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# A Novel Backstepping Method for the Three-dimensional Multi-UAVs Formation Control

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***Abstract* - In this paper, we propose a control method based on the backstepping technique and leader-follower strategy for the multiple Unmanned Aerial Vehicles (UAVs) formation control in three-dimensional space. Compared with the existing methods, the main contribution of the proposed algorithm is aiming to the dynamics of the UAV with the control inputs of the load factors rather than the angles of the rudders, which is commonly used in actual modern flight control. In addition, to solve the problem of the communication jam, we adopt the state estimation method based on the graph theory. Finally, the numerical simulations are performed to show the effectiveness of the proposed method.**

***Index Terms* - *UAV formation, backstepping method, load factor, state estimation.***

## I. INTRODUCTION

The multiple Unmanned Aerial Vehicles (UAVs) formation control has aroused many researchers' interests, because there are many advantages of the UAVs formation compared with a single UAV in military missions. For example, the UAVs formation can make use of the vortex to fly farther; the formation can also broaden the scale of reconnaissance, improve the hit rate and so on [1][2].

In general, the formation control problem includes two main issues. One is the formation construction and reconfiguration, namely, how to form a formation, how to split or rejoin it when accidental obstacles are in the way. The other is the formation preserving, including the shape keeping, switching, constricting and rotating in flight [2][3]. Our work in this paper is a part of the two issues. In recent decades, different methods have been proposed to solve these two issues in the formation control problems, such as the leader-follower strategy, the virtual structure strategy, the behavior-based strategy, the artificial field strategy and the graph-based strategy [4]. While, most of the existing strategies only concentrate on the plane model in two-dimensional space or base on the autopilot [5][6][7]. Few papers study the UAVs formation in three-dimensional space with nonlinear and coupling dynamics.

In this study, we consider the dynamics of the UAV in three-dimensional space with the control inputs of the load factors rather than the angles of the rudders, which is commonly used in the flight control of UAVs [10][11]. Based on the new dynamics, a distributed nonlinear formation controller is designed with aid of the backstepping technique and leader-follower strategy which proven stability. The proposed algorithms only required the leader's relative states (relative position and velocity), resulting in easily scaling up. But the leader's communication jam will happen with the number of follower increasing. So we adopt the method of state estimation to overcome the communication jam. In the state estimation method, every follower is not essential connected with the leader. Follower which is not directly connected with the leader just requires the information from other follower connected with it. All the followers acquire the leader's states almost at the same time. So, this method is better than the chain transfer which can result in the transmission errors and information delay. Combining the controller based on the backstepping method and the state estimation method, we obtain a new distributed controller.

The rest of the paper is organized as follows. In Section II, we reconstruct the formation dynamics model in three-dimensional space. The formation control design based on the backstepping is proposed in Section III. Further, in Section IV, we adopt the state estimation method to solve the communication jam, and the numerical simulations are performed to validate the controller in Section V.

## II. THE FORMATION MODEL

The definitions of the reference frame and the angles in this paper can be referred in [8].

The kinematics equations of the UAV [9] are:

$$\begin{cases} \dot{x}_i(\mathrm{t}) = V_i \cos\gamma_i \cos\chi_i \\ \dot{y}_i(\mathrm{t}) = V_i \cos\gamma_i \sin\chi_i \ , i = 1,2,...,n \ , \\ \dot{z}_i(\mathrm{t}) = -V_i \sin\gamma_i \end{cases} \tag{1}$$

where $V_i$ is the $i_{th}$ UAV's velocity in the wind coordinate frame, $\gamma_i, \chi_i$ are the $i_{th}$ UAV's flight path angle and the heading angle respectively, and $x_i, y_i, z_i$ are the $i_{th}$ UAV's positions in the Earth-surface inertial reference frame.

The load factor is the ratio between the sum force except gravity and the gravity in the coordinate reference frame $S_*$, as represented by $n_* = \left[n_x, n_y, n_z\right]^T$ [10][11].

We can obtain the $i_{th}$ UAV's dynamic equations in the wind coordinate frame are:

$$
\begin{cases}
\dot{V}_i = gn_x - g\sin\gamma_i \\
\dot{\gamma}_i = \dfrac{-g}{V_i}(n_z + \cos\gamma_i), i = 1,2,...,n \\
\dot{\chi}_i = \dfrac{g}{V_i\cos\gamma} n_y
\end{cases}, \tag{2}
$$

where $q = [\mathrm{V}_i, \gamma_i, \chi_i]^T$ is taken as the state variable. $u = [\mathrm{n}_x, \mathrm{n}_y, \mathrm{n}_z]^T$ is taken as the control variable.

Assume that the positions of the leader and the follower are depicted as Fig.1, where $V_f$ is the velocity vector of the follower, $V_l$ is the velocity vector of the leader, and $R$ is the relative location vector. The angle between $R$ and the $X$-$Y$ plane is $\xi$ ,and the angle between the projection of $R$ in the $X$-$Y$ plane and the $X$ axes is $\varphi$ .Their expected value are $\xi_d, \varphi_d$ which are constant. The positions of the leader and follower in the Earth-surface inertial reference frame are $(\mathrm{x}_l, \mathrm{y}_l, \mathrm{z}_l)$ and $(\mathrm{x}_f, \mathrm{y}_f, \mathrm{z}_f)$ .The follower's expected position $[x_{fd}, y_{fd}, z_{fd}]^T$ is given by:

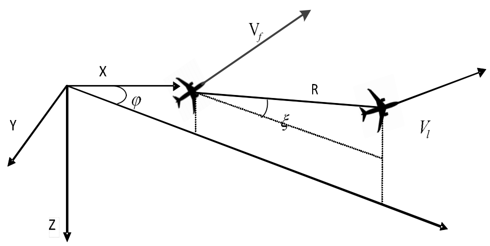



Fig. 1 the relative position between the leader and the follower

$$
\begin{bmatrix} x_{fd} \\ y_{fd} \\ z_{fd} \end{bmatrix} = \begin{bmatrix} x_l - R_d\cos\xi_d\cos\varphi_d \\ y_l - R_d\cos\xi_d\sin\varphi_d \\ z_l - R_d\sin\xi_d \end{bmatrix}. \tag{3}
$$

The tracking errors between the leader and follower can be defined as $x_e = [e_x, e_y, e_z]^T$ :

$$
x_e = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} x_f - x_{fd} \\ y_f - y_{fd} \\ z_f - z_{fd} \end{bmatrix}. \tag{4}
$$

The differential of (4) is:

$$
\dot{x}_e = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{bmatrix} = \begin{bmatrix} \dot{x}_f - \dot{x}_{fd} \\ \dot{y}_f - \dot{y}_{fd} \\ \dot{z}_f - \dot{z}_{fd} \end{bmatrix}. \tag{5}
$$

According to (1),we can get:

$$
\dot{x}_e = \begin{bmatrix} V_f\cos\gamma_f\cos\chi_f - \dot{x}_l \\ V_f\cos\gamma_f\sin\chi_f - \dot{y}_l \\ -V_f\sin\gamma_f - \dot{z}_l \end{bmatrix}. \tag{6}
$$

In the sequel, we will use the backstepping to design the formation controller to make the tracking errors asymptotically converge toward zero.

## III. The Formation Controller Based on Backstepping

Backstepping is a method of designing controller for the nonlinear system [12][13].

Combining (2) and (6), we can get the whole system equations:

$$
\begin{cases}
\dot{V}_f = gn_x - g\sin\gamma_f \\
\dot{\gamma}_f = -g(\mathrm{n}_z + \cos\gamma_f)/V_f \\
\dot{\chi}_f = gn_y/(V_f\cos\gamma_f) \\
\dot{e}_x = V_f\cos\gamma_f\cos\chi_f - \dot{x}_l \\
\dot{e}_y = V_f\cos\gamma_f\sin\chi_f - \dot{y}_l \\
\dot{e}_z = -V_f\sin\gamma_f - \dot{z}_l
\end{cases}. \tag{7}
$$

The objective is to design a controller with $n_x, \mathrm{n}_y, \mathrm{n}_z$ as the control variables to stabilize the $e_x, \mathrm{e}_y, e_z$ to 0.

Let $x_1 = x_e = [\mathrm{e}_x, \mathrm{e}_y, \mathrm{e}_z]^T$ , $x_2 = [\mathrm{V}_f, \gamma_f, \chi_f]^T$ , and $u = [\mathrm{n}_x, \mathrm{n}_y, \mathrm{n}_z]^T$ .

$$
\begin{cases}
\dot{x}_1 = f_1(\mathrm{x}_2) \\
\dot{x}_2 = f_2(\mathrm{x}_2) + \mathrm{g}_2(\mathrm{x}_2)\mathrm{u}
\end{cases}, \tag{8}
$$

$$
f_1(\mathrm{V}_f, \gamma_f, \chi_f) = \begin{bmatrix} V_f\cos\gamma_f\cos\chi_f - \dot{x}_l \\ V_f\cos\gamma_f\sin\chi_f - \dot{y}_l \\ -V_f\sin\gamma_f - \dot{z}_l \end{bmatrix},
$$

$$
f_2(\mathrm{V}_f, \gamma_f, \chi_f) = \begin{bmatrix} -g\sin\gamma_f \\ -g\cos\gamma_f/V_f \\ 0 \end{bmatrix},
$$

$$
g_2(\mathrm{V}_f, \gamma_f, \chi_f) = \begin{bmatrix} g & 0 & 0 \\ 0 & 0 & -g/V_f \\ 0 & g/(\mathrm{V}_f\cos\gamma_f) & 0 \end{bmatrix}.
$$

**Lemma 1:**The controller:

$$
u = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} (-k_v(V_f - V_f^*) + g\sin\gamma_f + \dot{V}_f^*)/g \\ ((-\mathrm{k}_\chi(\chi_f - \chi_f^*) + \dot{\chi}_f^*)\mathrm{V}_f\cos\gamma_f)/g \\ -(-k_\gamma(\gamma_f - \gamma_f^*) + \dot{\gamma}_f^*)V_f/g - \cos\gamma_f \end{bmatrix}
$$

can stabilize $e_x, \mathrm{e}_y, e_z$ to 0,where $k_v > 0, k_\chi > 0, k_\gamma > 0$ , and $V_f^*, \chi_f^*, \gamma_f^*$ are:

$$
\begin{bmatrix} V_f^* \\ \gamma_f^* \\ \chi_f^* \end{bmatrix} = \begin{bmatrix} \sqrt{(-\mathrm{k}_{11}\mathrm{e}_x + \dot{\mathrm{x}}_l)^2 + (-\mathrm{k}_{22}\mathrm{e}_y + \dot{\mathrm{y}}_l)^2 + (-\mathrm{k}_{33}\mathrm{e}_z + \dot{\mathrm{z}}_l)^2} \\ -\mathrm{asin}\dfrac{-k_{33}e_z + \dot{z}_l}{\sqrt{(-\mathrm{k}_{11}\mathrm{e}_x + \dot{\mathrm{x}}_l)^2 + (-\mathrm{k}_{22}\mathrm{e}_y + \dot{\mathrm{y}}_l)^2 + (-\mathrm{k}_{33}\mathrm{e}_z + \dot{\mathrm{z}}_l)^2}} \\ \mathrm{atan}\dfrac{-k_{22}e_y + \dot{y}_l}{-k_{11}e_x + \dot{x}_l} \end{bmatrix}.
$$

**Proof:**

**Step 1:**Let $\alpha(\mathrm{x}_1)=x_2^*=[V_f^*,\gamma_f^*,\chi_f^*]^T$ be the virtual control variable. We select the Lyapunov function $V_1=\frac{1}{2}\|x_1\|_2^2$, where $\|\bullet\|_2$ denotes the 2-norm of a vector. Then:

$$\begin{aligned}\dot{V}_1&=x_1^T\dot{x}_1\\&=e_x\dot{e}_x+e_y\dot{e}_y+e_z\dot{e}_z\\&=e_x(V_f\cos\gamma_f\cos\chi_f-\dot{x}_l)+\mathrm{e}_y(V_f\cos\gamma_f\sin\chi_f-\dot{y}_l)\\&+\mathrm{e}_z(V_f\sin\gamma_f-\dot{z}_l)\end{aligned}.$$

By observing the equation above, we can get that: if

$$\begin{bmatrix}V_f\cos\gamma_f\cos\chi_f-\dot{x}_l\\V_f\cos\gamma_f\sin\chi_f-\dot{y}_l\\-V_f\sin\gamma_f-\dot{z}_l\end{bmatrix}=\begin{bmatrix}-k_{11}&0&0\\0&-k_{22}&0\\0&0&-k_{33}\end{bmatrix}\begin{bmatrix}e_x\\e_y\\e_z\end{bmatrix},\qquad(9)$$

then $\dot{V}_1=-k_{11}e_x^2-k_{22}e_y^2-k_{33}e_z^2<0$ .So $e_x,e_y,e_z$ can be stabilized to 0 .

Through (9),we can get the virtual control variable $x_2^*$:

$$\begin{cases}V_f^*=\sqrt{(-\mathrm{k}_{11}\mathrm{e}_x+\dot{\mathrm{x}}_l)^2+(-\mathrm{k}_{22}\mathrm{e}_y+\dot{\mathrm{y}}_l)^2+(-\mathrm{k}_{33}\mathrm{e}_z+\dot{\mathrm{z}}_l)^2}\\\gamma_f^*=-\mathrm{asin}\dfrac{-k_{33}e_z+\dot{z}_l}{\sqrt{(-\mathrm{k}_{11}\mathrm{e}_x+\dot{\mathrm{x}}_l)^2+(-\mathrm{k}_{22}\mathrm{e}_y+\dot{\mathrm{y}}_l)^2+(-\mathrm{k}_{33}\mathrm{e}_z+\dot{\mathrm{z}}_l)^2}}\\\chi_f^*=\mathrm{atan}\dfrac{-k_{22}e_y+\dot{y}_l}{-k_{11}e_x+\dot{x}_l}\end{cases}.\ (10)$$

**Step 2:**The error between the real variable and the virtual variable is $z=x_2-\alpha(\mathrm{x}_1)$ ,so next we should design a controller to stabilize z to 0.

$$z=\begin{bmatrix}e_v\\e_\gamma\\e_\chi\end{bmatrix}=\begin{bmatrix}V_f-V_f^*\\\gamma_f-\gamma_f^*\\\chi_f-\chi_f^*\end{bmatrix}.\qquad(11)$$

We select the Lyapunov function $V_2=V_1+\frac{1}{2}\|z\|_2^2=\frac{1}{2}(\|x_1\|_2^2+\|z\|_2^2)$ .

Equation (8) can be written as follows:

$$\begin{cases}\dot{x}_1=f_1(\mathrm{z}+\alpha(\mathrm{x}_1))\\\dot{z}=\dot{x}_2-\dot{\alpha}(\mathrm{x}_1)=f_2(\mathrm{z}+\alpha(\mathrm{x}_1))+\mathrm{g}_2(\mathrm{z}+\alpha(\mathrm{x}_1))\mathrm{u}-\dot{\alpha}(\mathrm{x}_1)\end{cases},\qquad(12)$$

Then:

$$\begin{aligned}\dot{V}_2&=x_1^Tf_1(\mathrm{z}+\alpha(\mathrm{x}_1))+\mathrm{z}^T(f_2(\mathrm{z}+\alpha(\mathrm{x}_1))+\mathrm{g}_2(\mathrm{z}+\alpha(\mathrm{x}_1))\mathrm{u}-\dot{\alpha}(\mathrm{x}_1))\\&=((\mathrm{e}_v+\mathrm{V}_f^*)\cos(\mathrm{e}_\gamma+\gamma_f^*)\cos(\mathrm{e}_\chi+\chi_f^*)-\dot{\mathrm{x}}_l)e_x\\&+((\mathrm{e}_v+\mathrm{V}_f^*)\cos(\mathrm{e}_\gamma+\gamma_f^*)\sin(\mathrm{e}_\chi+\chi_f^*)-\dot{\mathrm{y}}_l)\mathrm{e}_y\\&+((\mathrm{e}_v+\mathrm{V}_f^*)\sin(\mathrm{e}_\gamma+\gamma_f^*)-\dot{\mathrm{z}}_l)e_z+(\mathrm{gn}_x-\mathrm{gsin}(\mathrm{e}_\gamma+\gamma_f^*)-\dot{\mathrm{V}}_f^*)\mathrm{e}_v\\&+(\frac{-g}{e_v+V_f^*}\mathrm{n}_z-\frac{g}{e_v+V_f^*}\cos(\mathrm{e}_\gamma+\gamma_f^*)-\dot{\gamma}_f^*)\mathrm{e}_\gamma\\&+(\frac{gn_y}{(\mathrm{e}_v+\mathrm{V}_f^*)\cos(\mathrm{e}_\gamma+\gamma^*)}-\dot{\chi}_f^*)e_\chi\end{aligned}.$$

We have known that :

$$\begin{aligned}&V_f^*\cos\gamma_f^*\cos\chi_f^*-\dot{x}_l=-k_{11}e_x\\&V_f^*\cos\gamma_f^*\sin\chi_f^*-\dot{y}_l=-k_{22}e_y\ ,\\&V_f^*\sin\gamma_f^*-\dot{z}_l=-k_{33}e_z\end{aligned}$$

So the objective is to design the controller:

$u=\left[n_x,n_y,n_z\right]^T=N(\mathrm{e}_v,\mathrm{e}_\gamma,\mathrm{e}_\chi,\mathrm{V}_f^*,\gamma_f^*,\chi_f^*,\dot{\mathrm{V}}_f^*,\dot{\gamma}_f^*,\dot{\chi}_f^*,\mathrm{V}_f,\gamma_f,\chi_f)$ to make $\dot{V}_2\le-W$,where $W$ is positive definite.

From $\dot{V}_2$ , we can get a controller:

$$u=\begin{bmatrix}n_x\\n_y\\n_z\end{bmatrix}=\begin{bmatrix}(-k_ve_v+g\sin\gamma_f+\dot{V}_f^*)/g\\((-\mathrm{k}_\chi\mathrm{e}_\chi+\dot{\chi}_f^*)\mathrm{V}_f\cos\gamma_f)/g\\-\dfrac{(-k_\gamma e_\gamma+\dot{\gamma}_f^*)V_f}{g}-\cos\gamma_f\end{bmatrix},\qquad(13)$$

$k_v>0,k_\chi>0,k_\gamma>0$ , $e_v,e_\chi,e_\gamma$ can be obtained from (11), and $\dot{V}_f^*,\dot{\chi}_f^*,\dot{\gamma}_f^*$ can be obtained from (10).

To prove that $\dot{V}_2$ is negative definite, we define another Lyapunov function $V_3=\frac{1}{2}\|z\|_2^2$,

$$\begin{aligned}\dot{V}_3&=z^T\dot{z}=z^T(f_2(\mathrm{z}+\alpha(\mathrm{x}_1))+\mathrm{g}_2(\mathrm{z}+\alpha(\mathrm{x}_1))\mathrm{u}-\dot{\alpha}(\mathrm{x}_1))\\&=(\mathrm{gn}_x-\mathrm{gsin}(\mathrm{e}_\gamma+\gamma_f^*)-\dot{\mathrm{V}}_f^*)\mathrm{e}_v\\&+(\frac{-g}{e_v+V_f^*}\mathrm{n}_z-\frac{g}{e_v+V_f^*}\cos(\mathrm{e}_\gamma+\gamma_f^*)-\dot{\gamma}_f^*)\mathrm{e}_\gamma\\&+(\frac{gn_y}{(\mathrm{e}_v+\mathrm{V}_f^*)\cos(\mathrm{e}_\gamma+\gamma^*)}-\dot{\chi}_f^*)e_\chi\end{aligned}.$$

According to controller (13), we can get $\dot{V}_3=-k_ve_v^2-k_\chi e_\chi^2-k_\gamma e_\gamma^2\le0$ ,so z is stable at 0. As the system runs with the work of the controller $u$ ,after time $t$ ,the follower's states outputs $V_f,\gamma_f,\chi_f$ can follow the expected inputs $V_f^*,\gamma_f^*,\chi_f^*$,that is $\left[e_v,e_\gamma,e_\chi\right]^T\to0$.

Then $\dot{V}_2$ can be rewritten as follows:

$$
\begin{aligned}
\dot{V}_2 &= (V_f^* \cos\gamma_f^* \cos\chi_f^* - \dot{x}_l)e_x + (V_f^* \cos\gamma_f^* \sin\chi_f^* - \dot{y}_l)e_y \\
&\quad + (V_f^* \sin\gamma_f^* - \dot{z}_l)e_z - k_v e_v^2 - k_\chi e_\chi^2 - k_\gamma e_\gamma^2 \\
&= -k_{11}e_x^2 - k_{22}e_y^2 - k_{33}e_z^2 - k_v e_v^2 - k_\chi e_\chi^2 - k_\gamma e_\gamma^2 \le 0
\end{aligned}
.
$$

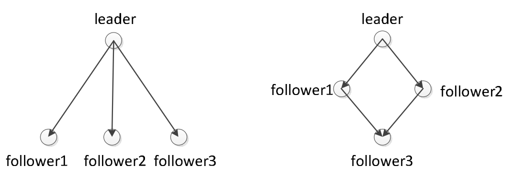
So $x_1$ and $z$ are asymptotically stable at 0.

**Remark**： By configuring the desired positions among the followers and the leader, we can realize the arbitrary formation. However, it is worth pointing out that the proposed controller requires the position and its differential of the leader. Assuming the followers accept the information from the leader by the communication links, as the number of the UAVs increases, communication jam will happen, so we use the state estimation to solve this problem.

## IV. STATE ESTIMATION

Assume that the leader only allows two other UAVs to connect with it. Obviously communication topology shown in Fig.2(a) is beyond the leader's communication ability. So we should make full use of the follower's communication resources. State estimation method can solve that problem. The communication topology shown in Fig.2(b) is designed based on the state estimation method which obviously can overcome the communication jam.

In this section, we expand the method of state estimation proposed in [15] to three-dimensional space to overcome the communication jam.




(a) (b)

Fig.2 The two different communication topologies

According to the graph theory [16],the whole formation can be regarded as a graph, and each UAV is a node of the graph. The communication links can be regarded as the edges. The node set is $V=\{v_1,v_2,\cdots,v_n\}$ ,and $|V|$ is the number of the nodes. The edge set is $\varepsilon$ , and the edge $(i,j)\in\varepsilon$ ,denotes that UAV $j$ can obtain information from UAV $i$. we can use the adjacency matrix $A$ to describe the graph, $A=[a_{ij}]\in R^{n\times n}$ ,and $n$ is the number of the UAVs.

$$
a_{ij}=\begin{cases} 1,(\mathrm{i,j})\in \mathrm{E}, \mathrm{i}\neq \mathrm{j} \\ 0,(\mathrm{i,j})\notin \mathrm{E}\, or\, \mathrm{i}=\mathrm{j} \end{cases}.
$$

The set of the communication links connected with the UAV $v_i$ is denoted as: $N_i=\{v_j \mid e_{ij}\in E, j\neq i\}$ .The directed path is formed by a series of edges $(i,j)(j,k)(k,p)$... .If every node has at least one directed path to all the other nodes, the graph is strongly connected. $\zeta_l=[x_l,y_l,z_l]$ is the real state of the leader, and $\zeta_i^l=[x_i^l,y_i^l,z_i^l]$ is the state estimation of the leader. Assume that there are $n$ followers and one leader. They form a strongly connected graph. $N_l$ is the set connected with the leader.

**Definition:**

$$
\omega_i=\begin{cases} 1, i\in N_l \\ 0, i\notin N_l \end{cases} \quad i=1,2,\cdots,\mathrm{n},
$$

$$
\boldsymbol{\omega}=[\omega_1,\omega_2,\cdots,\omega_n]^T,
$$

$$
G=[g_{ij}]=\begin{bmatrix} A & \boldsymbol{\omega} \\ \mathbf{0}^{\mathrm{T}} & 0 \end{bmatrix}\in R^{n+1}\times R^{n+1},
$$

Then the estimation algorithm is :

$$
\begin{cases}
\dot{\zeta}_i^l=\dfrac{\sum_{j=1}^{n+1} g_{ij}[\dot{\zeta}_j^l-\gamma(\zeta_i^l-\zeta_j^l)]}{\sum_{j=1}^{n+1} g_{ij}}, i=1,2,\cdots,n \\
\zeta_{n+1}^l=\zeta_l
\end{cases}, \qquad (14)
$$

**Theorem** : Combining the state estimation and controller (13),we get the whole controller for the three-dimensional formation. And the new controller:

$$
u=\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}=\begin{bmatrix} (-k_v(V_f-V_f^*)+g\sin\gamma_f+\dot{V}_f^*)/g \\ ((-\mathrm{k}_\chi(\chi_f-\chi_f^*)+\dot{\chi}_f^*)\mathrm{V}_f\cos\gamma_f)/g \\ \dfrac{(-k_\gamma(\gamma_f-\gamma_f^*)+\dot{\gamma}_f^*)V_f}{g}-\cos\gamma_f \end{bmatrix},
$$

$$
\begin{bmatrix} V_f^* \\ \gamma_f^* \\ \chi_f^* \end{bmatrix}=\begin{bmatrix} \sqrt{(-\mathrm{k}_{11}\mathrm{e}_x+\dot{\mathrm{x}}_i^l)^2+(-\mathrm{k}_{22}\mathrm{e}_y+\dot{\mathrm{y}}_i^l)^2+(-\mathrm{k}_{33}\mathrm{e}_z+\dot{\mathrm{z}}_i^l)^2} \\ -\mathrm{asin}\dfrac{-k_{33}e_z+\dot{z}_i^l}{\sqrt{(-\mathrm{k}_{11}\mathrm{e}_x+\dot{\mathrm{x}}_i^l)^2+(-\mathrm{k}_{22}\mathrm{e}_y+\dot{\mathrm{y}}_i^l)^2+(-\mathrm{k}_{33}\mathrm{e}_z+\dot{\mathrm{z}}_i^l)^2}} \\ \mathrm{atan}\dfrac{-k_{22}e_y+\dot{\mathrm{y}}_i^l}{-k_{11}e_x+\dot{\mathrm{x}}_i^l} \end{bmatrix},
$$

$$
\begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}=\begin{bmatrix} x_f-x_i^l+R_d\cos\xi_d\cos\varphi_d \\ y_f-y_i^l+R_d\cos\xi_d\sin\varphi_d \\ z_f-z_i^l+R_d\sin\xi_d \end{bmatrix}
$$

can still stabilize system (19),where $[x_i^l,y_i^l,z_i^l]^T$ and $[\dot{x}_i^l,\dot{y}_i^l,\dot{z}_i^l]$ denote the state estimations of the leader's positions and its differential , which can be obtained from (7).

**Proof:** Firstly, we should prove that the estimation algorithm (14) can make the state estimation of the leader asymptotically tend to the real state of the leader, which in math is

$$
\zeta_i^l \to \zeta_{n+1}^l=\zeta_l
$$

By multiplying $\sum_{j=1}^{n+1} g_{ij}$ at both sides of (14), we can get:

$$
\sum_{j=1}^{n+1} g_{ij}\dot{\zeta}_i^l=\sum_{j=1}^{n+1} g_{ij}[\dot{\zeta}_j^l-\gamma(\zeta_i^l-\zeta_j^l)].
$$

Rewriting the above equation, we can get $\sum_{j=1}^{n+1} g_{ij}(\dot{\zeta}_i^l-\dot{\zeta}_j^l)=-\gamma\sum_{j=1}^{n+1} g_{ij}(\zeta_i^l-\zeta_j^l)$ ,so over time, we can get:

$$\sum_{j=1}^{n+1} g_{ij}(\zeta_i^l - \zeta_j^l) \rightarrow 0 .$$

If $g_{ij} \neq 0$ ,then $\zeta_i^l \rightarrow \zeta_j^l$ . Because the communication topology is a strongly connected graph, there must be a series $[i, j, \cdots \text{k}, \text{n}+1]$ ,so that $g_{ij} = \cdots = g_{k(\text{n}+1)} = 1$ ,and therefore:

$$\zeta_i^l \rightarrow \zeta_{n+1}^l = \zeta_l , i = 1,2,3,\cdots,n .$$

Once the followers get the states of the leader, we can use the controller designed above based on the backstepping to follow the leader, and form the formation finally.

To combine the state estimation and controller (13), we just substitute the real state of the leader in $[V_f^*, \gamma_f^*, \chi_f^*]^T$ with the state estimation (14). It is already known that the state estimation of the leader can asymptotically tend to the real state of the leader, which means that as time goes, the estimation of the state can substitute the real state of the leader. Obviously, the system can keep stable.

## V. Simulation Results

### A. Simulation of Formation with Two UAVs

The trajectory of the leader is guided in advance and given in orders: $V_{lc}, \gamma_{lc}, \chi_{lc}$ .Assume that the leader plays the spire motion [14]:

$$V_{lc} = 30m/\sec$$

$$\gamma_{lc} = 10^{\circ} = \frac{10}{180}\pi(rad) ,$$

$$\chi_{lc} = \sin\frac{2\pi t V_{lc}}{L_{\sin}}(\text{rad})$$

$L_{\sin} = 200$ .Assume that the aircraft can be modeled by first-order dynamics. That is:

$$\begin{cases} \dot{V}_l = -k_{lv}(\text{V}_l - \text{V}_{lc}) \\ \dot{\gamma}_l = -k_{l\gamma}(\gamma_l - \gamma_{lc}) \\ \dot{\chi}_l = -k_{l\chi}(\chi_l - \chi_{lc}) \end{cases} ,$$

where $k_{lv}, k_{l\gamma}, k_{l\chi}$ are positive constants.

We use the Simulink block of Matlab R2011a to validate our controller. Fig.3 and Fig.4 show the simulation results.

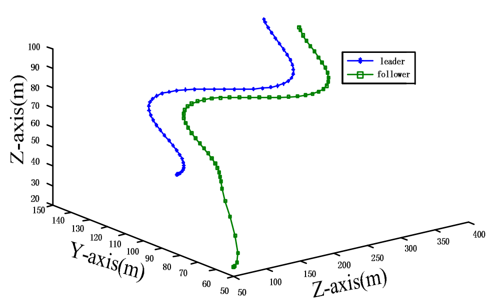



Fig. 3 Three-dimensional view paths of the leader and follower

Fig.3 indicates that the follower can asymptotically follow the leader when the leader plays the spire motion. Fig.4 indicates that the tracking errors in *X*-axis*, Y*-axis*, Z*-axis tend to 0 asymptotically.

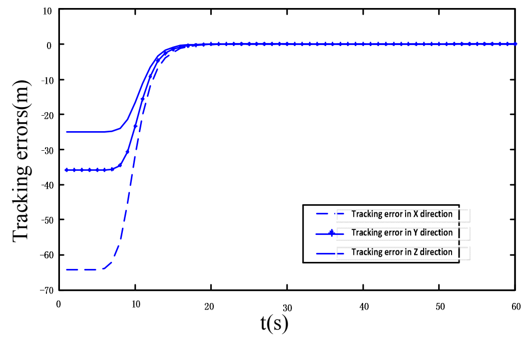



Fig. 4 The tracking errors in X-axis, Y-axis and Z-axis

The two figures indicate that our controller designed based on the backstepping method is viable.

### B. Simulation on Multi-UAVs Formation

In this simulation, we use four UAVs, including three followers and one leader. The communication topology is shown in Fig.5.

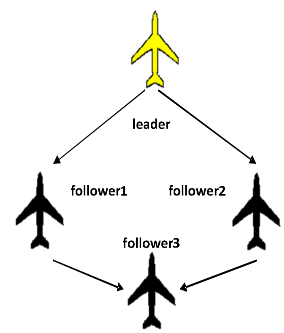


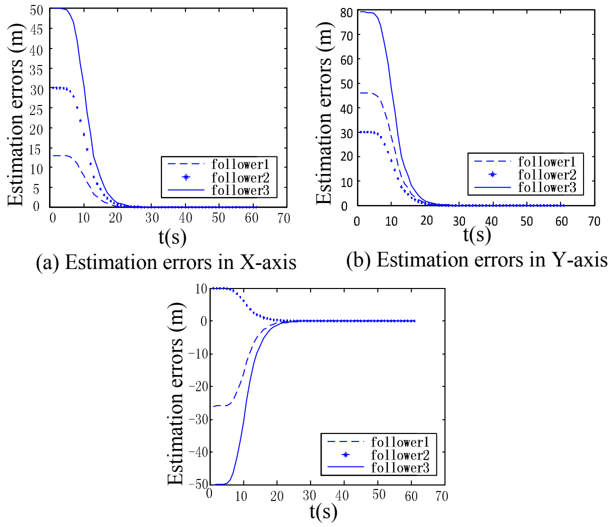

Fig.5 The formation communication topology

Followers 1 and 2 can receive information directly from the leader, but they still obtain the state information of the leader by using algorithm (14). Follower 3 estimates the states of the leader by receiving information from the followers 1 and 2. Obviously, the communication topology as shown in Fig.5 is strongly connected.

In the Multi-UAVs formation, followers get the state information by algorithm (14). So we firstly perform the simulation of state estimation to validate algorithm (14).

The estimation errors in *X*-axis, *Y*-axis and *Z*-axis are as shown in Fig.6(a)-6(c). From Fig.6(a)-6(c),we can know that though the three followers' initial state estimations are different, after some time, they asymptotically tend to the real leader's states.

After the followers obtain the leader's state, the formation controller will work. Next we perform the simulation to validate the theorem proposed in Section III. The simulation results are given in Fig.7.




(a) Estimation errors in X-axis (b) Estimation errors in Y-axis

(c) Estimation errors in Z-axis

Fig.6 The estimation errors in X-axis, Y-axis and Z-axis

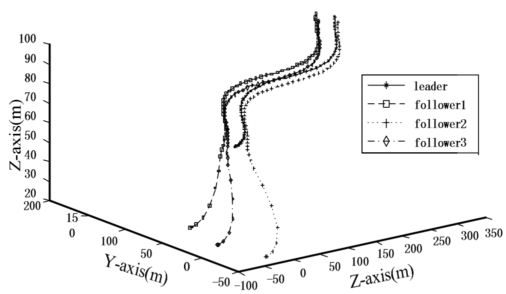



Fig.7 Three-dimensional view paths of the four UAVs

Fig.7 shows the paths in three-dimensional space of the leader and the three followers. We can see that the three followers track the leader tightly, and they form the desired formation while they are maneuvering.

Because the desired formation's altitudes are constant, so we can see the formation shape in *X*-*Y* plane clearly as shown in Fig.8. The desired formation shape is a lozenge. In Fig.8, we label the formation shape in the final simulation positions, and they form a lozenge exactly.

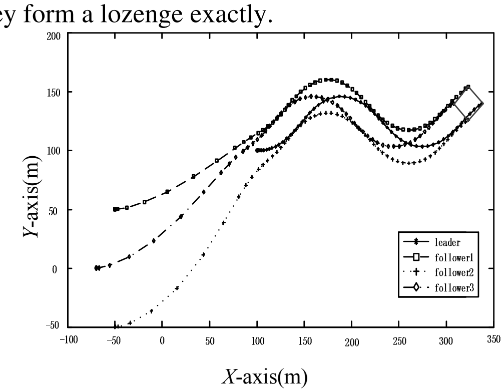



Fig.8 The paths in the *X*-*Y* plane

## VI. CONCLUSIONS

In this paper, we consider the dynamic equations of the UAV in three-dimensional space, and construct the error dynamics with load factor as the control variables .In general, the load factor decides the magnitude and the direction of the velocity. According to the constructed error dynamic equations, we use the backstepping method to design a novel controller. In order to overcome the communication jam, we adopt the state estimation method. Combining the backstepping method with the state estimation ,we design a new controller for the multi-UAVs formation in three-dimensional space. There are some advantages of our controller. Our controller is based on the nonlinear dynamic model, so it is more useful in real flight environment compared with the controller based on the PID method or autopilot. Because our controller is distributed, we can adjust the scale of the formation easily just by adding or deleting a communication link with another follower. We don't change the leader's communication links, so we needn't to consider the communication ability of the leader. Finally, we perform the simulations to validate our controller.

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