Nonlinear Consensus under Directed Graph via the Edge Laplacian

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Abstract: This paper considers a leaderless consensus problem for multi-agent system with inherently nonlinear dynamics under directed graph. To provide an effective perspective to handle the consensus problem, an innovative concept about the edge Laplacian is developed for directed graph. The algebraic properties of the graph Laplacian and edge Laplacian bare a lot in common, and the edge Laplacian has much potential to solve the leaderless consensus problem under the edge agreement framework. By using the proposed consensus protocol, the technical challenges caused by the inherently nonlinear dynamics can be well handled. Finally, the effectiveness of the theoretical results is demonstrated through simulation.

Key Words: Leaderless consensus, Edge Laplacian, Inherently Nonlinear Dynamics, Edge Agreement

1 INTRODUCTION

Over the last decade, the multi-agent distributed coordination problem has received much increasing attention from the systems and control community. Distributed dynamic systems are ubiquitous in diverse areas of science and engineering, such as formation flight [1], coordinated robotics [3], sensor fusion [9][7], and distributed computation [6]. One of the critical research problems is how to steer all agents to agree upon certain quantities of interest.

We are motivated to derive distributed leaderless consensus protocol that guarantee the multi-agent systems reach consensus on their states when the systems have only local interaction with their neighbours. Though admitting that the graph Laplacian [8][12] is convenient to describe the geometric interaction of networked agents, the edge Laplacian [18] has much potential to handle the leaderless consensus problem. Pioneering works on edge agreement protocol have provided totally new insights into how certain subgraphs, such as spanning trees and cycles effect the convergence properties. In addition, a novel systematic framework based on connected graph for analysing multi-agent systems in the edge perspective [17][19] has also been proposed. Learning multi-agent systems from the edge perspective is very promising, so that it is quite meaningful to extend edge Laplacian to directed graph.

Recently, consensus for multi-agent systems with nonlinear dynamics has received increasing attention, because most of physical systems are inherently nonlinear in nature. However, the leaderless consensus problem with inherently nonlinear dynamics has not been extensively addressed due to many technical challenges. To our best knowledge, most of the existing works focus on solving the nonlinear leaderless consensus under the undirected graphs[10][5][2] and scarcely any of them considered the directed topology. Only a small amount of them use the notions of generalized algebraic connectivity[16], nonsmooth analysis based on the work of Moreau [4] and limit-set-based approach [14] to solve the state agreement problem with nonlinear dynamics under directed graph.

The contributions of this paper lie on three aspects. Firstly, the edge Laplacian for directed graph is originally proposed, which extends the concepts of its undirected counterpart. The algebraic properties of the edge Laplacian are explored and as a matter of fact the spectra properties of the edge Laplacian play a key role in the subsequent analysis. Secondly, the technical challenges caused by inherently nonlinear dynamics can be well handled by using the proposed distributed consensus protocol. Thirdly, the edge agreement model open the way for building an universal problem solving framework to deal with the consensus issues.

The rest of the paper is organized as follows. In Section 2, a brief overview of the basic notions and results in graph theory are presented. The algebraic properties of the edge Laplacian for directed graph are proposed in Section 3. The main result about the consensus problem under inherently nonlinear dynamics is proposed in Section 4. The simulation results are given in Section 5. The last section draws the conclusions and proposes some future work.

2 Preliminaries and Notations

The matrix theoretic notion used in this paper is as follows: $|\cdot|$ and $\|\cdot\|_2$ denote the Euclidean norm and Frobenius norm for vectors and matrices, respectively. The notion D represents a diagonal matrix with d_i denoting the i-th entry on the diagonal. Let $G = (v, \varepsilon)$ be a directed graph specified by a node set $v = \{v_1, v_2, \cdots, v_N\}$ and an edge set $\varepsilon = \{e_1, e_2, \cdots, e_L\}$ whose elements characterize the incidence relation between distinct pairs of v, i.e., $\varepsilon \in v \times v$. For an edge e, we let $v_{\otimes}(e)$ denotes its initial node and $v_{\odot}(e)$ refers to its terminal node. $\sigma(v_i, v_k) = (v_i v_j \cdots v_k)$ indicates a path from v_i to v_k. $A(G) = [a_{ij}]$ with nonnegative adjacency elements a_{ij} is a weighted adjacency ma-

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trix. The adjacency elements associated with the edges of the graph are positive, i.e., $(v_j, v_i) \in \varepsilon \Leftrightarrow a_{ij} > 0$ (for simplicity, we choose $a_{ij} = 1$), otherwise, $a_{ij} = 0$. The in-degree and out-degree of node i are defined as $\deg_{in}(v_i)$ and $\deg_{out}(v_i)$, respectively. The diagonal in-degree matrix $D(G)$ of the graph contains the in-degree of each node on its diagonal. The set of neighbors of node i is denoted by $N_i = \{j : e_{ij} \in \varepsilon\}$.

The corresponding graph Laplacian of G is defined as

$$L_n(G) := D(G) - A(G), \tag{1}$$

whose eigenvalues are real and will be ordered as $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$.

The $N \times L$ incidence matrix $E(G)$ for a directed graph is a $\{0, \pm 1\}$-matrix with rows and columns indexed by nodes and edges of G, i.e.,

$$[E(G)]_{ik} = \begin{cases} +1 & \text{if } i \text{ is the initial node of edge } e_k, \\ -1 & \text{if } i \text{ is the terminal node of edge } e_k, \\ 0 & \text{otherwise}, \end{cases}$$

which implies that each column of E contains exactly two nonzero entries indicating the initial node and the terminal node.

A directed graph is called strongly connected if and only if any two distinct nodes of G can be connected via a directed path; weakly connected if G is connected when viewed as a disoriented directed graph; quasi-strongly connected if G has a center from which any other node is reachable. Actually, if a directed graph is quasi-strongly connected, it has a spanning tree ([15]). Specifically, for convenience, when we call a directed graph as quasi-strongly connected, it does not include these tree graphs.

3 THE EDGE LAPLACIAN FOR DIRECTED GRAPH

As known, the graph Laplacian contribute significantly in the analysis and synthesis of networked multi-agent systems and have been extensively explored in previous literatures [8][12]. Contrary to these works, the main graph-theoretic tool involved refers to an edge variant of the graph Laplacian, which was first formally named as the edge Laplacian and analyzed in [18]. The edge Laplacian is adopted to be natural interpretations of the information flow and has much potential to deal with the leaderless consensus problem. However, until now the notion of the edge Laplcian is only valid for undirected graph. Unquestionability, extending the concept to directed graph and exploring its relative algebraic properties are quite meaningful. For clarity, we give the definition of the in-incidence matrix and out-incidence matrix at first.

Definition 1 (In-incidence and Out-incidence Matrix)**.** *The $N \times L$ in-incidence matrix $E_\odot(G)$ for a directed graph G is a $\{0, -1\}$ matrix with rows and columns indexed by nodes and edges of G, respectively, such that*

$$[E_\odot(G)]_{ik} := \begin{cases} -1 & \textit{if } i \textit{ is the terminal node of edge } e_k \\ 0 & \textit{otherwise.} \end{cases}$$

and the out-incidence matrix is a $\{0, +1\}$ matrix, defined as

$$[E_\otimes(G)]_{ik} := \begin{cases} +1 & \textit{if } i \textit{ is the initial node of edge } e_k \\ 0 & \textit{otherwise.} \end{cases}$$

In comparison with the definition of the incidence matrix, $E_\odot$ can be easily obtained by taking off the $+1$ elements from E and $E_\otimes$ can be obtained by eliminating the -1 elements from E. Besides, we can write E in the following way,

$$E(G) = E_\odot(G) + E_\otimes(G). \tag{2}$$

On the other hand, each row of the in-incidence matrix can be viewed as decomposition of the in-degree from a node to each specific edge. Actually, this property, next, will lead us to find out the relationship between the in-incidence matrix and the graph Laplacian.

In the following discussions, we simply use E, $E_\odot$ and $E_\otimes$ instead of $E(G)$, $E_\odot(G)$ and $E_\otimes(G)$.

Lemma 1. *Consider a directed graph G with the incidence matrix E and in-incidence matrix $E_\odot$, the graph Laplacian of G have the following expression*

$$L_n = E_\odot E^T \tag{3}$$

Proof. Actually, by using (2), we have

$$E_\odot E^T = E_\odot E_\odot^T + E_\odot E_\otimes^T. \tag{4}$$

According to the preceding definition of $E_\odot$, it's clear that,

$$E_{\odot i} E_{\odot i}^T = d_i, \tag{5}$$

where $E_{\odot i}$ and $E_{\odot i}^T$ represent the ith row of $E_\odot$ and $E_\odot^T$, respectively. Additionally,

$$E_{\odot_i} E_{\odot_j}^T = 0, \text{ if } i \neq j. \tag{6}$$

In this way, we have

$$E_\odot E_\odot^T = D. \tag{7}$$

Besides, we also have

$$E_{\odot i} E_{\otimes j}^T = \begin{cases} -1 & \text{if } j \in N_i \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

we collect it as

$$\begin{bmatrix} 0 & E_{\odot 1} E_{\otimes 2}^T & \cdots & E_{\odot 1} E_{\otimes N}^T \\ E_{\odot 2} E_{\otimes 1}^T & 0 & \cdots & E_{\odot 2} E_{\otimes N}^T \\ \vdots & \vdots & \ddots & \vdots \\ E_{\odot N} E_{\otimes 1}^T & E_{\odot N} E_{\otimes 2}^T & \cdots & 0 \end{bmatrix} = -A(G).$$

According to the definition of the graph Laplacian, we have

$$E_\odot E^T = E_\odot E_\odot^T + E_\odot E_\otimes^T = D - A = L_n.$$

The proof is end. $\square$

Definition 2 (Edge Laplacian)**.** *The edge Laplacian of a directed graph is defined as*

$$L_e := E^T E_{\odot} \tag{9}$$

with $L \times L$ elements.

As we known, if a digraph has a spanning tree, then it's graph Laplacian L_n has a simple zero eigenvalue associated with an eigenvector $\mathbf{1}$, $rank(L_n) = N - 1$ and all the other eigenvalues have positive real parts [11]. To provide a deep insights that the edge Laplacian L_e representation offers in the analysis and synthesis of multi-agent systems, we give the following lemma.

Lemma 2. *The nonzero eigenvalues of edge Laplacian L_e and the graph Laplacian L_n are equal and $rank(L_e) = rank(L_n)$.*

Proof. Suppose $\lambda \neq 0$ is an eigenvalue of graph Laplacian L_n, which is associated with a nonzero eigenvector p. Clearly, we have,

$$L_n p = E_{\odot} E^T p = \lambda p. \tag{10}$$

Notice that

$$\lambda p = E_{\odot} E^T p \neq 0. \tag{11}$$

So it's clear that

$$E^T p \neq 0. \tag{12}$$

We denote $E^T p$ as $\bar{p}$. By left-multiplying E^T of both sides of (10), we get

$$\begin{aligned} & E^T E_{\odot} E^T p = E^T \lambda p, \\ \Rightarrow & L_e E^T p = \lambda E^T p, \qquad (13) \\ \Rightarrow & L_e \bar{p} = \lambda \bar{p}, \qquad (14) \end{aligned}$$

which means λ is also an eigenvalue of L_e and $\bar{p}$ is a nonzero eigenvector respect to λ. Namely, the nonzero eigenvalues of L_n and L_e are equal to each other. Besides, since L_n and L_e have the same nonzero eigenvalues, the rank of them are equal too. □

Lemma 3. *Consider a directed graph G containing a spanning tree with N vertexes, the edge Laplacian L_e contains $N - 1$ nonzero eigenvalues and all of them are in the open right-half plane, and the rank of L_e is $N - 1$. Furthermore, if G is quasi-strongly connected, L_e has at least one zero eigenvalue and zero is a simple root of the minimal polynomial of L_e.*

Proof. Denote the number of the nonzero eigenvalues of a arbitrary matrix A as $\mu(A)$. By Lemma 3.3 in [12], for the graph Laplacian L_n of a directed graph with a spanning tree, we have $\mu(L_n) = N - 1$ and all the nonzero eigenvalues are in the open right-half plane. According to Lemma 2, it's clear that, we have $\mu(L_e) = N - 1$ as well. As we know,

$$rank(L_e) \geq \mu(L_e) = N - 1. \tag{15}$$

From [15], for a directed graph having a spanning tree, we have

$$rank(E) = rank(E^T) = N - 1, \tag{16}$$

and $L_e = E^T E_{\odot}$, so that

$$rank(L_e) \leq rank(E^T) = N - 1. \tag{17}$$

Combine (15) and (17), then obtain

$$rank(L_e) = N - 1. \tag{18}$$

Additionally, for a quasi-strongly connected graph, L_e has $N - 1$ nonzero eigenvalues and $L - N + 1$ ($L \geq N$) zero eigenvalues. To prove zero is a simple root of the minimal polynomial of L_e, we just need to check if the algebraic multiplicity and the geometric multiplicity of $\lambda = 0$ of L_e are equal. Notice that $\mu(L_e) = N - 1$, so the algebraic multiplicity of $\lambda = 0$ of L_e is $m = L - N + 1$. Besides, assume p is the corresponding eigenvector of $\lambda = 0$, then

$$L_e p = 0. \tag{19}$$

By (18), the dimension of the null space of L_e is

$$dim\, N(L_e) = L - N + 1, \tag{20}$$

in other words, the geometric multiplicity of $\lambda = 0$ is $n = L - N + 1$. Clearly, the algebraic multiplicity and the geometric multiplicity of $\lambda = 0$ of L_e are equal, then we come to the conclusion that zero is a simple root of the minimal polynomial of L_e. □

4 Leaderless consensus with Inherently Nonlinear Dynamics

Consider a multi-agent system consisting of N agents under quasi-strongly connected graph G. The dynamics of each agent is represented by

$$\dot{x}_i = f(t, x_i) + u_i, \tag{21}$$

where $x_i \in \mathbb{R}^n$ is the state vector, $u_i \in \mathbb{R}^n$ is the control input, and $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ denotes a Lipschitz continuous function.

The following consensus protocol will be used for the i-th agent:

$$u_i = -k \sum_{j \in N_i} a_{ij}(x_i - x_j),\ i = 1, 2, \cdots, N. \tag{22}$$

Let x be the state collection of all nodes and assume $a_{ij} = 1, j \in N_i$ for simplicity, then we have

$$\dot{x} = -kL_n x + F, \tag{23}$$

where F is the column stack vector of $f(t, x_i)$.

The objective of leaderless consensus is achieved, if

$$\lim_{t \to 0}(x_i(t) - x_j(t)) = 0, \text{ for } i \neq j \tag{24}$$

Assumption 1. *For the nonlinear function f in (21), there exists a nonnegative constant ξ such that*

$$|f(t, x_1) - f(t, x_2)| \leq \xi |x_1 - x_2|, \forall x_1, x_2 \in \mathbb{R}^n; t \geq 0. \tag{25}$$

4.1 Edge Agreement Model

To solve the leaderless consensus problem in edge perspective, in the beginning of this section we firstly introduce the edge state $\tilde{x}$ and show that the asymptotic stability of $\tilde{x}$ implies consensus.

We consider the k-th edge e_k and suppose the edge state $\tilde{x}_{e_k}$ represents the difference of two states which is associated with e_k, then we have

$$\tilde{x}_{e_k} = x_{v_{\otimes}(e_k)} - x_{v_{\odot}(e_k)}. \tag{26}$$

In this avenue, we obtain

$$\tilde{x} = E^T x, \tag{27}$$

where $\tilde{x}$ is the collection of $\tilde{x}_{e_k}$.

Lemma 4. *If the directed graph G has a spanning tree, the asymptotic stability of $\tilde{x}$ implies consensus.*

Proof. If G has a directed spanning tree, as we known, for an arbitrary node pair (v_i, v_j), $i \neq j$ in G, there always exist two distinct paths $\sigma(v^*, v_i)$ and $\sigma(v^*, v_j)$, where v^* refers to the root. We denote the corresponding state of the root as x^* and it is not hard to see while the asymptotic stability is achieving, namely

$$\lim_{t\to\infty} \tilde{x}(t) = 0, \tag{28}$$

x_i and x_j will agree on the joint value x^* according to (26). Obviously, we have

$$\lim_{t\to\infty} (x_i(t) - x^* + x^* - x_j(t)) = 0. \tag{29}$$

The asymptotic stability of $\tilde{x}$ leads to consensus. □

Taking the derivative of (27) and by using (23), we have

$$\begin{aligned}\dot{\tilde{x}} &= -kE^T L_n x + E^T F \\ &= -kL_e\tilde{x} + E^T F\end{aligned} \tag{30}$$

where $L_e = E^T E_{\odot}$ is the edge Laplacian of G. Next, we will study the edge agreement model $\tilde{x}$ instead of the original system.

4.2 Main Result

In this sequel, the edge Laplacian and the edge agreement model mentioned above play an important role in the synthesis of the consensus problem. Consider the following Lyapunov function candidate,

$$V_{\tilde{x}} = \frac{1}{2}\tilde{x}^T P \tilde{x}. \tag{31}$$

where the matrix P is positive definite.

Before moving on, we need the following lemma.

Lemma 5. *If the underlying digraph is quasi-strongly connected and associated with the edge Laplacian L_e, then there exists a symmetric positive definite matrix P satisfying the equation*

$$PL_e + L_e^T P = Q \geq 0 \tag{32}$$

where Q is a positive semi-definite matrix.

Proof. Let J be the Jordan form of L_e, i.e., $L_e = T^{-1}JT$. If G has a spanning tree we can choose a suitable T that $J = \{\mathbf{0}, J_1\}$, where the rank of $\mathbf{0}$ is $L - N + 1$ and $-J_1$ is a $(N-1)\times(N-1)$ Hurwitz matrix. With the similar trick in the proof of Lemma 1 in [20], the result can be obtained. □

Theorem 1. *Suppose G is quasi-strongly connected and associated with the edge Laplacian L_e, for a positive semi-definite matrix Q, the matrix P can be obtained by Lemma 5. If*

$$\xi \|P\|_2 - \frac{1}{2}k\lambda_2(Q) < 0, \tag{33}$$

is satisfied, where $k > 0$ and $\lambda_2(Q)$ is denoted as the smallest nonzero eigenvalue, then the leaderless consensus is achieved by using the consensus protocol (22).

Proof. Consider the Lyapunov fuction candidate defined in (31) and differentiating $V_{\tilde{x}}$ along the trajectories (30), we have

$$\nabla V_{\tilde{x}} = \frac{1}{2}\left(\tilde{x}^T P\dot{\tilde{x}} + \dot{\tilde{x}}^T P\tilde{x}\right) \tag{34}$$

$$= \frac{1}{2}\Big(-k\tilde{x}^T\left(PL_e + L_e^T P\right)\tilde{x} + \tilde{x}^T P E^T F \tag{35}$$

$$+ F^T E P\tilde{x}\Big) \tag{36}$$

$$\leq -\frac{1}{2}k\lambda_2(Q)|\tilde{x}|^2 + \frac{1}{2}\xi\left|\tilde{x}^T\right|\|P\|_2|\tilde{x}| \tag{37}$$

$$+ \frac{1}{2}\xi|\tilde{x}|\|P\|_2\left|\tilde{x}^T\right| \tag{38}$$

$$\leq -\frac{1}{2}k\lambda_2(Q)|\tilde{x}|^2 + \xi\|P\|_2|\tilde{x}|^2 \tag{39}$$

$$\tag{40}$$

If $\xi\|P\|_2 - \frac{1}{2}k\lambda_2(Q) < 0$, we have

$$\nabla V_{\tilde{x}} < 0. \tag{41}$$

By Lemma 4, the asymptotic stability of $\tilde{x}$ implies consensus. The proof is concluded. □

Remark 1. *In comparison with the leader-follower consensus problem (or consensus tracking problem), the leaderless one is always considered as a much more challenge problem. In this paper, the new graph-theoretic tool, i.e., the edge Laplacian is employed to mainly deal with the leaderless consensus problem, however it can be also used to analysis and synthesis of the consensus tracking problem under edge agreement model.*

5 Simulation and Results

Assume the group has six agents and the directed graph has a spanning tree as shown in Figure 1. The dynamics of the i-th agent is assumed to be as

$$\dot{x}_i = f(t, x_i) + u_i, i = 1, 2, \cdots, 6. \tag{42}$$

where $x_i(t), u_i(t) \in \mathbb{R}^3$, with the inherent nonlinear dynamics $f_i : \mathbb{R}\times\mathbb{R}^3 \to \mathbb{R}^3$ described by Chua's circuit

$$\begin{aligned}f_i(t, x_i) = (&\zeta(-x_{i1} + x_{i2} - l(x_{i1})), \\ &x_{i1} - x_{i2} + x_{i3}, -\chi x_{i2})^T\end{aligned} \tag{43}$$

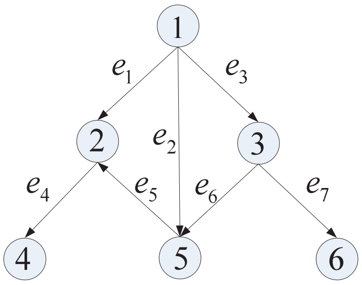

Figure 1: The quasi-strongly connected digraph

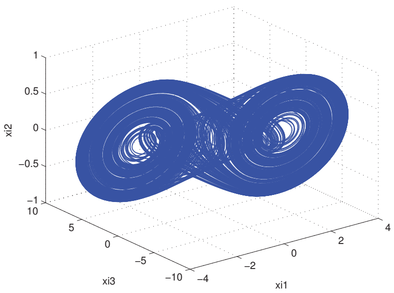

Figure 2: Chua's circuit: chaos with 2 attractors.

where $l(x_{i1}) = bx_{i1} + 0.5(a-b)(|x_{i1}+1| - |x_{i1}-1|)$. The system (43) is chaotic as shown in Figure 2, when $\zeta = 10$, $\chi = 18$, $a = -4/3$ and $b = -3/4$, with the Lipschitz constant $\xi = 4.3871$ [16].
The incident matrix E and the edge Laplacian L_e are

$$E = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$L_e = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

By choosing $Q1$ [20] as $\boldsymbol{I_5}$, we can obtain $\lambda_2(Q) = 0.1752$ and $\|P\|_2 = 14.7231$ by directed calculation. To satisfy (33), we can choose the feedback gain as $k = 7.38$. The initial positions are set as $x_1(0) = [150\ 100\ -80]$, $x_2(0) = [-500\ -100\ 150]$, $x_3(0) = [250\ -200\ 120]$, $x_4(0) = [-250\ 100\ -350]$, $x_5(0) = [200\ 300\ -200]$ and $x_6(0) = [-500\ 150\ 150]$. By using the consensus protocol (22), the state trajectories of six agents reaches consensus. The simulation results are shown in Figure 3, Figure 4 and Figure 5.

6 Conclusion and Future Work

In this paper, the leaderless consensus problem with inherently nonlinear dynamics under directed topology has been solved. The notion of the edge Laplacian for directed graph has been developed to describe the agreement in the edge setting, which shows an enormous potential on solving the consensus problem. A sufficient condition has been derived to ensure the leaderless consensus in multi-agent systems with inherently nonlinear dynamics under directed topology. We are now working on the theoretic analysis of the system's performance using both H_2 and H_∞ norm under edge agreement model.

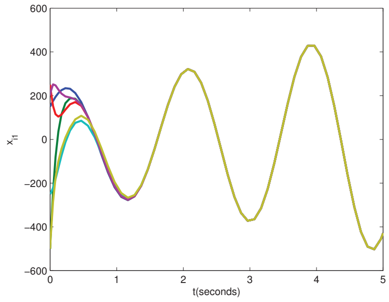

Figure 3: The evolutions of x_{i1}

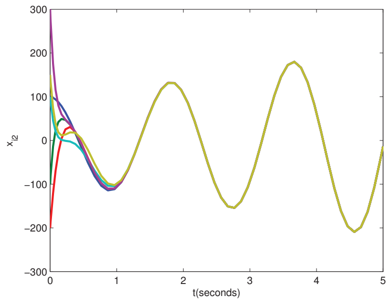

Figure 4: The evolutions of x_{i2}

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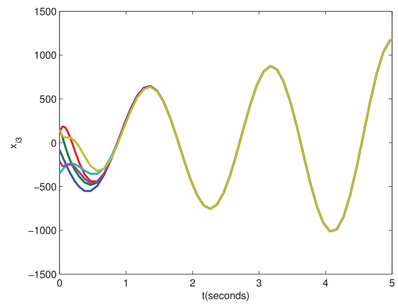

Figure 5: The evolutions of x_{i3}

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