed out in [10], even though the Zeno behavior is ruled out theoretically, it is still troublesome from an implementation viewpoint, if the physical hardware cannot match the speed of actions required by the protocol. In other words, ensuring a system does not exist the Zeno behavior may not be enough to guarantee the protocol can be implemented on a physical system. As an expected feature, the triggering rule (5) designed in this paper guarantees that the interval between different triggering instants in finite time is not less than a strictly positive constant. Besides, the hybrid triggering functions (5) including the state term $-\delta\sum_{j=1}^{N} a_{ij}(t)\|G(\tilde{x}_i - \tilde{x}_j)\|^2$ and the time term $-\mu e^{-\nu t}$ are more propitious to reduce communication frequency compared to the ones in [27] when the time t becomes very long or even as $t \to \infty$.

Remark 4: Theorems 1 and 2 show that the presented event-triggered algorithm is applicable to switching networks satisfying the jointly connected condition. According to the triggering rule (4), communications only take place when the triggering function (5) is violated or the topology switches. It should be noted when $\tau \to +\infty$, the event-based protocol here is reduced to the one for fixed graphs as a special case. If τ is too small, there is no need to check whether the triggering function (5) is violated or not and communications are not required until the next switch of the topologies takes place.

III. Event-Based Output Consensus of Heterogeneous Multiagent Systems

A. Problem Formulation

In this section, we consider N heterogeneous linear agents, whose dynamics can be described by

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i u_i + E_i w_0\\ y_i &= C_i x_i + F_i w_0,\ \ i = 1, \ldots, N\end{aligned} \quad (31)$$

where $x_i \in \mathbf{R}^{n_i}$ denotes the state, $u_i \in \mathbf{R}^{p_i}$ represents the control input, $y_i \in \mathbf{R}^{q_i}$ is the output, and $A_i \in \mathbf{R}^{n_i \times n_i}$, $B_i \in \mathbf{R}^{n_i \times m_i}$, $C_i \in \mathbf{R}^{p_i \times n_i}$, $E_i \in \mathbf{R}^{q \times n_i}$, and $F_i \in \mathbf{R}^{q \times p_i}$ are constant matrices. The exogenous signal $w_0 \in \mathbf{R}^q$, which can be treated as a reference input or an external disturbance, satisfies the following dynamics:

$$\dot{w}_0 = S w_0 \tag{32}$$

where $S \in \mathbf{R}^{q \times q}$.

The objective here is to design distributed event-based algorithms under which all subsystems described by (31) converge to a common output and Zeno behaviors can be eliminated.

Similarly as in [38], we can view the exosystem (32) as a leader, indexed by 0, and the N subsystems (31) as followers, indexed by $1, \ldots, N$. Denote $\Delta_\theta \triangleq \operatorname{diag}\{a_{10}(t), \ldots, \delta_{N0}(t)\}$, where $a_{i0}(t) = 1$ if the leader is a neighbor of i currently and $a_{i0}(t) = 0$ otherwise. Use $\bar{\mathcal{G}}_\theta$ to denote the leader–follower graph and let $\mathcal{H}_\theta = \mathcal{L}_\theta + \Delta_\theta$. The leader has directed paths to all followers during $[\bar{t}_k, \bar{t}_{k+1})$, if the union graph $\bigcup_{t \in [\bar{t}_k, \bar{t}_{k+1})} \mathcal{G}_{\theta(t)}$ contains a directed spanning tree with the leader as the root node.

Assumption 3: The pairs (A_i, B_i), $\forall i \in \mathcal{V}$, are stabilizable.

Assumption 4: S has no eigenvalues with positive real parts.

Assumption 5: For all $\lambda \in \sigma(S)$, where $\sigma(S)$ represents the spectrum of S, $\operatorname{rank}\left(\begin{bmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{bmatrix}\right) = n_i + p_i$.

Assumption 6: There exist solutions $R \in \mathbf{R}^{q \times p_i}$ such that the following regulator equations have solutions $\Pi_i \in \mathbf{R}^{n_i \times q}$ and $U_i \in \mathbf{R}^{m_i \times q}$:

$$\begin{aligned} \Pi_i S &= A_i \Pi_i + B_i U_i + E_i \\ R &= C_i \Pi_i + F_i, \ \ i = 1, \ldots, N. \end{aligned} \tag{33}$$

Assumption 7: The leader has directed paths to all followers in the union graph $\bigcup_{t \in [\bar{t}_k, \bar{t}_{k+1})} \mathcal{G}_{\theta(t)}$.

Remark 5: Assumptions 3–6 are often used in the output consensus or regulation control of heterogeneous networks [34], [36], [47], [48]. According to Assumption 5, the transmission zeros of the system (31) do not coincide with the eigenvalues of the matrix S, which is often called the transmission zeros condition [48]. Assumption 6 gives a characterization of the control objective in terms of the solvability of a set of linear matrix equations. This characterization allows the linear output consensus problem to be studied using the familiar mathematic tool of linear algebra.

B. Event-Based Estimates of the Exogenous Signal

Since the exogenous signal (32) is available to only a subset of followers, we first design a distributed event-based observer for each follower as

$$\dot{w}_i = S w_i + c \sum_{j=0}^{N} a_{ij}(t) \big(\tilde{w}_i - \tilde{w}_j\big), \ \ \forall i \in \mathcal{V} \tag{34}$$

where $c > 0$, $w_i(t)$ represents the estimate of the exogenous signal $w_0(t)$, and $\tilde{w}_i(t) = e^{S(t - t_k^i)} w_i(t_k^i)$. Denote $z_i = w_i - w_0$ and $\tilde{z}_i = \tilde{w}_i - \tilde{w}_0$, $i = 1, \ldots, N$. Let $z = [z_1^T, \ldots, z_N^T]^T$ and $\tilde{z} = [\tilde{z}_1^T, \ldots, \tilde{z}_N^T]^T$. Let $z_0 = 0$ and $\tilde{z}_0 = 0$. Then, it follows that $z = 0$ if and only if $w_0 = w_1 = \cdots = w_N$. Thus, z_i satisfies the following dynamics:

$$\dot{z}_i = S z_i - c \sum_{j=0}^{N} a_{ij}(t) \big(\tilde{z}_i - \tilde{z}_j\big), \ \ \forall i \in \mathcal{V}. \tag{35}$$

Rewrite (35) as

$$\dot{z}(t) = (I_N \otimes S) z - \big(c \mathcal{H}_\theta \otimes I_q\big) \tilde{z}. \tag{36}$$

Let $\varphi = [\varphi_1^T, \ldots, \varphi_N^T]^T = (I_N \otimes e^{-St}) z$ and $\tilde{\varphi} = [\tilde{\varphi}_1^T, \ldots, \tilde{\varphi}_N^T]^T = (I_N \otimes e^{-St}) \tilde{z}$ with $\varphi(0) = z(0)$ and $\tilde{\varphi}(0) = \tilde{z}(0)$. It then follows from (36) that:

$$\begin{aligned} \dot{\varphi} &= -\Big(I_N \otimes S e^{-St}\Big) z + \Big(I_N \otimes e^{-St}\Big) \dot{z} \\ &= -\big(c \mathcal{H}_\theta \otimes I_q\big) \tilde{\varphi}. \end{aligned} \tag{37}$$

Lemma 3: If $\varphi(t)$ converges to 0 exponentially, so does $z(t)$.

Proof: Based on the convergency of φ, we can choose constants μ_1 and μ_2 such that

$$\|\varphi(t)\| \leq \mu_1 \|\varphi(0)\| e^{-\mu_2 t}.$$

According to Assumption 4, there exists a polynomial $\Omega(t)$ satisfying

$$\left\| \Big(I_N \otimes e^{St}\Big) \right\| \leq \Omega(t).$$

Since $\varphi = (I_N \otimes e^{-St}) z$, we get

$$\|z(t)\| \leq \left\| \Big(I_N \otimes e^{St}\Big) \right\| \cdot \|\varphi(t)\| \leq \mu_1 \|z(0)\| \Omega(t) e^{-\mu_2 t}.$$

This means if $\varphi(t)$ converges to 0 exponentially, so does $z(t)$. ■

Define the measurement error as

$$e_i \triangleq \tilde{w}_i - w_i, \ \ i = 1, \ldots, N. \tag{38}$$

Let $\epsilon = [\epsilon_1^T, \ldots, \epsilon_N^T]^T$ with $\epsilon_i \triangleq e^{-St} e_i(t)$, $i = 1, \ldots, N$. Event triggering instants are determined by (4) where

$$f_i(t) = d_i(t) \|\epsilon_i\|^2 - \frac{1}{4} \sum_{j=0}^{N} a_{ij}(t) \big\|\tilde{w}_i - \tilde{w}_j\big\|^2 - \mu e^{-\nu t} \tag{39}$$

with $\tilde{w}_0 \triangleq w_0$ and $d_i(t)$ being the degree of agent i associated with the subgraph $\mathcal{G}_{\theta(t)}$.

Theorem 3: The observers (34) with $c > 0$ can track the exogenous signal $w_0(t)$ under the triggering function (39). Moreover, there does not exist the Zeno behavior.

Proof: Construct the Lyapunov function candidate as

$$V_4 = \frac{1}{2} \varphi^T \varphi. \tag{40}$$

Evidently, V_4 is positive definite, whose derivative is given by

$$\begin{aligned} \dot{V}_4 &= -\varphi^T \big(c \mathcal{H}_\theta \otimes I_q\big) \tilde{\varphi} \\ &= -c \sum_{i=1}^{N} a_{i0}(t) \varphi_i^T \tilde{\varphi}_i - c \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(t) \varphi_i^T \big(\tilde{\varphi}_i - \tilde{\varphi}_j\big). \end{aligned} \tag{41}$$

It is easy to verify that

$$-\sum_{i=1}^{N} a_{i0}(t)\varphi_i^T\tilde{\varphi}_i = -\frac{1}{2}\sum_{i=1}^{N} a_{i0}(t)\varphi_i^T\varphi_i - \frac{1}{2}\sum_{i=1}^{N} a_{i0}(t)\tilde{\varphi}_i^T\tilde{\varphi}_i + \frac{1}{2}\sum_{i=1}^{N} a_{i0}(t)\epsilon_i^T\epsilon_i \tag{42}$$

and

$$\begin{aligned}
-\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\varphi_i^T\big(\tilde{\varphi}_i-\tilde{\varphi}_j\big)
&= -\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\big(\varphi_i-\varphi_j\big)^T\big(\varphi_i-\varphi_j\big)\\
&\quad -\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\big(\tilde{\varphi}_i-\tilde{\varphi}_j\big)^T\big(\tilde{\varphi}_i-\tilde{\varphi}_j\big)\\
&\quad +\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\big(\epsilon_i-\epsilon_j\big)^T\big(\epsilon_i-\epsilon_j\big).
\end{aligned} \tag{43}$$

Using the Young's inequality gives

$$\begin{aligned}
\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\big(\epsilon_i-\epsilon_j\big)^T\big(\epsilon_i-\epsilon_j\big)
&\le 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\epsilon_i^T\epsilon_i + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\epsilon_j^T\epsilon_j\\
&= 4\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\epsilon_i^T\epsilon_i.
\end{aligned} \tag{44}$$

Denote $\tilde{\varphi}_0 = 0$. Substituting (35) and (42)–(44) into (41) yields

$$\begin{aligned}
\dot{V}_4 &\le -\frac{c}{2}\varphi^T\big(\mathcal{H}_\theta\otimes I_q\big)\varphi - \frac{c}{2}\sum_{i=1}^{N} a_{i0}(t)\|\tilde{\varphi}_i\|^2\\
&\quad + \frac{c}{2}\sum_{i=1}^{N} a_{i0}(t)\|\epsilon_i\|^2 - \frac{c}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\big\|\tilde{\varphi}_i-\tilde{\varphi}_j\big\|^2\\
&\quad + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(t)\|\epsilon_i\|^2\\
&\le -\frac{c}{2}\varphi^T\big(\mathcal{H}_\theta\otimes I_q\big)\varphi\\
&\quad + c\sum_{i=1}^{N}\left\{d_i(t)\|\epsilon_i\|^2 - \frac{1}{4}\sum_{j=0}^{N} a_{ij}(t)\big\|\tilde{\varphi}_i-\tilde{\varphi}_j\big\|^2\right\}\\
&\le -\frac{c}{2}\varphi^T\big(\mathcal{H}_\theta\otimes I_q\big)\varphi + c\mu N e^{-\nu t}
\end{aligned} \tag{45}$$

where we have used the triggering function (39) to get the last inequality.

Similarly as in the proof of Theorem 1, we can prove that $\lim_{t\to\infty}\varphi(t) = 0$. According to Lemma 3, the observers (34) can track the exogenous signal $w_0(t)$.

Zeno behaviors can be similarly eliminated as in the proof of Theorem 2. ■

C. Distributed Control Inputs

Upon the basis of the designed observer (34), we present the following controller:

$$u_i = K_{1i}x_i + K_{2i}w_i,\ \ i = 1,\ldots,N \tag{46}$$

where w_i is defined in (34), and K_{1i} and K_{2i} are feedback matrices to be designed. Substituting (46) into (31) gives the following closed-loop dynamics:

$$\begin{aligned}
\dot{x}_i &= (A_i + B_iK_{1i})x_i + B_iK_{2i}w_i + E_iw_0\\
y_i &= C_ix_i + F_iw_0,\ \ i = 1,\ldots,N.
\end{aligned} \tag{47}$$

Theorem 4: Select K_{1i} such that $A_i+B_iK_{1i}$ are Hurwitz and $K_{2i} = U_i - K_{1i}\Pi_i$, $i = 1,\ldots,N$, where (X_i, Π_i, R_i) are unique solutions to regulator (33). Output consensus is achieved under the event-based observer (34), the triggering function (39), and the local controller (46).

Proof: Let $\phi_i = x_i - \Pi_iw_0$. Noting that (33) and $K_{2i} = U_i - K_{1i}\Pi_i$, we can rewrite (47) as

$$\begin{aligned}
\dot{\phi}_i &= (A_i + B_iK_{1i})\phi_i - (E_i - \Pi_iS)z_i\\
y_i &= C_ix_i + F_iw_0,\ \ i = 1,\ldots,N.
\end{aligned} \tag{48}$$

According to Theorem 3, we have $\lim_{t\to\infty} z_i(t) = 0$. Thus, if we choose K_{1i}, $i = 1,\ldots,N$, such that $A_i+B_iK_{1i}$ are Hurwitz, it is not difficult to obtain the result that $\lim_{t\to\infty}\phi_i(t) = 0$, which further leads to

$$\begin{aligned}
\lim_{t\to\infty}\big(y_i(t)-y_j(t)\big) &= \lim_{t\to\infty}\big[(C_ix_i+F_iw_0) - \big(C_jx_j+F_jw_0\big)\big]\\
&= \lim_{t\to\infty}\big[(C_i\Pi_i+F_i)w_0 - \big(C_j\Pi_j+F_j\big)w_0\big]\\
&\quad + \lim_{t\to\infty} C_i\phi_i - \lim_{t\to\infty} C_j\phi_j\\
&= \lim_{t\to\infty}(R-R)w_0\\
&= 0.
\end{aligned}$$

In conclusion, the output consensus of heterogeneous systems (31) is achieved. ■

Remark 6: Theorems 3 and 4 show that the proposed protocol (34), (39), and (46) is able to solve the event-driven output consensus control problem of heterogeneous networks. In particular, the state consensus of homogeneous agents considered in Section III can be treated as a special case here, if we let $A_i = A$, $B_i = B$, $C_i = I$, $E_i = 0$, and $F_i = 0$, $\forall i \in \mathcal{V}$.

Remark 7: Compared to [38], where output consensus of heterogeneous networks with continuous communications is considered, the event-based protocol given in this paper does not require continuous communications either between sensors and controllers or among neighboring agents. For each agent, both the control input and the triggering function are only based on state estimates of neighboring agents $\tilde{w}_j$ (or $\tilde{x}_j$) but not their real state w_j (or x_j). As for $e_i(t) = \tilde{w}_i(t) - w_i(t)$ (or $e_i(t) = \tilde{x}_i(t) - x_i(t)$), it can be computed according to its own information rather than neighbors' one. In other words, discrete information of neighbors at event instants rather than continuous one is required for control laws' updating and triggering functions' monitoring. Thus, the event-based protocols proposed in this paper are able to reduce communication frequency when implemented on practical systems.

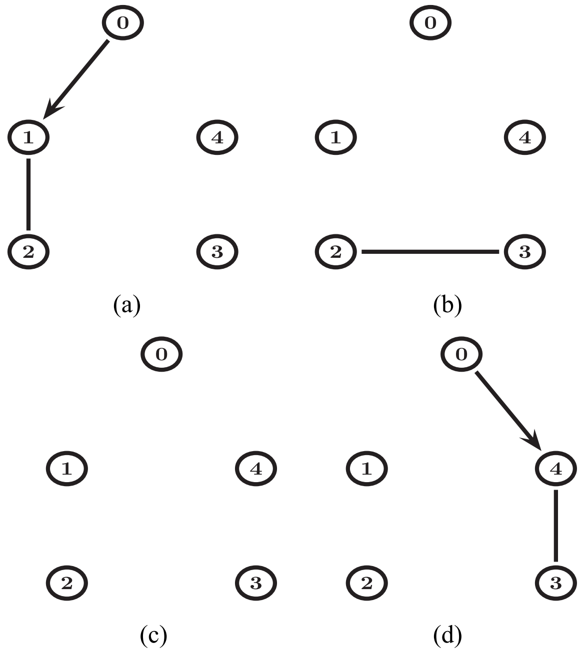

Fig. 1. Possible interaction topologies. (a) $\bar{\mathcal{G}}_1$. (b) $\bar{\mathcal{G}}_2$. (c) $\bar{\mathcal{G}}_3$. (d) $\bar{\mathcal{G}}_4$.

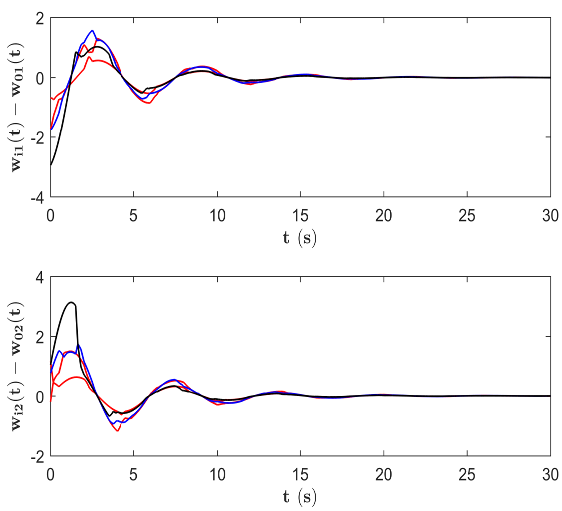

Fig. 2. Estimate errors $w_i - w_0$, $i = 1, \ldots, 4$.

IV. Simulation Examples

In this section, numerical simulations are introduced to demonstrate the effectiveness of the presented algorithms. The leader's dynamics satisfies (32) with $S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and the dynamics of followers are described by (31) with $A_i = \begin{bmatrix} -1 & 1 \\ 0 & -i \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ i \end{bmatrix}$, $C_i = \begin{bmatrix} i \\ 0 \end{bmatrix}$, $E_i = I_2$, $F_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $i = 1, \ldots, 4$. All agents' initial values are randomly chosen. Suppose that possible interaction topologies shown in Fig. 1 switches as $\bar{\mathcal{G}}_1 \to \bar{\mathcal{G}}_2 \to \bar{\mathcal{G}}_3 \to \bar{\mathcal{G}}_4 \to \bar{\mathcal{G}}_1 \to \cdots$, with the dwelling time $\tau = 0.5$ s. Note that node 0 represents the leader and nodes 1–4 denote followers. It is not difficult to find that Assumptions 3–7 are satisfied.

To achieve output consensus, we utilize the event-triggered protocol (34), (39), and (46). Solving the regulation (33) gives $\Pi_i = \begin{bmatrix} 1/i & 1/i \\ -1 & 2/i \end{bmatrix}$, $U_i = \begin{bmatrix} -1 - 2/i^2 & 0 \end{bmatrix}$, and $R = \begin{bmatrix} 2 & 1 \end{bmatrix}$, $i = 1, \ldots, 4$. Other parameters in this protocol are chosen as $c = 2$, $K_{1i} = \begin{bmatrix} -1 & -1 \end{bmatrix}$, and $K_{2i} = \begin{bmatrix} -2 - 1/i - 2/i^2 & 3/i \end{bmatrix}$, $i = 1, \ldots, 4$.

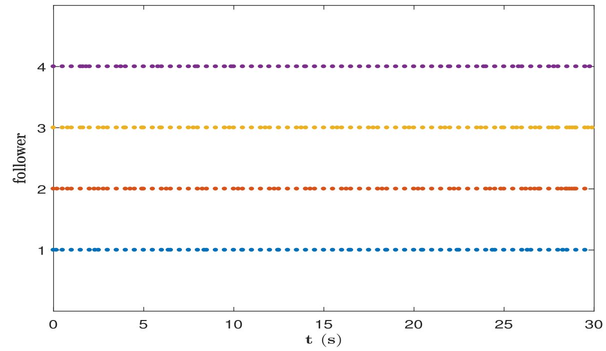

Fig. 3. Triggering instants of each follower.

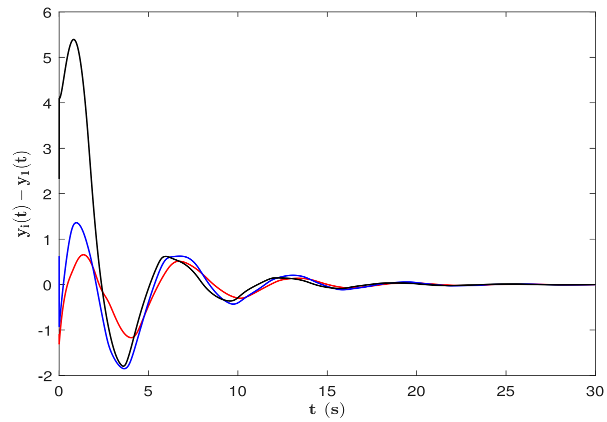

Fig. 4. Output errors $y_i - y_1$, $i = 2, 3, 4$.

The estimate errors $w_i - w_0$, $i = 1, \ldots, 4$, for t from 0 to 30 s, are depicted in Fig. 2, implying that the observers (34) can track the exogenous signal $w_0(t)$. Event instants of all followers are shown in Fig. 3, indicating that there exist no Zeno behaviors. The output errors $y_i - y_1$, $i = 2, 3, 4$, are depicted in Fig. 4, implying the achievement of output consensus.

V. Conclusion

In this paper, distributed event-driven consensus algorithms have been proposed for homogeneous and heterogeneous linear networks with jointly connected switching topologies. These protocols can be explicitly constructed and utilized in a completely distributed manner. It is shown that the proposed protocols are able to guarantee the achievement of consensus and a strictly positive lower bound for the interval between different triggering instants. Extending these results to general directed switching graphs or fixed-time consensus [49], [50] is an interesting work in the future.

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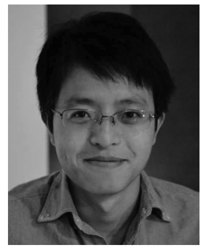

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