Cooperative Output Regulation of Heterogeneous Multi-Agent Systems With Adaptive Edge-Event-Triggered Strategies

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***Abstract*—This brief considers the distributed output regulation problem of multiple heterogeneous agents in the presence of an exosystem and constrained with limited communication bandwidth. To estimate the exosystem, we devise adaptive edge-based event-triggered dynamic observers for all followers. Then, we devise both distributed state feedback control inputs and output feedback inputs to guarantee that the regulated output asymptotically converges to zero. The presented protocols composed of distributed observers and local control inputs are completely distributed and can reduce the communication frequency.**

***Index Terms*—Heterogeneous multi-agent systems, output regulation, event-triggered control, adaptive control.**

I. Introduction

In recent years, cooperative control of multi-agent systems (MASs) has attracted compelling attention as a result of its broad applications on unmanned air vehicles, distributed sensor networks, robot teams, etc. [1]–[5]. Each subsystem of practical MASs often has different dynamics, so researchers are interested in the cooperative output regulation problem of heterogeneous MASs. It aims at driving a team of heterogeneous agents to asymptotically track some prescribed trajectory or reject unexpected disturbances.

On the basis of the consensus problem of homogeneous MASs, such as [6]–[9], many papers are progressively published on the output regulation problem. Reference [10] systematically introduces the output regulation problem and demonstrates its applications. In [11], [12], researches consider output regulation control of heterogeneous linear MASs. To avoid using the global information, [13] further utilize the adaptive control technique for such a problem. However, in the above references about the output regulation problem, continuous communications are required to design the protocols. Due to limited bandwidth and power source of agents, it is vital to refrain from continuous communications and reduce the frequency of information transmission among neighboring agents. It has been proven that event-triggered control can save precious energy and reduce action numbers. Its core idea is to design appropriate triggering functions to carry out communications only when they are needed. For homogeneous MASs, including low-order and high-order linear agents, many event-triggered algorithms are designed in published papers, like [14]–[19]. The authors of [20] present event-driven algorithms to achieve tracking control of MASs with a dynamic leader. As for heterogeneous MASs, [21], [22] put forward event-based algorithms in order to regulate the outputs. However, the event-driven algorithms of the above papers require global values of the whole graph and hence cannot be used in the practical occasions where these values are not available. To avoid using global values, we design adaptive event-triggered protocols for homogeneous MASs in our earlier work [23]. We also consider the existence of a dynamic leader or external disturbances in MASs constrained with event-triggered communications and lack of global information in [24], [25].

In this brief, we aim at further studying cooperative output regulation of heterogeneous MASs limited by communication bandwidth. Different from our existing results, the heterogeneity nature of agents renders this problem much more challenging. We must put forward novel event-based algorithms to overcome the heterogeneity, estimate the exosystem, and regulate the output, using limited communication bandwidth and calculation resources.

To handle the existence of the exosystem, we devise adaptive edge-based event-triggered observers for the followers. It is shown that the observers asymptotically converge to the state of the exosystem and the Zeno behavior does not exist. For the case where subsystems' states are available, we devise local state feedback control inputs for agents to regulate their outputs to zero. If subsystems' states are not available, we need to estimate them based on measurement output and then give the local inputs. We show that the output regulation problem of heterogeneous MASs is solved by the presented algorithms composed of distributed observers and local control inputs. It is to be emphasized that these algorithms are completely distributed, because they do not need any global values.

Manuscript received August 27, 2019; revised October 10, 2019; accepted October 31, 2019. Date of publication November 18, 2019; date of current version October 5, 2020. This work was supported in part by the National Natural Science Foundation of China under Grant 61973006, Grant 61973309, and Grant U1713223, in part by the Beijing Nova Program under Grant 2018047, and in part by the Joint Fund of Ministry of Education of China for Equipment Pre-Research. This brief was recommended by Associate Editor X. Wu. *(Corresponding author: Zhongkui Li.)*

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Digital Object Identifier 10.1109/TCSII.2019.2953930

Meanwhile, they can reduce the communication frequency and save energy resources. With these advantages, the adaptive event-triggered algorithms are applicable to more practical and complex occasions.

The rest of this brief is organized as follows. The output regulation problem with event-triggered communications is formulated in Section II. Main results are given in Section III. Section IV concludes this brief. The proofs of the theorems are presented in the Appendix.

II. Problem Formulation

Consider N heterogeneous agents, whose dynamics are as follows:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v, \\ y_{mi} &= C_{mi} x_i + D_{mi} v, \\ e_i &= C_i x_i + D_i v, \ \ i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $x_i \in \mathbf{R}^{n_i}$, $u_i \in \mathbf{R}^{m_i}$, $y_{mi} \in \mathbf{R}^{p_{mi}}$, and $e_i \in \mathbf{R}^{p_i}$ are, respectively, the state, the control input, the measurement output, and the regulated output of the i-th agent, and A_i, B_i, C_i, C_{mi}, D_i, D_{mi}, and E_i are with compatible dimensions. And $v \in \mathbf{R}^q$ as an exogenous signal can be either a reference signal or a disturbance, and satisfies the following dynamics:

$$\dot{v} = Sv, \tag{2}$$

where $S \in \mathbf{R}^{q \times q}$.

Because the exosystem (2) never receives information from other subsystems, we call it a virtual leader, indexed by 0, and other N subsystems followers, indexed by $1, \ldots, N$. We represent the information flow among the leader-follower network by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V} = \{0, 1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denote the node set and the edge set, respectively. If $(i, j) \in \mathcal{E}$, we call node i a neighbor of node j. For graph $\mathcal{G}$, the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$ is defined as $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{(N+1)\times(N+1)}$ is defined as $l_{ii} = \sum_{j=0}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$.

Assumption 1: The subgraph $\mathcal{G}_s$ associated with the followers is undirected and the graph $\mathcal{G}$ contains a directed spanning tree with the leader as the root.

Then, we partition the Laplacian matrix as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$, where $\mathcal{L}_1 \in \mathbf{R}^{N\times N}$ and $\mathcal{L}_2 \in \mathbf{R}^{N\times 1}$.

Lemma 1 [13]: Under Assumption 1, $\mathcal{L}_1$ is symmetric and positive-definite.

The aim of this brief is to present distributed event-based algorithms using local information available to each follower such that (i) the closed-loop system is asymptotically stable when $v = 0$; (ii) for any initial conditions, $\lim_{t\to\infty} e_i(t) = 0$; (iii) there does not exist the Zeno behavior, i.e., an infinite number of events never happen during a finite period of time.

In order to achieve the above objectives, the following standard assumptions [10] are required.

Assumption 2: Each pair (A_i, B_i) is stabilizable.

Assumption 3: Each pair (A_i, C_{mi}) is detectable.

Assumption 4: The matrix S has no eigenvalues with negative real parts.

Assumption 5: For all $\lambda \in \sigma(S)$, in which $\sigma(S)$ denotes the spectrum of S, $\operatorname{rank}\left(\begin{bmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{bmatrix}\right) = n_i + p_i$.

Assumption 6: There exist solution pairs ($X_i \in \mathbf{R}^{n_i\times q}$, $U_i \in \mathbf{R}^{m_i\times q}$) for the following regulation equations:

$$\begin{aligned} X_i S &= A_i X_i + B_i U_i + E_i, \\ 0 &= C_i X_i + D_i, \ \ i = 1, \ldots, N. \end{aligned} \tag{3}$$

III. Main Results

A. Adaptive Event-Triggered Dynamic Observers

Because a subset of followers can directly obtain the state v of the exosystem (2), we first devise adaptive event-triggered observers for followers:

$$\begin{aligned} \dot{v}_i &= S v_i + K \sum_{j=0}^{N} c_{ij} a_{ij} \tilde{\eta}_{ij}, \\ \dot{c}_{ij} &= \kappa_{ij} a_{ij} \tilde{\eta}_{ij}^T \Gamma \tilde{\eta}_{ij}, \ \ i = 1, \ldots, N, \end{aligned} \tag{4}$$

where v_i is the estimate of v, $\tilde{\eta}_{i0} = \tilde{v}_{i0} - v$, $\tilde{\eta}_{ij} = \tilde{v}_{ij} - \tilde{v}_{ji}$, $j = 1, \ldots, N$, $\tilde{v}_{ij}(t) = e^{S(t - t_k^{ij})} v_i(t_k^{ij})$, $c_{ij}(t)$ is a time variant gain for edge (i, j) with $c_{ij}(0) = c_{ji}(0)$, $\kappa_{ij} = \kappa_{ji} > 0$, and $K \in \mathbf{R}^{q\times q}$ and $\Gamma \in \mathbf{R}^{q\times q}$ are constant matrices. And t_k^{ij} is the k-th event instant of edge (i, j).

To determine event instants t_0^{ij}, t_1^{ij}, …, we should propose appropriate triggering functions for edge (i, j), and define a measurement error: $e_{ij}(t) \triangleq \tilde{v}_{ij}(t) - v_i(t)$. The triggering functions $f_{ij}(t)$ and $f_{ji}(t)$ for edges (i, j) and (j, i) can be respectively designed as

$$f_{ij}(t) = (1 + 2\delta c_{ij}) e_{ij}^T \Gamma e_{ij} - \frac{1}{4} \tilde{\eta}_{ij}^T \Gamma \tilde{\eta}_{ij} - \mu_{ij} e^{-\nu_{ij} t}, \tag{5a}$$

$$f_{ji}(t) = (1 + 2\delta c_{ji}) e_{ji}^T \Gamma e_{ji} - \frac{1}{4} \tilde{\eta}_{ji}^T \Gamma \tilde{\eta}_{ji} - \mu_{ji} e^{-\nu_{ji} t}, \tag{5b}$$

where δ, μ_{ij}, μ_{ji}, ν_{ij}, and ν_{ji} are positive constants. Then, event instants of edge (i, j) can be computed by using

$$t_{k+1}^{ij} \triangleq \{t_k^{ij} + \min\{\tau_k^{ij}, \tau\}\}, \tag{6}$$

where τ denotes the upper bound of event intervals, and $\tau_k^{i0} \triangleq \{t - t_k^{i0} > 0 \mid f_{i0}(t) \geq 0\}$ and $\tau_k^{ij} \triangleq \{t - t_k^{ij} > 0 \mid f_{ij}(t) \geq 0 \text{ or } f_{ji}(t) \geq 0\}$, $j = 1, \ldots, N$. We let $t_0^{ij} \triangleq 0$, $\forall (i, j) \in \mathcal{E}$, which means communications are needed at the initial instant. At each triggering instant, for example, t_k^{ij}, agents i and j exchange the state information of observers. Meanwhile, the variables $\tilde{v}_{ij}$ and $\tilde{v}_{ji}$ should be updated to newest values v_i and v_j, respectively.

Remark 1: Note that the triggering functions (5) are designed for each edge in this brief and such edge-based triggering functions have some advantages compared to the node-based ones of previous works, such as [15]–[17]. Generally speaking, the edge-event-triggered algorithms are more propitious to reduce the communication frequency and easier to compute. Interested readers can refer to [25], [26] for more detailed explanations.

Denote $\xi = [\xi_1^T, \ldots, \xi_N^T]^T$, where $\xi_i = v_i - v$. It is obvious that $\xi = 0$ if and only if $v_1 = \cdots = v_N = v$. We then obtain the dynamics of ξ:

$$\dot{\xi}_i = S\xi_i + K\sum_{j=0}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij},$$
$$\dot{c}_{ij} = \kappa_{ij}a_{ij}\tilde{\eta}_{ij}^T\Gamma\tilde{\eta}_{ij}. \tag{7}$$

The following theorem establishes the convergence of the event-triggered observer (4).

Theorem 1: Let $K = -P$ and $\Gamma = P^2$, where $P > 0$ is a solution of the algebraic Riccati equation: $PS+S^TP-P^2+I=0$. Then, the event-based observers composed of (4) and (5) can track the state v of the exosystem (2). Meanwhile, the coupling gains $c_{ij}(t)$ converge to finite steady-state values.

Remark 2: Under the proposed triggering rule, communications are not required until triggering conditions are satisfied. The parameter τ in (6) provides convenience for the boundedness proof of $\dot{\xi}$. We can avoid the conservatism of the existence of τ by choosing a large value for it. We also want to emphasize that when an edge is triggered, the two agents connected by the edge should share observer information with each other synchronously. The advantage of the synchronous edge-event-triggered mechanism is also providing convenience to show the boundedness of $\dot{\xi}$. Only after that, the Barbalat's lemma can be used to show that the designed observer asymptotically converges to the exosystem. In addition, we can extend the synchronous edge-event-triggered protocol given in this brief to asynchronous ones by referring to [25].

Remark 3: Note that the time variant gains $c_{ij}(t)$ are included by both the observer (4) and triggering function (5). Benefitting from this, the adaptive observer relies on no global values and is independent of the network size. Such an adaptive event-triggered idea for heterogeneous MASs is partly inspired by our previous works [23], [25], where we study the consensus problem of homogeneous agents.

In some case, if we know the smallest nonzero eigenvalue $\lambda_2(\mathcal{L})$ of the Laplacian matrix, we can simplify the observer and the triggering function by using a static gain c to replace the time-varying gains $c_{ij}(t)$. Specifically, we can design the event-based observer and the triggering function as

$$\dot{v}_i = Sv_i + cK\sum_{j=0}^{N} a_{ij}\tilde{\eta}_{ij},\ \ i = 1, \ldots, N,$$
$$f_{ij}(t) = e_{ij}^T\Gamma e_{ij} - \frac{1}{4}\tilde{\eta}_{ij}^T\Gamma\tilde{\eta}_{ij} - \mu_{ij}e^{-\nu_{ij}t}. \tag{8}$$

In this case, triggering instants can be defined as $t_{k+1}^{ij} \triangleq \{t_k^{ij} + \tau_k^{ij}\}$, where $\tau_k^{ij} \triangleq \{t - t_k^{ij} > 0 \mid f_{ij}(t) \geq 0\}$.

Proposition 1: The event-triggered static observers (8) can track the state v of the exosystem (2) with $c \geq \frac{2}{\lambda_2(\mathcal{L})}$, and the other parameters are set as in Theorem 1.

Remark 4: Note that the static edge-event-triggered observer can be used only when $\lambda_2(\mathcal{L})$ is available (the agent number and topology structure are required). Once the number or the structure is changed, users must recompute the value $\lambda_2(\mathcal{L})$. It is also a time-consuming task to compute that value especially for large-scale networks. Furthermore, for each agent, it is hard to obtain the global values. As a comparison, the adaptive edge-event-triggered observer composed of (4) and (5) does not need to compute $\lambda_2(\mathcal{L})$ and thereby can be utilized in a completely distributed fashion.

The next theorem guarantees the feasibility of the proposed adaptive edge-event-triggered observers.

Theorem 2: The closed-loop system (7) does not exist the Zeno behavior.

B. Two Kinds of Local Controllers for Subsystems

Based on the dynamic observer (4), we will devise local controllers for each subsystem. Here, we consider two cases: (i) the state of agent x_i is available; (ii) x_i is not available and only the measurement output y_{mi} can be used.

For case (i), we design the following state feedback controller:

$$u_i = K_{1i}x_i + K_{2i}v_i, \tag{9}$$

where v_i is the observer of the exosystem defined in (4), and K_{1i} and K_{2i} are both design matrices.

Theorem 3: Select K_{1i} such that $A_i + B_iK_{1i}$ are Hurwitz, and $K_{2i} = U_i - K_{1i}X_i$, where (X_i, U_i) satisfy (3). The output regulation problem can be solved under the protocol composed of (4), (5), and (9).

Remark 5: The protocol given in Theorem 3 can be designed according to the following steps:

1) Solve the ARE: $PS + S^TP - P^2 + I = 0$ to get P.
2) Compute $K = -P$ and $\Gamma = P^2$.
3) Solve the regulation equations (3) to get X_i and U_i.
4) Select K_{1i} such that $A_i+B_iK_{1i}$ are Hurwitz, and compute $K_{2i} = U_i - K_{1i}X_i$.
5) Select $c_{ij}(0) = c_{ji}(0)$, $\kappa_{ij} = \kappa_{ji}$, δ, τ, μ_{ij}, and ν_{ij} to be any positive constants.

For case (ii), we devise an observer-based output feedback controller:

$$u_i = K_{1i}\hat{x}_i + K_{2i}v_i,$$
$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_iu_i + E_iv_i + H_i(C_{mi}\hat{x}_i + D_{mi}v_i - y_{mi}), \tag{10}$$

where $\hat{x}_i$ is the observer of state x_i, H_i is a design matrix, and other parameters are the same as in (9).

We have the following result, which can be proved by using similar lines in the proof of Theorem 3.

Theorem 4: Select H_i such that $A_i+H_iC_{mi}$ are Hurwitz, and other parameters are set as in Theorem 3. Then, the output regulation problem can be solved by the protocol composed of (4), (5), and (10).

Remark 6: We admit that the controller (9) is partly inspired by [12], [13], which study cooperative output regulation control of heterogenous MASs using continuous local information. Different from [12], [13], the event-triggered protocols of the current brief can save the bandwidth of communication. It has been shown that these protocols require no global values and are completely distributed. This fully distribution feature is a big improvement compared to the existing works considering event-triggered output regulation problem, such as [21], [22].

IV. Conclusion

In this brief, we have presented distributed adaptive edge-event-triggered algorithms to achieve output regulation of heterogeneous MASs. In contrast to the existing related works, the protocols presented in this brief are completely distributed relying on no global values related to the whole graph. Under the given event-based rule, there are no Zeno behaviors and the frequency of communication can be reduced. Following the current brief, we are going to further consider the case of general directed topologies in the future work.

Appendix

A. Proof of Theorem 1

Choose a Lyapunov function candidate as

$$V = \frac{1}{2}\sum_{i=1}^{N}\xi_i^T P\xi_i + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}-\alpha)^2}{8\kappa_{ij}} + \sum_{i=1}^{N}\frac{(c_{i0}-\alpha)^2}{4\kappa_{i0}}, \quad (11)$$

where $\alpha = \max\{\frac{4}{\lambda_2(\mathcal{L})}, \frac{1}{\delta}\}$. Obviously, V is positive definite and its time derivative is:

$$\dot{V} = \sum_{i=1}^{N}\xi_i^T PS\xi_i + \sum_{i=1}^{N}\sum_{j=0}^{N} c_{ij}a_{ij}\xi_i^T PK\tilde{\eta}_{ij} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}-\alpha)}{4\kappa_{ij}}\dot{c}_{ij} + \sum_{i=1}^{N}\frac{(c_{i0}-\alpha)}{2\kappa_{i0}}\dot{c}_{i0}. \quad (12)$$

Because $c_{ij} = c_{ji}$, $i, j = 1, \ldots, N$, we have

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=0}^{N} c_{ij}a_{ij}\xi_i^T PK\tilde{\eta}_{ij}\\
&= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}a_{ij}(\xi_i-\xi_j)^T P^2\tilde{\eta}_{ij} - \sum_{i=1}^{N} c_{i0}a_{i0}\xi_i^T P^2\tilde{\eta}_{i0}\\
&= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij} - \sum_{i=1}^{N} c_{i0}a_{i0}\tilde{\eta}_{i0}^T P^2\tilde{\eta}_{i0}\\
&\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}a_{ij}(e_{ij}-e_{ji})^T P^2\tilde{\eta}_{ij} + \sum_{i=1}^{N} c_{i0}a_{i0}e_{i0}^T P^2\tilde{\eta}_{i0}\\
&\leq -\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij} + \sum_{i=1}^{N}\sum_{j=0}^{N} c_{ij}a_{ij}e_{ij}^T P^2 e_{ij}\\
&\quad - \frac{1}{2}\sum_{i=1}^{N} c_{i0}a_{i0}\tilde{\eta}_{i0}^T P^2\tilde{\eta}_{i0},
\end{aligned} \quad (13)$$

where the last inequality can be obtained by using the Young's inequality [27].

Using (4), we get that

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{c_{ij}-\alpha}{4\kappa_{ij}}\dot{c}_{ij} = \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} a_{ij}\frac{c_{ij}-\alpha}{4}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij}\\
&\leq \frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{ij}^T P^2 e_{ij} + \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij}\\
&\quad - \frac{\alpha}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij} - \frac{\alpha}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\xi_i^T P^2(\xi_i-\xi_j),
\end{aligned} \quad (14)$$

and

$$\begin{aligned}
\sum_{i=1}^{N}\frac{c_{i0}-\alpha}{2\kappa_{i0}}\dot{c}_{i0} &\leq \frac{1}{2}\sum_{i=1}^{N} c_{i0}a_{i0}\tilde{\eta}_{i0}^T P^2\tilde{\eta}_{i0} - \frac{\alpha}{4}\sum_{i=1}^{N} a_{i0}\tilde{\eta}_{i0}^T P^2\tilde{\eta}_{i0}\\
&\quad - \frac{\alpha}{8}\sum_{i=1}^{N}\xi_i^T P^2\xi_i + \frac{\alpha}{2}\sum_{i=1}^{N} a_{i0}e_{i0}^T P^2 e_{i0}.
\end{aligned} \quad (15)$$

Since $\alpha = \max\{\frac{4}{\lambda_2(\mathcal{L})}, \frac{1}{\delta}\}$, substituting (13), (14), and (15) into (12) yields

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^{N}\xi_i^T PS\xi_i - \frac{\alpha}{8}\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\xi_i^T P^2(\xi_i-\xi_j)\\
&\quad + \frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\Big[(1+2\delta c_{ij})e_{ij}^T P^2 e_{ij} - \frac{1}{4}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij}\Big]\\
&\leq -\frac{1}{2}\xi^T\Big[I_N\otimes(PS+S^T P) - \frac{\alpha}{4}\mathcal{L}_1\otimes P^2\Big]\xi\\
&\quad + \frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\Big[(1+2\delta c_{ij})e_{ij}^T P^2 e_{ij} - \frac{1}{4}\tilde{\eta}_{ij}^T P^2\tilde{\eta}_{ij}\Big]\\
&\leq -\frac{1}{2}\xi^T\xi + \frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\mu_{ij}e^{-\nu_{ij}t},
\end{aligned} \quad (16)$$

where we supplementarily define $\xi_0 \triangleq 0$, and have used Lemma II and the triggering functions (5) to get the last inequality.

It follows from (11) and (16) that $0 \leq V(t) \leq V(0) + \frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\frac{\mu_{ij}}{\nu_{ij}}$, implying that V is bounded. With (11), ξ and c_{ij} are all bounded. It follows from (7) that $\dot{\xi}_i = S\xi_i + \sum_{j=0}^{N} c_{ij}a_{ij}K(\tilde{v}_{ij}-\tilde{v}_{ji})$. Since S and ξ are all bounded, we then prove the boundedness of $\tilde{v}_{ij}-\tilde{v}_{ji}$. Note that $\tilde{v}_{ij}(t)-\tilde{v}_{ji}(t) = e^{S(t-t_k^{ij})}[v_i(t_k^{ij}) - v_j(t_k^{ij})]$. Because ξ is bounded, $v_i(t_k^{ij}) - v_j(t_k^{ij})$ is also bounded. Because event intervals are no more than the constant parameter τ, $t - t_k^{ij}$ is bounded. Thus, $\tilde{v}_{ij}(t)-\tilde{v}_{ji}(t)$ is bounded and so is $\dot{\xi}$. From (16), we can obtain that

$$\begin{aligned}
\int_0^{+\infty}\xi^T\xi dt &\leq \int_0^{+\infty} 2\Big(\frac{\alpha}{2}\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\mu_{ij}e^{-\nu_{ij}t} - \dot{V}(t)\Big)dt\\
&= 2(V(0)-V(+\infty)) + \alpha\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\frac{\mu_{ij}}{\nu_{ij}}.
\end{aligned}$$

Thus, $\int_0^{+\infty}\xi^T\xi dt$ is bounded. Also remembering that both ξ and $\dot{\xi}$ are bounded, we can utilize the Barbalat's lemma [28] to get that $\xi \to 0$ as $t \to +\infty$.

Since c_{ij} are bounded and also monotonically increasing, they converge to finite steady-state values. ■

B. Proof of Theorem 2

We rule out Zeno behaviors by providing a contradiction argument. We first do this brief for the edges among the followers. Generally, suppose edge $(i, j), i, j = 1, \ldots, N$ exists Zeno behaviors. Based on the definition of the Zeno behavior [25], $\exists T < +\infty \Rightarrow \lim_{k\to\infty} t_k^{ij} = T$. Then, for any $\varepsilon_0 > 0$, $\exists M_0 \in \mathbf{Z}_{\geq 0} \Rightarrow T - \varepsilon_0 < t_m^{ij} \leq T$, for any $m \geq M_0$.

The right-hand Dini derivative of e_{ij} is computed by: $D^+e_{ij} = Se_{ij} + P\sum_{j=0}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}$. Since $\tilde{\eta}_{ij}$ and c_{ij} are bounded, we have $D^+\|e_{ij}\| \leq \|S\|\|e_{ij}\| + \sigma_{ij}$, where σ_{ij} denotes the upper bound of $\|P\sum_{j=0}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}\|$ for $t \in [t_k^{ij}, t_{k+1}^{ij})$.

Define a function $\phi : [0, \infty) \to \mathbf{R}_{\geq 0}$, satisfying

$$\dot{\phi} = \|S\|\phi + \sigma_{ij}, \;\; \phi(0) = \|e_{ij}(t_k^{ij})\| = 0. \tag{17}$$

Therefore, $\|e_{ij}(t)\| \leq \phi(t - t_k^{ij})$ and $\phi(t) = \frac{\sigma_{ij}}{\|S\|}(e^{\|S\|t} - 1)$ satisfies (17).

Under the triggering rule, edge (i, j) will not be triggered if the following conditions hold at the same time:

$$\|e_{ij}\| \leq \frac{\sqrt{\mu_{ij}}}{\|P\|}e^{-\frac{v_{ij}}{2}T}, \quad \|e_{ji}\| \leq \frac{\sqrt{\mu_{ji}}}{\|P\|}e^{-\frac{v_{ji}}{2}T}. \tag{18}$$

Let $\varepsilon_{ij} = \frac{1}{2\|S\|}\ln(1 + \frac{\|S\|\sqrt{\mu_{ij}}}{\sigma_{ij}\|P\|}e^{-\frac{v_{ij}}{2}T})$ and $\varepsilon_{ji} = \frac{1}{2\|S\|}\ln(1 + \frac{\|S\|\sqrt{\mu_{ji}}}{\sigma_{ji}\|P\|}e^{-\frac{v_{ji}}{2}T})$. Denote $\varepsilon_0 \triangleq \min\{\varepsilon_{ij}, \varepsilon_{ji}\}$. Then, according to (18), the time interval between t_k^{ij} and t_{k+1}^{ij} satisfies $t_{k+1}^{ij} - t_k^{ij} \geq 2\varepsilon_0$, which further implies that $t_{k+1}^{ij} \geq t_k^{ij} + 2\varepsilon_0 > T + \varepsilon_0$. It does not consist with $T - \varepsilon_0 < t_{k+1}^{ij} \leq T$. So, Zeno behaviors are ruled out for the edges among followers.

Then, we can similarly rule out Zeno behaviors from the leader to informed followers. Consequently, there are no Zeno behaviors for the whole MASs. ■

C. Proof of Theorem 3

Let $\phi_i = x_i - X_i v_i$. Noting that (3) and $K_{2i} = U_i - K_{1i}X_i$, then combining (1) with (4) yields

$$\begin{aligned}\dot{\phi}_i &= (A_i + B_iK_{1i})\phi_i + (A_iX_i + B_iU_i - X_iS)v_i \\ &\quad + B_i(K_{2i} - U_i + K_{1i}X_i)v_i + \psi_i \\ &= (A_i + B_iK_{1i})\phi_i + \psi_i,\end{aligned}$$

where $\psi_i = -X_iK\sum_{j=0}^{N} c_{ij}a_{ij}\tilde{\eta}_{ij}$. According to Theorem 1, $\lim_{t\to+\infty}\psi_i(t) = 0$. So, we further obtain that $\lim_{t\to+\infty}\phi_i(t) = 0$, which further yields

$$\lim_{t\to+\infty} x_i = \lim_{t\to+\infty}(\phi_i + X_iv_i) = \lim_{t\to+\infty} X_iv_i = \lim_{t\to+\infty} X_iv. \tag{19}$$

Noting that $C_iX_i + D_i = 0$, we have

$$\lim_{t\to+\infty} e_i = \lim_{t\to+\infty}(C_ix_i + D_iv) = \lim_{t\to+\infty}(C_iX_i + D_i)v = 0.$$

And when $v = 0$, it follows from (19) that $\lim_{t\to+\infty} x_i(t) = 0$, i.e., the closed-loop system is asymptotically stable. Therefore, output regulation can be achieved for the heterogeneous MASs composed of (1) and (2). ■

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[2] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control," *IEEE Control Syst. Mag.*, vol. 27, no. 2, pp. 71–82, Apr. 2007.

[3] X. Wang *et al.*, "Coordinated flight control of miniature fixed-wing UAV swarms: Methods and experiments," *Sci. China Inf. Sci.*, vol. 62, no. 11, 2019, Art. no. 212204.

[4] R. Niu, X. Wu, J.-A. Lu, and J. Lü, "Adaptive diffusion processes of time-varying local information on networks," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 66, no. 9, pp. 1592–1596, Sep. 2019.

[5] X. Wang, Z. Zeng, and Y. Cong, "Multi-agent distributed coordination control: Developments and directions via graph viewpoint," *Neurocomputing*, vol. 199, pp. 204–218, Jul. 2016.

[6] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 57, no. 1, pp. 213–224, Jan. 2010.

[7] Y. Zhao, Y. Liu, G. Wen, W. Ren, and G. Chen, "Designing distributed specified-time consensus protocols for linear multiagent systems over directed graphs," *IEEE Trans. Autom. Control*, vol. 64, no. 7, pp. 2945–2952, Jul. 2019.

[8] Y. Liu, Y. Zhao, W. Ren, and G. Chen, "Appointed-time consensus: Accurate and practical designs," *Automatica*, vol. 89, pp. 425–429, Mar. 2018.

[9] Z. Li and J. Chen, "Robust consensus of linear feedback protocols over uncertain network graphs," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4251–4258, Aug. 2017.

[10] J. Huang, *Nonlinear Output Regulation: Theorem and Applications*. Philadelphia, PA, USA: SIAM, 2004.

[11] J. Xiang, W. Wei, and Y. Li, "Synchronized output regulation of linear networked systems," *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1336–1341, Jun. 2009.

[12] Y. Su and J. Huang, "Cooperative output regulation of linear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 57, no. 4, pp. 1062–1066, Apr. 2012.

[13] Z. Li, M. Z. Q. Chen, and Z. Ding, "Distributed adaptive controllers for cooperative output regulation of heterogeneous agents over directed graphs," *Automatica*, vol. 68, pp. 179–183, Jun. 2016.

[14] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.

[15] E. Garcia, Y. Cao, H. Yu, A. Giua, P. Antsaklis, and D. Casbeer, "Decentralised event-triggered cooperative control with limited communication," *Int. J. Control*, vol. 86, no. 9, pp. 1479–1488, 2013.

[16] X. Meng and T. Chen, "Event based agreement protocols for multi-agent networks," *Automatica*, vol. 49, no. 7, pp. 2125–2132, 2013.

[17] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," *Automatica*, vol. 49, no. 1, pp. 245–252, 2013.

[18] N. Mu, X. Liao, and T. Huang, "Leader-following consensus in second-order multiagent systems via event-triggered control with nonperiodic sampled data," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 62, no. 10, pp. 1007–1011, Oct. 2015.

[19] L.-N. Zhao, H.-J. Ma, L.-X. Xu, and X. Wang, "Observer-based adaptive sampled-data event-triggered distributed control for multi-agent systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, to be published, doi: 10.1109/TCSII.2019.2900545.

[20] Y. Cheng and V. Ugrinovskii, "Event-triggered leader-following tracking control for multivariable multi-agent systems," *Automatica*, vol. 70, pp. 204–210, Aug. 2016.

[21] W. Hu and L. Liu, "Cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control," *IEEE Trans. Cybern.*, vol. 47, no. 1, pp. 105–116, Jan. 2017.

[22] W. Hu, L. Liu, and G. Feng, "Output consensus of heterogeneous linear multi-agent systems by distributed event-triggered/self-triggered strategy," *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 1914–1924, Aug. 2017.

[23] B. Cheng and Z. Li, "Fully distributed event-triggered protocols for linear multiagent networks," *IEEE Trans. Autom. Control*, vol. 64, no. 4, pp. 1655–1662, Apr. 2019.

[24] B. Cheng and Z. Li, "Designing fully distributed adaptive event-triggered controllers for networked linear systems with matched uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2018.2868986.

[25] B. Cheng and Z. Li, "Coordinated tracking control with asynchronous edge-based event-triggered communications," *IEEE Trans. Autom. Control*, vol. 64, no. 10, pp. 4321–4328, Oct. 2019.

[26] B. Cheng, Z. Wu, and Z. Li, "Distributed edge-based event-triggered formation control," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2019.2910131.

[27] D. S. Bernstein, *Matrix Mathematics: Theory, Facts, and Formulas*. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[28] P. Ioannou and J. Sun, *Robust Adaptive Control*. New York, NY, USA: Prentice-Hall, 1996.