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Decision-making of UAV for Tracking Moving Target via Information Geometry

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Abstract:
In this paper, we consider the unmanned aerial vehicle (UAV) action decision-making problem for tracking a ground moving target. The studied problem was formulated as a Partially Observable Markov Decision Process (POMDP), and following a Kalman filter is derived to estimate the Markov chain system state. Exploiting an innovations approach, the total information gain in the UAV-to-target observation is used as an optimization criterion in a partially observable Markov decision process formulation to optimize the control strategy. In our method, we make UAV command decision to maximize the Fisher information in information geometry. The computer simulations validate that, comparing the simulation results with conventional methods, our proposed method has less time cost and smaller tracking error.

Key Words: Information Geometry, Fisher Information, Decision-making, POMDP, Target Tracking

1 Introduction

Tracking a moving ground target from a unmanned aerial vehicle (UAV) is a key requirement for many information, surveillance, target acquisition, and reconnaissance (ISTAR) [1] applications, which is one of the many uses of UAVs. In target tracking, the observability of target is achieved by adequate maneuvers of the sensor mounted on the UAV. Since the sensing capabilities of UAVs are constrained by the UAV dynamics and the onboard sensor performances, one of the major research challenges is UAV action decision-making, which means how to plan ahead for the sensor platform movement to maximize target localization accuracy at a future time.

In the recent past, partially observable Markov decision process (POMDP) has been used to design decision-making algorithms for UAVs in tracking problems, as the dynamic of the target is assumed to follow a Markov process and the UAV state and dynamics are known and can be controlled. The approach in [2, 3] uses POMDP incorporating the dynamic constraints on UAV motion for designing a UAV guidance algorithm for target tracking, but it differs from the method in this study in the cost criterion. However, solving POMDP optimally has been proven PSPACE-hard [4]. In [5], in order to minimize the location uncertainty of a mobile target, the author proposes an off-line policy iteration algorithm to find an optimal UAV path in a coarse discretized state space, followed by an on-line policy iteration algorithm that applies a finer grid MDP to the region of interest to find the final optimal policy for fixed-wing UAV. The composed method can improve the real-time performance.

In our study, we use the Fisher information to modify the criterion function to decrease the computation cost and the location uncertainty. According to the relevant knowledge of information geometry, the Fisher information matrix (FIM) is derived through the model of radar detection. In this method, tracking performances can be evaluated more accurate, and the calculation is faster. The following is the structure of this paper.

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In the next section, we describe the problem of UAV decision-making for target tracking via a track-while-scan (TWS) radar system. The UAV action choosing driven by POMDP formulation and accumulative information is presented in Section 3, where we use the distribution of the predicted target state for the evaluation of the Fisher information FIM i.e.,the accumulative information. Simulation and results are presented in Section 4, which is followed by the conclusions in Section 5.

2 Problem Specification

We use a partially observable Markov decision process to find the optimal action of a UAV based on the current states of the UAV and the target. The decision-making problem is to select a sequence of actions for the UAV that maximize the total information gain to reduce the uncertainty of target localization.

The targets move on the ground in two-dimensional. The motion of the UAV is a simplified kinematic model [3]. The 2-D position coordinate of the UAV is varied by applying the controls forward acceleration (which controls the speed) and bank angle (which controls the heading). The values of these control variables are restricted to lie within certain minimum and maximum limits. The UAV is mounted with an onboard TWS radar that generates the position measurements of the target. These measurements are corrupted by random errors that are spatially varying, i.e., we assume the measurement error covariance of a target is proportional to the distance from the UAV to the target. depends on the locations of UAV and the target [3].

Based on POMDP, the decision-making program collects measurements from the sensor (mounted on the UAV), constructs the trackers, and makes the control decisions for the UAV. In order to obtain more information of the target through the radar, we should make appropriate decision based on some criterions. The difference from other algorithms is that our reward criterion is the accumulative information. We use the FIM to represent the accumulative information in information geometry.

3 POMDP Formulation and Information Geometry Theory

The POMDP is a controlled dynamical process in discrete time. It's objective is to find a policy such that the expected cumulative reward is maximized. This section introduce the POMDP formulation of the tracking problem and the reward function based on information geometry.

3.1 The Formulation of the System based on POMDP

We poses the UAV decision-making problem as a partially observable Markov decision process. The term "partial" indicates that the state of the whole system cannot be sensed directly i.e., the UAV is likely to suffer from uncertainty (such as noise and limited view of the environment) in its sensor. The followings define the key components of POMDP with special respect to UAV tracking a target.

- **States**. Three subsystems are defined as follows: the UAV, the target, and the tracker. The state at time k is given by $\boldsymbol{s}_k = (\boldsymbol{x}_k^o, \boldsymbol{x}_k, \boldsymbol{\mu}_k, \boldsymbol{P}_k)$, where $\boldsymbol{x}_k^o$ represents the UAV state, $\boldsymbol{x}_k$ represents the target state, and $(\boldsymbol{\mu}_k, \boldsymbol{P}_k)$ represents the tracker state. The UAV and the target states include the locations and velocities of the UAV and the target, respectively. The tracker state is a standard in the Kalman filter [6], where $\boldsymbol{\mu}_k$ is the posterior mean vector and $\boldsymbol{P}_k$ is the posterior covariance matrix. Let the relative target state be

$$\mathbf{x}_k = \boldsymbol{x}_k - \boldsymbol{x}_k^o.$$

- **Actions**. In our system, the control command of the UAV is the actions including the forward accelerations and the bank angles. More specifically, the action at time k is given by $\boldsymbol{u}_k = [a_k, \phi_k]^{\mathrm{T}}$, where a_k and ϕ_k are the forward acceleration and the bank angle, respectively.
- **State-transition law**. Since there are three subsystems, it is convenient to define the state-transition law for each subsystem separately.
The UAV state evolves based on $\boldsymbol{x}_{k+1}^o = f(\boldsymbol{x}_k^o, \boldsymbol{u}_k)$, where the mapping function f can be specified as a collection of simple kinematic equations that govern the UAV motion. The states of the UAV at time k is given by $\boldsymbol{x}_k^o = [x_k^o, y_k^o, V_k^o, \theta_k^o]^{\mathrm{T}}$, where (x_k^o, y_k^o) represents the position coordinates, V_k^o represents the speed, and θ_k^o represents the heading angle. The kinematic equations of the UAV motion (Geiger et al. 2006) are as follows:

$$\begin{cases} x_{k+1}^o = x_k^o + V_k^o T \cos\theta_k^o \\ y_{k+1}^o = y_k^o + V_k^o T \sin\theta_k^o \\ V_{k+1}^o = [V_k^o + a_k T]_{V_{\min}}^{V_{\max}} \\ \theta_{k+1}^o = \theta_k^o + (a_k T \tan(\phi_k)/V_k^o) \end{cases} \tag{1}$$

where $[v]_{V_{\min}}^{V_{\max}} = \max\{V_{\min}, \min(V_{\max}, v)\}$, $V_{\min}$ and $V_{\max}$ are the minimum and the maximum limits on the speed of the UAV, g is the acceleration due to gravity, and T is the length of the time step.

We consider that the target state evolves as a linearized target motion model with zero-mean noise. as given below:

$$\boldsymbol{x}_{k+1} = \mathbf{A}\boldsymbol{x}_k + \boldsymbol{v}_k, \boldsymbol{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k) \tag{2}$$

where the state of the target at time k is given by $\boldsymbol{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^{\mathrm{T}}$, and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

Finally, the tracker state evolves according to the Kalman filter equations
- **Observations and observation law**. The UAV and the tracker states are assumed to be fully observable. Sensors (mounted on the UAV) used for target tracking provide measurements of a target in a natural sensor coordinate system (CS). In our case, this CS is polar in 2-D with range r, bearing φ. The noisy measurements of the target measured at time k satisfy

$$\mathbf{z}_k = \begin{bmatrix} r_k \\ \varphi_k \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}_k) \\ h_2(\mathbf{x}_k) \end{bmatrix} + \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix} \tag{4}$$

where $(h_1(\mathbf{x}_k), h_2(\mathbf{x}_k))$ denotes the error-free true target position in the sensor polar coordinates, and w_{1k}, w_{2k} are the respective random measurement errors. It is normally assumed that these measurement errors in the sensor CS are zero-mean with Gaussian distributed,i.e.,

$$\mathbf{z}_k | \mathbf{x}_k \sim \mathcal{N}(\boldsymbol{h}(\mathbf{x}_k), \mathbf{C}(\mathbf{x}_k)).$$

For scan-while-track radar, the true target position, that is the mean can be represented as

$$\begin{aligned} \boldsymbol{h}(\mathbf{x}_k) &= \begin{bmatrix} h_1(\mathbf{x}_k) \\ h_2(\mathbf{x}_k) \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{(x_k - x_k^o)^2 + (y_k - y_k^o)^2} \\ \arctan\left((y_k - y_k^o)/(x_k - x_k^o)\right) \end{bmatrix} \end{aligned} \tag{5}$$

and the covariance matrix $\mathbf{C}(\mathbf{x}_k)$ depends on the the distance between the UAV and the target. The specific form is

$$\mathbf{C}(\mathbf{x}_k) = \begin{bmatrix} \hat{r}_k^2 \sigma_r^2 & 0 \\ 0 & \hat{r}_k^2 \sigma_\varphi^2 \end{bmatrix} \tag{6}$$

where $\hat{r}_k \sigma_r$ and $\hat{r}_k \sigma_\varphi$ are the standard deviations of the range and bearing, respectively. Apparently, $\hat{r}_k$ is the estimated range between the UAV and the target, which is equal to $h_1(\mathbf{x}_k)$. Range and bearing measurements may have vastly different accuracies. In this measurement model, we assume that

$$\sigma_r / \sigma_\varphi = \mathcal{O}(10^2).$$

In target tracking,Target motion is best described in a Cartesian CS, but measurements are available physically in the sensor CS. A method commonly used is to convert measurements from sensor CS to Cartesian CS, and do tracking entirely in the Cartesian CS. Once the noisy measurements of the target position are converted to the Cartesian coordinates, then a linear Kalman filter can be applied if the dynamics is linear. After linearized conversion, the measurement model in Cartesian coordinates has the form

$$\mathbf{z}_k^{'} = \mathbf{H}\mathbf{x}_k + \boldsymbol{w}_k^{'}, \boldsymbol{w}_k^{'} \sim \mathcal{N}(0, \mathbf{R}_k(\mathbf{x}_k)) \tag{7}$$

where the measurement matrix is

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{8}$$

The standard approach treats $\boldsymbol{w}_k^{'}$ approximately as zero-mean with covariance determined by a first-order Taylor series expansion.

$$\mathbf{R}(\mathbf{x}_k) = \mathbf{J}(\mathbf{x}_k)\mathbf{C}(\mathbf{x}_k)\mathbf{J}(\mathbf{x}_k)^{\mathrm{T}} \tag{9}$$

where in 2-D coordinate we have

$$\mathbf{J}(\mathbf{x}_k) = \begin{bmatrix} \cos(\varphi_k) & -\sin(\varphi_k) \\ \sin(\varphi_k) & \cos(\varphi_k) \end{bmatrix}$$

We emphasize that the measurement noise $\boldsymbol{w}_k^{'}$ is in general not only coupled across coordinates, non-Gaussian, but also state dependent. More seriously, this linearized conversion ignores the bias in the converted measurements, which may lead to substantially degraded performance and even filtering divergence. As a result, it is still an illusion that the application of the Kalman filter here in the case of linear dynamics yields optimal results. However, if we use information geometry to evaluate the accuracy the predicted states, the above problems can be solved.

- **Belief state**. In our circumstances, it is unable to identify the current state of the target unambiguously. However, we can infer a probability distribution over states as a belief state and the entire probability space as the belief space from the past history of the control actions and the current observation information. A belief state is a probability distribution over all states, which is $\boldsymbol{b}_k = \left(\boldsymbol{b}_k^{\boldsymbol{x}^o}, \boldsymbol{b}_k^{\boldsymbol{x}}, \boldsymbol{b}_k^{\boldsymbol{\mu}}, \boldsymbol{b}_k^{\boldsymbol{P}}\right)$ at time k. Since the sensor and the tracker states are fully observable. The belief states of them are $\boldsymbol{b}_k^{\boldsymbol{x}^o} = \delta\left(\boldsymbol{x}^o - \boldsymbol{x}_k^o\right)$, and $\boldsymbol{b}_k^{\boldsymbol{\mu}} = \delta\left(\boldsymbol{\mu} - \boldsymbol{\mu}_k\right), \boldsymbol{b}_k^{\boldsymbol{P}} = \delta\left(\boldsymbol{P} - \boldsymbol{P}_k\right)$. $\boldsymbol{b}_k^{\boldsymbol{x}}$ is the posterior distribution of the target state. The probability distributions are assumed to be Gaussian, the target belief state can be expressed approximatly as $\boldsymbol{b}_k^{\boldsymbol{x}} \sim \mathcal{N}\left(\boldsymbol{\mu}_k, \mathbf{P}_k\right)$.
In our system, if an optimal choice of action decisions exists, then there exists an optimal sequence of actions that depend only on the belief-state feedback [7].
- **Reward (Cost) function**. According to the nominal belief-state optimization (NBO) method [7], the objective function is approximated as follows:

$$J_H\left(\boldsymbol{b}_0\right) \approx \sum_{k=0}^{H-1} c\left(\hat{\boldsymbol{b}}_k, \boldsymbol{u}_k\right) \tag{10}$$

where $\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, ..., \hat{\boldsymbol{b}}_{H-1}$ is a nominal belief-state sequence and the optimization is over an action sequence $\boldsymbol{u}_0, \boldsymbol{u}_1, ..., \boldsymbol{u}_{H-1}$. The nominal target belief-state sequence can be identified with the nominal tracks $(\hat{\boldsymbol{\mu}}_k, \hat{\mathbf{P}}_k)$, which are obtained from the Kalman filter equations.
In some literatures, the cost function is used to represent the uncertainty of the target lacation, i.e., the mean-squared error between the tracks and the targets, can be written as

$$c\left(\hat{\boldsymbol{b}}_k, \boldsymbol{u}_k\right) = Tr\hat{\mathbf{P}}_{k+1} \tag{11}$$

The bias of the above linearized conversion can be considerable in some tracking applications and needs to be compensated. Since then, in this paper, we use Fisher information distance (explained later) on a statistical manifold as the criterion of POMDP to improve the accuracy of the expected future reward.

When we study the tracking problem in a POMDP framework, it boils down to an optimization problem, where the objective is to find sequence of actions over a time horizon H such that the expected cumulative reward is maximized or the cost is minmized.

3.2 Cumulative Information in Information Geometry

Information geometry [8] offers comprehensive results about statistical models simply by considering them as geometrical objects and the statistical structures as geometrical structures. From the information geometry point of view, the ensemble of measurement distribution parameterized by target state forms the statistical manifold of a particular geometric structure. The UAV-tracking-target process is to optimize the structure of the statistical manifold such that the target information observed by the sensor at a future time horizon is maximized.The amount of information which the sensor may acquired from the target at a particular time is characterized by the information distance. In a discrete measurement sampling scenario, the information distance is given by the sum of the FIM determinants.

In this paper, we address the POMDP optimization problem using the criterion of Fisher information distance on a statistical manifold. The determinant of Fisher information matrix is used to characterize the amount of information obtained by sensors. We establish the target state prediction uncertainty statistical model to include the uncertainty of the target state into measurement Fisher information matrix, then through calculating the cumulative amount of information each time through iterations, we can optimize UAV action decision-making. The following introduces the derivation process of the discrete Fisher information matrix.

Fisher information matrix at time k is defined [8, 9] by

$$\mathbf{G}_k = E\left[\left(\nabla_x \ln p(\mathbf{z}_{1:k} \left|\mathbf{x}_k\right.)\right)\left(\nabla_x \ln p(\mathbf{z}_{1:k} \left|\mathbf{x}_k\right.)\right)^T\right] \tag{12}$$

where $p\left(\mathbf{z}_{1:k} \left|\mathbf{x}_k\right.\right)$ is the batch measurement likelihood defined as formula (13). where $\boldsymbol{h}(\mathbf{x}_k, i)$ denotes the measurement model (5) evaluated at time i in terms of the state $\mathbf{x}_k$ at k. In view of (5) and (13) (in the top of the next page), the gradient term in (12) is taken with respect to $\mathbf{x}_k$ as (14). Noting that

$$E\left[\left(r_i - h_1(\mathbf{x}_k, i)\right)^4\right] = 3\hat{r}_i^4\sigma_r^4$$

and

$$E\left[\left(\varphi_i - h_2(\mathbf{x}_k, i)\right)^4\right] = 3\hat{r}_i^4\sigma_\varphi^4,$$

$$p\left(\mathbf{z}_{1:k}\left|\mathbf{x}_k\right.\right)=p\left(\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_k\left|\mathbf{x}_k\right.\right)=\prod_{i=1}^{k}\frac{1}{\sqrt{2\pi|\mathbf{C}(\mathbf{x}_k,i)|}}\exp\left(-\frac{1}{2}(\mathbf{z}_i-\boldsymbol{h}(\mathbf{x}_k,i))^T\mathbf{C}^{-1}(\mathbf{x}_k,i)(\mathbf{z}_i-\boldsymbol{h}(\mathbf{x}_k,i))\right) \quad (13)$$

$$\begin{aligned}\nabla_x\ln p\left(\mathbf{z}_{1:k}\left|\mathbf{x}_k\right.\right)&=\nabla_x\left(\sum_{i=1}^{k}\left(\ln\frac{1}{\sqrt{|\mathbf{C}(\mathbf{x}_k,i)|}}-\frac{1}{2}(\mathbf{z}_i-\boldsymbol{h}(\mathbf{x}_k,i))^T\mathbf{C}^{-1}(\mathbf{x}_k,i)\left(\mathbf{z}_i-\boldsymbol{h}(\mathbf{x}_k,i)\right)\right)\right)\\&=\nabla_x\left(\sum_{i=1}^{k}\left(\ln\frac{1}{\hat{r}_i^2}-\frac{1}{2}\left(\frac{(r_i-h_1(\mathbf{x}_k,i))^2}{\hat{r}_i^2\sigma_r^2}+\frac{(\varphi_i-h_2(\mathbf{x}_k,i))^2}{\hat{r}_i^2\sigma_\varphi^2}\right)\right)\right)\\&=\sum_{i=1}^{k}\left(-\frac{2}{\hat{r}_i}\nabla_x\hat{r}_i+\frac{(r_i-h_1(\mathbf{x}_k,i))^2}{\hat{r}_i^3\sigma_r^2}\nabla_x\hat{r}_i-\frac{(r_i-h_1(\mathbf{x}_k,i))}{\hat{r}_i^2\sigma_r^2}\nabla_x h_1(\mathbf{x}_k,i)\right.\\&\left.+\frac{(\varphi_i-h_2(\mathbf{x}_k,i))^2}{\hat{r}_i^3\sigma_\varphi^2}\nabla_x\hat{r}_i-\frac{(\varphi_i-h_2(\mathbf{x}_k,i))}{\hat{r}_i^2\sigma_\varphi^2}\nabla_x h_2(\mathbf{x}_k,i)\right)\end{aligned} \quad (14)$$

substitute (14) into (12), the Fisher information matrix can be calculated in a recursive form as (15). where

$$\Delta\mathbf{G}_{rk}=\begin{bmatrix}\frac{(x_k-x_k^o)^2}{\hat{r}_k^2} & \frac{(x_k-x_k^o)(y_k-y_k^o)}{\hat{r}_k^2}\\ \frac{(x_k-x_k^o)(y_k-y_k^o)}{\hat{r}_k^2} & \frac{(y_k-y_k^o)^2}{\hat{r}_k^2}\end{bmatrix} \quad (16)$$

$$\Delta\mathbf{G}_{\varphi k}=\begin{bmatrix}\frac{(y_k-y_k^o)^2}{\hat{r}_k^4} & -\frac{(x_k-x_k^o)(y_k-y_k^o)}{\hat{r}_k^4}\\ -\frac{(x_k-x_k^o)(y_k-y_k^o)}{\hat{r}_k^4} & \frac{(x_k-x_k^o)^2}{\hat{r}_k^4}\end{bmatrix} \quad (17)$$

$\hat{r}_i$ is the estimated range between the UAV and the target, which is

$$\hat{r}_i=h_1(\mathbf{x}_k,i)=\sqrt{(x_i-x_i^o)^2+(y_i-y_i^o)^2}.$$

Since the underlying sensor collects target information only at discrete sampling points, the accumulative information should only take into account those points where the sensor will take measurements. Therefore, the sum of determinants of FIM [10] at H sampling locations will be used to approximate the accumulative information along the UAV trajectory $(\boldsymbol{x}_1^o,\boldsymbol{x}_2^o,...,\boldsymbol{x}_H^o)$, i.e.,

$$\mathcal{D}(\boldsymbol{x}_1^o,\boldsymbol{x}_2^o,...,\boldsymbol{x}_H^o)\approx\sum_{i=1}^{H}|\mathbf{G}_i| \quad (18)$$

The value of the determinant of a FIM represents the volume of the amount of information that can be acquired by the sensor. Our decision-making process is to find the optimal sequence of actions which maximized the amount of cumulative information.

4 Simulation Results

This section shows the simulations to demonstrate the ability of UAV with a TWS radar to track a target using the proposed algorithm, comparing the simulation results with other method. The comparative algorithm treats the trace of the covariance in Kalman filter as the objective function. We compare of these two method from the aspect of the tracking performance and the computation time cost.

We implement our method in MATLAB, where the policy is obtained from the MATLAB command $fmincon$ to minimize the objective function. We perform 1000-step simulations and every step is $0.1\ s$. In our simulations, the time horizon H is set to be $H=6$ similar to [2]. It means that at the current time-step, we plan for 6 time-steps into the future, then implement the first control action for the current time-step, and discard the next 5 planned actions. At the very next time-step, we replan for 6 time-steps, and so forth. In addition, we set the speed of UAV in all simulations was between $25\ m/s$ to $30\ m/s$ and all other parameters are shown in Table 1. The target is moving in constant velocity in the first and second simulation scenarios, and in the other scenario, the target is following a circle trajectory. We also compare them under the condition of different measurement standard deviation. The following figures show the simulation results.

Table 1: The parameters of simulation experiments

No.	LST	STD of Range	STD of Bearing
1	$10\ m/s$	$10^{-1}\ m$	$10^{-3}\pi\ rad$
2	$10\ m/s$	$10^{-3}\ m$	$10^{-5}\pi\ rad$
3	$10\ m/s$	$10^{-1}\ m$	$10^{-3}\pi\ rad$
4	$10\ m/s$	$10^{-3}\ m$	$10^{-5}\pi\ rad$

[a] LST is short for the linear speed of the target.

In Figs. 1, 3, 5, and 7, the trajectory of a target is represented by red line and the trajectory of the UAV is represented by green and blue line. We define the distance error as one of the tracking performance metrics. At every time step, we compute the squared distance (squared error) between the actual target location and the estimated target location from each UAV. Figs. 2, 4, 6, and 8 show the distance error at every time step.

In the first scenario, the UAV's initial position is $(0,50)$, and the initial position of the target is $(0,0)$. Figs. 1, 2 and Figs. 3, 4 show the UAV using Kalman filter and information

$$
\begin{aligned}
\mathbf{G}_k &= \sum_{i=1}^{k}\left(\frac{4}{\hat{r}_i^2}\nabla_x\hat{r}_i(\nabla_x\hat{r}_i)^T + \frac{1}{\hat{r}_i^2\sigma_r^2}\nabla_x h_1(\mathbf{x}_k,i)(\nabla_x h_1(\mathbf{x}_k,i))^T + \frac{1}{\hat{r}_i^2\sigma_\varphi^2}\nabla_x h_2(\mathbf{x}_k,i)(\nabla_x h_2(\mathbf{x}_k,i))^T\right) \\
&= \sum_{i=1}^{k}\left(\left(\frac{4}{\hat{r}_i^2}+\frac{1}{\hat{r}_i^2\sigma_r^2}\right)\nabla_x h_1(\mathbf{x}_k,i)(\nabla_x h_1(\mathbf{x}_k,i))^T + \frac{1}{\hat{r}_i^2\sigma_\varphi^2}\nabla_x h_2(\mathbf{x}_k,i)(\nabla_x h_2(\mathbf{x}_k,i))^T\right) \\
&= \mathbf{G}_{k-1} + \left(\frac{4}{\hat{r}_k^2}+\frac{1}{\hat{r}_k^2\sigma_r^2}\right)\nabla_x h_1(\mathbf{x}_k,k)(\nabla_x h_1(\mathbf{x}_k,k))^T + \frac{1}{\hat{r}_i^2\sigma_\varphi^2}\nabla_x h_2(\mathbf{x}_k,k)(\nabla_x h_2(\mathbf{x}_k,k))^T \\
&= \mathbf{G}_{k-1} + \left(\frac{4}{\hat{r}_k^2}+\frac{1}{\hat{r}_k^2\sigma_r^2}\right)\Delta\mathbf{G}_{rk} + \frac{1}{\hat{r}_k^2\sigma_\varphi^2}\Delta\mathbf{G}_{\varphi k}
\end{aligned}
\tag{15}
$$

geometty to track the linear motion target in the higher and lower radar measurement accuracy, respectively.

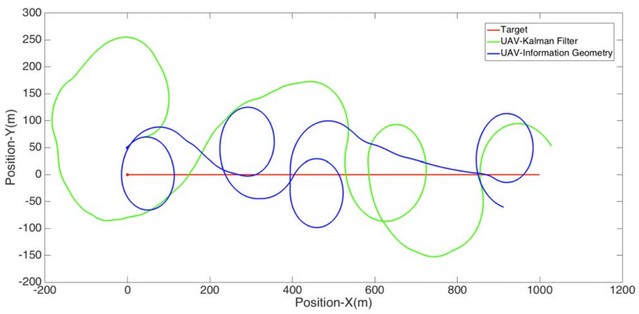

Fig. 1: UAV tracking a target on a linear path, $\sigma_r = 10^{-1}\ m$, $\sigma_\varphi = 10^{-3}\pi\ rad$

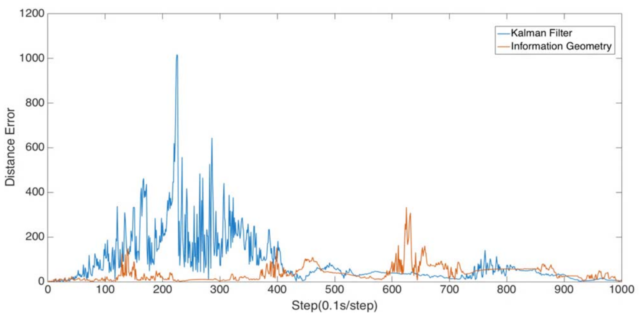

Fig. 2: Distance error of tracking a target on a linear path, $\sigma_r = 10^{-1}\ m$, $\sigma_\varphi = 10^{-3}\pi\ rad$

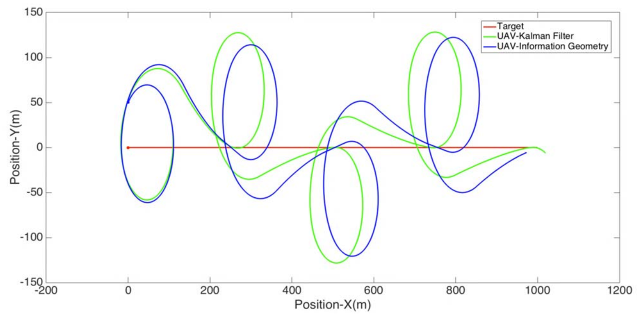

Fig. 3: UAV tracking a target on a linear path, $\sigma_r = 10^{-3}\ m$, $\sigma_\varphi = 10^{-5}\pi\ rad$

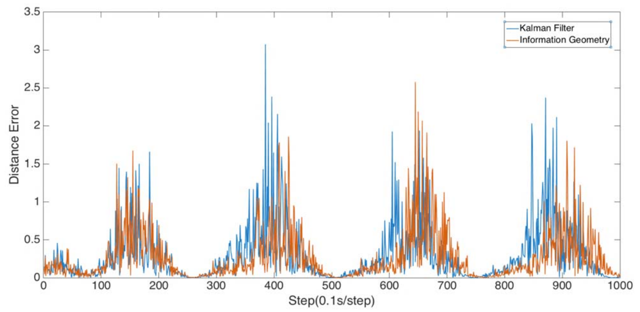

Fig. 4: Distance error of tracking a target on a linear path, $\sigma_r = 10^{-3}\ m$, $\sigma_\varphi = 10^{-5}\pi\ rad$

As can be seen from the tracking results of the first scenario, the proposed method using the cumulative information as the objective function perform better intuitively. Indeed, the tracking distance error is smaller.

In the second scenario, the initial position of the UAV is $(0, 100)$, and the target's initial position is $(0, 50)$. Figs. 5, 6 and Figs. 7, 8 also illustrates tracking the circular motion target results in different measurement accuracy.

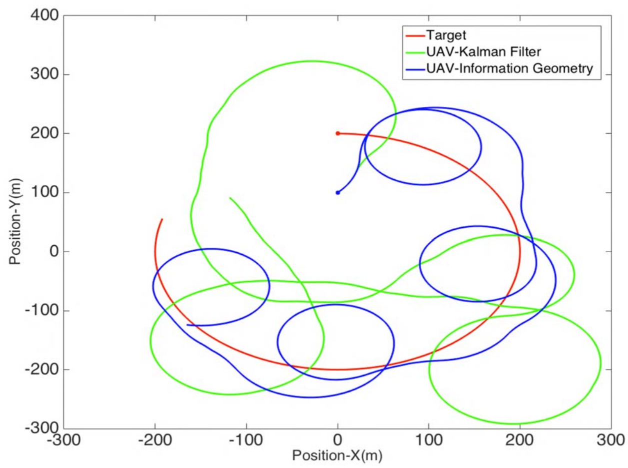

Fig. 5: UAV tracking a target on a circle path, $\sigma_r = 10^{-1}\ m$, $\sigma_\varphi = 10^{-3}\pi\ rad$

As it can be seen from the all result figures, information geometry-based approach has a better decision results. Besides, with the improvement of measurement accuracy, the performance of these two method is approaching. Then analyzing the reasons, when we using the Kalman filter co-

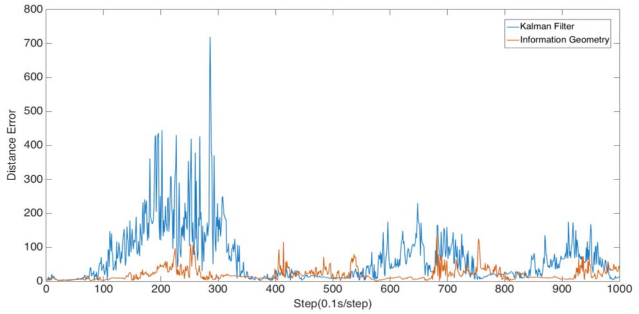

Fig. 6: Distance error of tracking a target on a circle path, $\sigma_r = 10^{-1}\ m$, $\sigma_\varphi = 10^{-3}\pi\ rad$

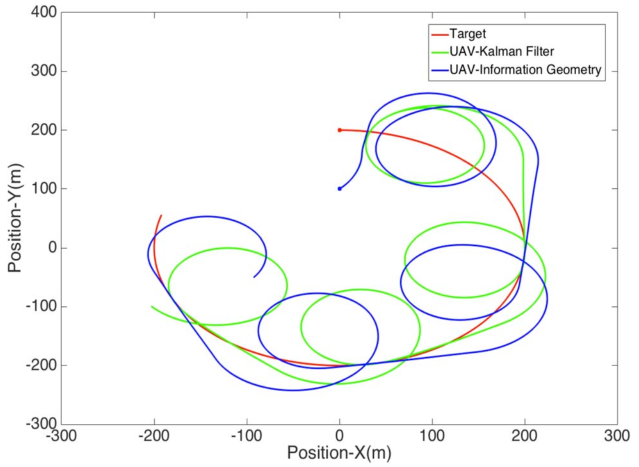

Fig. 7: UAV tracking a target on a circle path, $\sigma_r = 10^{-3}\ m$, $\sigma_\varphi = 10^{-5}\pi\ rad$

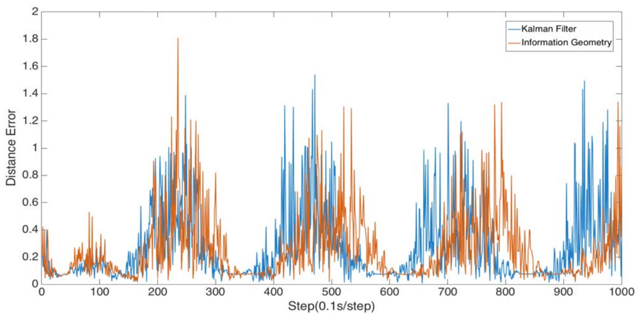

Fig. 8: Distance error of tracking a target on a circle path, $\sigma_r = 10^{-3}\ m$, $\sigma_\varphi = 10^{-5}\pi\ rad$

variance to predict the tracking error, it has a linear process. During this process, the emergence of the first order truncation error leads to the inaccurate prediction. On the contrary, if using the cumulative information to characterize the tracking error, there not exists such problem. Meanwhile, the improvement of the measuring accuracy can reduce the impact of the truncation error.

We also calculated the average distance error and average computation time for each simulation, the results in the table below. We summate these distance errors over the simulation runtime, and the mean of these errors (from each Monte Carlo run) is called the average target-location error. The average computation time is computed as follows. At every step of the simulation runtime, we record the time costs of

Table 2: The results of simulation experiments

No.	ACT		ADE	
	KF	IG	KF	IG
1	$0.5978\ s$	$0.1017\ s$	$87.6183\ m$	$37.3619\ m$
2	$0.3734\ s$	$0.1777\ s$	$0.2914\ m$	$0.2652\ m$
3	$0.5417\ s$	$0.1093\ s$	$68.2240\ m$	$20.4651\ m$
4	$0.3997\ s$	$0.1465\ s$	$0.2704\ m$	$0.2564\ m$

[a] ACT is short for the average computation time
[b] ADE is short for the average distance error
[c] KF is short for Kalman filter
[d] IG is short for information geometry

researching the optimal strategy, and the mean of these costs (from each Monte Carlo run) is called the average computation cost.

Since the presence of the matrix inversion calculation in Kalman filter, it costs more time than our method. In other words, the proposed algorithm in this paper has about 50% decrease in computation time than ones resulted using conventional methods.

5 Conclusion and Future Work

In this paper, we have shown a new UAV action decision-making method to track a mobile ground target. Our method uses the theory of POMDP select an optimal sequence of UAV control actions to follow the surface target while maximizing accumulative information of the target from the TWS radar. The simulation results show that our method has a better tracking performance and less computation time cost.

In the future, we tend to develop more accurate models for target motion and use them for intelligent UAV tracking. Besides, we attempt to apply this method to multi-UAVs tracking multi-targets.

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