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Bearing-only circumnavigation control of the multi-agent system around a moving target

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Abstract: The bearing-based coordinated circumnavigation control for networked multi-agent systems with only bearing measurements in the presence of moving target is studied. First, a distributed algorithm is proposed to estimate the target's velocity and the distances between the agents and the target, based on the velocity and the bearing information of the local network. Then, a distributed circumnavigation algorithm is designed to drive the agents to circumnavigate around a moving target at a desired distance. The stability of the proposed control algorithm is proven by employing the newly developed bearing rigidity theory. Finally, the simulation experiment is performed based on the Gazebo simulator to illustrate the effectiveness of the proposed circumnavigation control law.

Nomenclature

I_{d}	identity matrix of dimension d
R_{b}	bearing rigidity matrix
p_i	position of agent i
H	incidence matrix of a graph
P	orthogonal projection operator
$\mathcal{N}_i$	set of agent i's neighbouring agents
$\otimes$	Kronecker product
$\mathbf{1}_n$	n-dimension column vector whose elements are all 1
$\lVert \cdot \rVert$	Euclidian norm of a vector or the spectral norm of a matrix
n	number of agents
r_{d}	desired radius of the predefined polygon configuration for the multi-agent system
$\lambda_2(A)$	second smallest eigenvalue of a positive semi-definite matrix A
Null(.)	kernel space of a matrix
dim(.)	dimension of a space

1 Introduction

In recent years, applications like the surveillance and target assessment [1, 2], satellite formation flying [3], and source seeking [4], are of significant potential in civilian and military domains. Thus a variety of methods are proposed to execute these tasks. Among various scenarios, a typical way is to monitor a single or a group of targets of interest by circumnavigating them on a circular trajectory of the prescribed radius. To solve such a circumnavigation problem, one needs to design a control law to drive the agents to enclose the target and then circle around it in an agreed sense with a prescribed radius [5, 6].

The study of the circumnavigation control initially stems from the N-bugs problem [7]. Inspired by the ‘bugs’ problem from mathematics, Marshall *et al.* [8] studies the geometry for formations of multi-vehicle systems under cyclic pursuit. The authors study the general circumnavigation for a team of unicycle-type agents using a similar cyclic pursuit strategy [9, 10]. Some other prior works dealing with the circumnavigation control include [11–13] and the references therein.

According to the availability of the target information, the circumnavigation problem can be divided into two types: the cooperative circumnavigation and non-cooperative circumnavigation. The cooperative circumnavigation corresponds to the case where the position of the target is available to the circumnavigating agents. This case is relatively easy to accomplish since the position of the target is known and a relatively simple control law can drive the agents to rotate around the target [14]. In practical scenarios, however, the position of the target is often inaccessible, especially in some adversarial environments, such as some GPS-denied environments due to jamming or spoofing. In order to tackle these issues, the non-cooperative circumnavigation problem is raised and intensively studied in recent years. Actually, the non-cooperative circumnavigation problem is a dual control problem [15] in which the identification of the target's position and the control of agents should be achieved simultaneously.

According to different kinds of implemented sensors, the dual problem of target estimation and circumnavigation can be mainly categorised into three types: based on the relative position measurement, the range-only, and the bearing-only. In the relative position measurement based case, the agents can detect the relative position with respect to the target as well as parts of other agents, which has been previously studied in [16, 17]. However, the relative position of the target is often not easy to obtain. It is preferred to find a local circumnavigation algorithm that depends on less sensor knowledge. Therefore, the range-only based [18, 19] and bearing-based control methods [20–22] have become a hot research field and many related works have been reported. Compared with relative position measurement, the relative range or the relative bearing measurement is obviously easier to acquire and less demanding in terms of the capacity of transmission and storage, which is of vital importance in platforms whose communication and computation capacities are limited, such as the unmanned aerial vehicles (UAVs). As for the circumnavigation control with bearing information, an estimator for localising an either stationary or slow moving single target is proposed in [21], and a corresponding controller is designed to force the agent to move on a circular trajectory around the centre of the target. The algorithm in [21] is expanded to the case of multiple targets in [23]. Deghat *et al.* [21, 23] address the problem of circumnavigation for a single agent. It should be stressed that the performance of a single agent, such as the adaptability to the variations of environments or the unexpected incidents, is generally inferior to the multi-agent system.

Despite the advantages in the robustness and invulnerability to unexpected events, a multi-agent system also imposes a much higher difficulty in information transmission compared with a single agent system. Meanwhile, it is also a tricky problem for coordinating the different agents during the circumnavigation, such as avoiding collision between agents. In order to tackle these issues, some previous work has been published. For instance, the range sensor information is utilised in [14] to develop a control strategy such that a robot can orbit around a stationary target at a desired distance, larger than the maximum communication range between the robot and the target. The authors in [24] use the bearing information to develop a controller to drive a team of robots to circumnavigate an arbitrary distribution of target points at a desired radius from the targets. Some distance information is required in [24]. In our previous work [25], a similar multi-agent encirclement problem is considered, in which the multiple agents coordinate to enclose a target and finally capture it [26, 27]. However, both the relative position between agents and the target's velocity are utilised in [25]. To the best of our knowledge, there still do no exist circumnavigation algorithms without demanding position information. In previous works such as [20–22], the agents' global position as well as the bearing information is required. However, without the aid of external positioning methods like GPS, it is hard to get access to the global positions of the multiple agents in a shared coordinate.

In this paper, we address the bearing-based coordinated circumnavigation control problem for networked multi-agent systems with only bearing measurements in the presence of a moving target. The objective of this paper is to design a distributed controller which only uses the onboard bearing and velocity information. Compared to the position information, the bearing information and velocity information are easier to obtain and do not need a shared origin point. The main contribution of this paper is that we propose a bearing-based coordinated circumnavigation control scheme, based on the networked agents' coordinated estimation algorithm for the velocity of the target as well as the distance towards the target. In particular, an estimation algorithm is first designed to estimate the target's velocity and the distance between the agent and the target, based on the local velocity and bearing information. The topology condition for the observability of the target's velocity is further provided. On the basis of the proposed estimation algorithm, a circumnavigation algorithm is proposed to track a moving target coordinately without the aid of the position information. Moreover, the stability of the proposed algorithm is proven by employing the newly developed bearing rigidity theory. The effectiveness of the proposed algorithm is verified in a simulation environment called Gazebo by using five simulated UAVs and a moving target.

The rest of this paper is organised as follows. The circumnavigation control problem is formulated in Section 2. Section 3 presents the distributed estimation algorithm and Section 4 designs the distributed circumnavigation control law. Simulation results are presented in Section 5. Conclusions and future works are given in Section 6.

2 Background and problem statement

2.1 Bearing rigidity theory

For a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, the vertex set is denoted as $\mathscr{V} = \{v_1, \dots, v_n\}$ and the edge set is denoted as $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ with $m = |\mathscr{E}|$. The Cartesian coordination of vertex v_i in graph $\mathscr{G}$ is denoted as $p_i \in \mathbb{R}^d, (d \geq 2)$. Then vector $p = [(p_1)^\mathrm{T}, \dots, (p_n)^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{nd}$ is called a configuration of graph $\mathscr{G}$ in $\mathbb{R}^d$. A framework in $\mathbb{R}^d$, denoted as $\mathscr{G}(p)$, is a combination of a graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ and a configuration p. In graph $\mathscr{G}(p)$, we define the edge vector ϕ_k and the bearing vector g_k for the kth edge composed of the tail node i and the head node j as follows:

$$\phi_k \triangleq p_j - p_i,\ g_k \triangleq \phi_k / \parallel \phi_k \parallel, \quad \forall k \in 1, \dots, m. \tag{1}$$

In the bearing rigidity theory, two vectors represent the same bearing as long as they are parallel or collinear. In order to verify whether two vectors are parallel or collinear, an orthogonal projection operator $P\colon \mathbb{R}^d \to \mathbb{R}^{d \times d}$ is defined as follows:

$$P(\vec{v}) \triangleq I_\mathrm{d} - \frac{\vec{v}}{\parallel \vec{v} \parallel} \frac{\vec{v}^\mathrm{T}}{\parallel \vec{v} \parallel},$$

where $\vec{v} \in \mathbb{R}^d (d \geq 2)$ is a non-zero vector. The vector $\vec{v}$ will be projected onto its orthogonal compliment after the operation P. For notational simplicity, define $P_{\vec{v}} = P(\vec{v})$. Two non-zero vectors $\vec{v}, \vec{u} \in \mathbb{R}^d$ are parallel or collinear if and only if $P_{\vec{v}} \vec{u} = 0$ (or equivalently $P_{\vec{u}} \vec{v} = 0$) [28]. It can be easily verified that $P_{\vec{v}}$ satisfies $P_{\vec{v}}^2 = P_{\vec{v}}$ and $P_{\vec{v}}^\mathrm{T} = P_{\vec{v}}$.

Then, in order to describe all the bearings in the framework, a bearing function: $F_\mathrm{B}\colon \mathbb{R}^{dn} \to \mathbb{R}^{dm}$ is defined as follows:

$$F_\mathrm{B}(p) \triangleq [g_1^\mathrm{T}, \dots, g_m^\mathrm{T}] \in \mathbb{R}^{dm}. \tag{2}$$

As revealed in (2), the bearing function $F_\mathrm{B}(p)$ describes all the bearings in the framework. Based on $F_\mathrm{B}(p)$, a special matrix $R_\mathrm{b}(p) \in \mathbb{R}^{dm \times dn}$ called *bearing rigidity matrix* [28] is defined as the Jacobian of the bearing function:

$$R_\mathrm{b}(p) \triangleq \frac{\partial F_\mathrm{B}(p)}{\partial p} = \mathrm{diag}\left(\frac{P_{g_k}}{\parallel \phi_k \parallel}\right)(H \otimes I_\mathrm{d}), \tag{3}$$

where the incidence matrix $H \in \mathbb{R}^{m \times n}$ represents a $\{0, \pm 1\}$-matrix for a directed graph whose rows are indexed by the edges and columns by the vertices. The element of H is defined as follows:

$$[H]_{ki} = \begin{cases} 1 & \text{if vertex} i \text{is the head of edge } k, \\ -1 & \text{if vertex } i \text{ is the tail of edge } k, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Let δp be a variation of the configuration p. If $R(p)\delta p = 0$, then δp is called an *infinitesimal bearing motion* of $\mathscr{G}(p)$. Then, the definition of *infinitesimal bearing rigidity* mentioned in [28] is given as follows:

Definition 1 (Infinitesimal Bearing Rigidity): A framework is infinitesimally bearing rigid if all the infinitesimal bearing motions are trivial.

According to the bearing rigidity theory, an infinitesimally bearing rigid framework can be uniquely determined up to a translational and a scaling factor [29]. The infinitesimal bearing rigidity is determined by the bearing rigidity matrix $R_\mathrm{b}(p)$ and the following statements are equivalent:

(a) $\mathscr{G}$ is infinitesimally bearing rigid;
(b) $\mathrm{rank}(R(p)) = dn - d - 1$;
(c) $\mathrm{Null}(R(p)) = \mathrm{span}\{\mathbf{1}_n \otimes I_\mathrm{d}, p\} = \mathrm{span}\{\mathbf{1}_n \otimes I_\mathrm{d}, p - \mathbf{1}_n \otimes \overline{p}\}$, where $\overline{p} = (\mathbf{1}_n \otimes I_\mathrm{d})^\mathrm{T} p / n$ is the centroid of $\{p_i\}_{i \in \mathscr{V}}$.

2.2 Problem formulation

Inspired by the circumnavigation technique that bottlenose dolphins use to entrap fishes, we are trying to explore a technique to achieve the multi-agent circumnavigation around the target just like dolphins. In this section, we will try to describe this process in a mathematical manner.

Considering a group of n agents and each agent is described as a single integrator model:

$$\dot{p}_i = \mu_i, \quad i = 1, 2, \dots, n, \tag{5}$$

where $p_i \in \mathbb{R}^d$ and $\mu_i \in \mathbb{R}^d$ represent the position and the control input of agent i, respectively. Let $p = [p_1^\mathrm{T}, \dots, p_n^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{nd}$ and

$\mu = [\mu_1^{\mathrm{T}}, \ldots, \mu_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nd}$. Further we define the centre $c(t)$ and the scale of the multi-agent system $s(t)$ as follows:

$$c(t) \triangleq \frac{1}{n}\sum_{i=1}^{n} p_i = \frac{1}{n}(\mathbf{1}_n \otimes I_{\mathrm{d}})^{\mathrm{T}} p, \tag{6}$$

$$s(t) \triangleq \sqrt{\frac{1}{n}\sum_{i=1}^{n} \| p_i - c(t) \|^2} = \frac{1}{\sqrt{n}} \| p - \mathbf{1}_n \otimes c(t) \| . \tag{7}$$

The dynamics, as well as the position of the target, is unknown to the agents. Each agent can only obtain the bearing information with respect to the target. In this paper, we need the following assumption.

Assumption 1: All agents cannot acquire their global positions directly, but can only obtain its own velocity information and the relative bearing information with respect to their neighbour agents and the target T. Besides, neighbouring agents can exchange their sensor information via communication.

The communication topology among the agents is modelled as a graph $\mathscr{G}$. Each agent is represented by a node in $\mathscr{G}$ while each communication link between two agents is represented by an edge. Based on graph $\mathscr{G}$, we can obtain the augmented graph $\hat{\mathscr{G}}$ combined with target T, as illustrated in Fig. 1. The target is also modelled as a node and the information flow between the agents and the target is modelled as edges. The corresponding node set of augmented graph is $\hat{\mathscr{G}}$ is $\hat{\mathscr{V}} = \{v_1, \ldots, v_n, v_{n+1}\}$ and the edge set is $\hat{\mathscr{E}} = \{e_1, \ldots, e_m, e_{m+1}, \ldots, e_{m+n}\}$. After assigning an arbitrary direction for each edge in the graph, the edge vectors, and bearing vectors can be denoted as $\phi = [\phi_1^{\mathrm{T}}, \ldots, \phi_m^{\mathrm{T}}]^{\mathrm{T}}$ and $g = [g_1^{\mathrm{T}}, \ldots, g_m^{\mathrm{T}}]^{\mathrm{T}}$. A pair $(i, j) \in \mathscr{E}$ if and only if the relative bearing information of agent i to agent j, namely g_{ij}, can be acquired by agent i. The agents having a communication link with agent i are called the neighbours of agent i, denoted as $\mathscr{N}_i \triangleq \{j \in \mathscr{V} : (i, j) \in \mathscr{E}\}$. Thus, at time t, the set of relative bearing information of agent i with respect to its neighbours can be denoted as $g_{ij}(t) = (p_j - p_i)/\| p_j - p_i \|$. Considering a moving target T, its position and velocity are represented as p_T and $\dot{p}_T$, respectively. If target T can be detected by agent i, the relative bearing of agent i to target T can be represented as $g_{iT} = (p_T - p_i)/\| p_T - p_i \|$.

In this paper, the objective is to design distributed control laws to achieve the following goals:

- The centre of the formation $c(t)$ is guaranteed to converge to the position of the target $p_T(t)$;
- The agents in the formation circumnavigate around the target at the same angular speed $\hat{\omega}$;
- The angle of any two neighbouring agents of the target ψ converge to an identical constant $\overline{\psi}$, namely $\psi_i \to \overline{\psi}$;
- Every agent in the formation keep the desired distance r_{d} with respect to the target.

Similar to our previous work in [25], the geometry of the multi-agent system satisfying the control goals should be a regular polygon as illustrated in Fig. 1. Every agent can only sense the bearing relative to two neighbouring agents and the target, and have communication links with its two neighbours. Let g_{ij}^* denote the desired bearing between agent i and agent j and g_{iT}^* denote the desired bearing between agent i and target T. Then, by predefining a desired regular polygon geometry, the desired g_{ij}^* and g_{iT}^* can be obtained. On the other hand, the augmented graph $\hat{\mathscr{G}}$ constrained by g_{ij}^* and g_{iT}^* is infinitesimally bearing rigid according to the definition of bearing rigidity in Definition 1. When the bearing constraints of every pair of neighbouring nodes are given, the geometry configuration is determined up to a translational and a scaling factor, namely, the target's position and the formation's scale. Therefore, the multi-agent circumnavigation control problem can be transferred into the following desired formation problem:

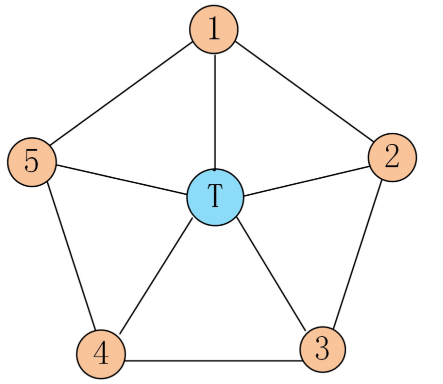

Fig. 1 *Augmented graph $\hat{\mathscr{G}}$ with six agents and target T*

Definition 2 (Desired formation): If $\mathscr{G}(p(t))$ is the desired formation graph, then it should satisfy following constraints:

1. Bearing constraint:$g_{ij}//g_{ij}^*$, $g_{iT}//g_{iT}^*$,
2. Formation's centre constraint:$\lim_{t \to \infty} c(t) \to p_T(t)$,
3. Formation's scale constraint:$\lim_{t \to \infty} s(t) = r_{\mathrm{d}}$,

where // represents that two non-zero vectors are parallel.

In order to drive the whole system to rotate around the target, the desired bearing $g_{ij}^*(t)$ and $g_{iT}^*(t)$ should be time-varying functions and they can be designed as follows.

First, the agents are assigned on the unit circle evenly, and then the initial desired inter-neighbour bearing among agents $g_{ij}^*(0)$ and initial desired bearing between agents and target $g_{iT}^*(0)$ can be calculated according to (1). Later, by introducing a rotation matrix $R(\theta)$ as

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

the desired bearings at time t are described as:

$$g_{ij}^*(t) = R(\theta) g_{ij}^*(0), \tag{8}$$

$$g_{iT}^*(t) = R(\theta) g_{iT}^*(0), \tag{9}$$

where $\theta = \overline{w}t$ and $\overline{w}$ is the given angular velocity of the formation. If $\overline{w} > 0$, the formation rotates anticlockwise. Otherwise, the formation rotates clockwise.

3 Distributed estimation algorithms

In this section, a distributed algorithm will be proposed to estimate the target's speed, based on the available local information, namely, the agent's own velocity, the target's bearing, neighbouring agents' velocities and their relative bearing with respect to the target.

3.1 Estimation of the distance with respect to the target

According to the assumption, the accessible information from onboard sensors for each agent is only the bearing information and its own velocity. In order to estimate the target's velocity, the information about the distances between the agents and the target is needed. Therefore, an algorithm for each agent to estimate its distance with respect to the target is firstly by cooperating with agent 1. As illustrated in Fig. 2, agent 1, agent 2, and the target form a triangle. According to the well-known sine theorem, we have the following equation:

$$\frac{d_{2T}}{\sin\theta_{2T}} = \frac{d_{12}}{\sin\theta_{12}}, \tag{10}$$

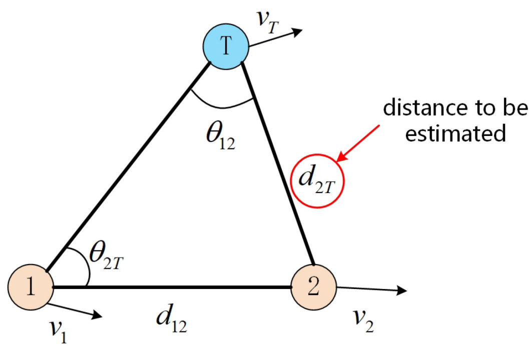

Fig. 2 *Agent estimates its distance with respect to the target with the aid of a neighbouring agent*

where d_{12} denotes the distance between agents 1 and 2, θ_{12} denotes the angle between the edge from agent 1 to the target and the edge from agent 2 to the target, and θ_{2T} is defined accordingly. Let v_1, v_2, and v_3 denote the speed vectors of the agent 1, agent 2, and the target T, respectively.

It can be seen from (10) that for agent 2, its distance to agent 1 is needed in order to estimate its distance to the target. However, based on the assumption given in Section 2, agent 2 can only get access to its relative bearing g_{21} with respect to agent 1 as well as their velocities. Therefore, the following part will discuss how to estimate the mutual distance between any two agents using the velocity and bearing information.

For any vector $\vec{v} \in \mathscr{R}^d$, the time derivative of the unit vector $\vec{v}/\parallel \vec{v} \parallel$ is obtained as follows:

$$\begin{aligned}\frac{\mathrm{d}(\vec{v}/\parallel \vec{v} \parallel)}{\mathrm{d}t} &= \frac{1}{\parallel \vec{v} \parallel^2}\left(\parallel \vec{v} \parallel \dot{\vec{v}} - \frac{\vec{v}^{\mathrm{T}}\dot{\vec{v}}}{\parallel \vec{v} \parallel}\vec{v}\right) \\ &= \frac{1}{\parallel \vec{v} \parallel}\left(I_{\mathrm{d}} - \frac{\vec{v}}{\parallel \vec{v} \parallel}\frac{\vec{v}^{\mathrm{T}}}{\parallel \vec{v} \parallel}\right)\dot{\vec{v}} = \frac{1}{\parallel \vec{v} \parallel}P_{\vec{v}}\dot{\vec{v}}.\end{aligned} \tag{11}$$

Let $\vec{v} = p_2 - p_1$, then $\dot{\vec{v}} = \boldsymbol{v}_2 - \boldsymbol{v}_1$, where $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ are the velocity of the agent 1 and the agent 2, respectively. Further, it is obvious that $d_{12} = \parallel \vec{v} \parallel$ and $g_{21} = \vec{v}/\parallel \vec{v} \parallel$. Thus from (11) we can obtain the distance d_{12} between agent 1 and agent 2 as:

$$d_{12} = \frac{\parallel P_{g_{21}}(\boldsymbol{v}_2 - \boldsymbol{v}_1) \parallel}{\parallel \dot{g}_{21} \parallel}. \tag{12}$$

It has been proved in [29] that for the angle $\theta \in [0, \pi]$ between two non-zero vectors $x, y \in \mathscr{R}^d$ (i.e. $x^{\mathrm{T}}y = \parallel x \parallel \parallel y \parallel \cos\theta$), $\parallel P_x - P_y \parallel = \sin\theta$ always exists. Then, $\sin\theta_{2T}$ and $\sin\theta_{12}$ can be rewritten as

$$\sin\theta_{2T} = \parallel P_{g_{1T}} - P_{g_{12}} \parallel, \quad \sin\theta_{12} = \parallel P_{g_{1T}} - P_{g_{2T}} \parallel. \tag{13}$$

Combined with (10), (12), and (13), the estimation for d_{2T} is obtained as follows:

$$d_{2T} = \frac{\parallel P_{g_{1T}} - P_{g_{12}} \parallel}{\parallel P_{g_{1T}} - P_{g_{2T}} \parallel}\frac{\parallel P_{g_{21}}(\boldsymbol{v}_2 - \boldsymbol{v}_1) \parallel}{\parallel \dot{g}_{21} \parallel}.$$

Then, considering that each agent may have multiple connections with other agents, any agent i can estimate the distance to the target d_{iT}, by cooperating with any other neighbouring agent simultaneously. Therefore, in order to reduce the error caused by the noise, the final estimation of d_{iT} is the average of the obtained $|\mathscr{N}_i|$ distances to the target with the aid of $|\mathscr{N}_i|$ other agents. Therefore, the agent i's estimation of the distance d_{iT} between itself and the target can be obtained by

$$d_{iT} = \frac{1}{|\mathscr{N}_i|}\sum_{j \in \mathscr{N}_i}\frac{\parallel P_{g_{iT}} - P_{g_{ij}} \parallel}{\parallel P_{g_{jT}} - P_{g_{iT}} \parallel}\frac{\parallel P_{g_{ij}}(\boldsymbol{v}_i - \boldsymbol{v}_j) \parallel}{\parallel \dot{g}_{ij} \parallel}. \tag{14}$$

3.2 Estimation of target's velocity

On the basis of the proposed estimation algorithm for the distance between each agent and the target, the estimation of the target's speed will be further discussed. Firstly, for arbitrary agent i, according to (11), we have

$$\dot{g}_{iT} = \frac{P_{iT}(\boldsymbol{v}_T - \boldsymbol{v}_i)}{d_{iT}}. \tag{15}$$

Let $\tilde{\mathscr{N}}_i$ denote the set composed of agent i and its neighbours, i.e. $\tilde{\mathscr{N}}_i \triangleq \{i, \mathscr{N}_i\}$. Then, on the basis of (15), the following equation is always established for any agent i:

$$\sum_{j \in \mathscr{N}_i}\dot{g}_{jT} = \sum_{j \in \mathscr{N}_i}\frac{P_{jT}(\boldsymbol{v}_T - \boldsymbol{v}_j)}{d_{jT}}.$$

Further, we obtain:

$$\left(\sum_{j \in \mathscr{N}_i}\frac{P_{g_{jT}}}{d_{jT}}\right)\boldsymbol{v}_T = \sum_{j \in \mathscr{N}_i}\left(\frac{P_{g_{jT}}\boldsymbol{v}_j}{d_{jT}} + \dot{g}_{jT}\right). \tag{16}$$

For agent i, obviously its estimation for the target's speed will have an analytical solution if and only if the matrix $\sum_{j \in \tilde{\mathscr{N}}_i}(P_{g_{jT}}/d_{jT})$ is non-singular. If the matrix $\sum_{j \in \tilde{\mathscr{N}}_i}(P_{g_{jT}}/d_{jT})$ is non-singular, then we call the target's speed is observable for agent i. Then, a theorem is summarised as follows:

Theorem 1: In a multi-agent network $\mathscr{G}_p$, for agent i, the target T's speed is observable if and only if the matrix $\sum_{j \in \tilde{\mathscr{N}}_i}(P_{g_{jT}}/d_{jT})$ is non-singular. If the target T's speed is observable for agent i, then its estimation $\hat{v}_{Ti}$ can be expressed as

$$\hat{\boldsymbol{v}}_{Ti} = \left(\sum_{j \in \mathscr{N}_i}\frac{P_{g_{jT}}}{d_{jT}}\right)^{-1}\sum_{j \in \mathscr{N}_i}\left(\frac{P_{g_{jT}}\boldsymbol{v}_j}{d_{jT}} + \dot{g}_{jT}\right), \tag{17}$$

where the distance d_{jT} between agent j and target T can be estimated as

$$d_{jT} = \frac{1}{|\mathscr{N}_j|}\sum_{k \in \mathscr{N}_j}\frac{\parallel P_{g_{jT}} - P_{g_{jk}} \parallel}{\parallel P_{g_{kt}} - P_{g_{jt}} \parallel}\frac{\parallel P_{g_{jk}}(\boldsymbol{v}_j - \boldsymbol{v}_k) \parallel}{\parallel \dot{g}_{jk} \parallel}.$$

One may wonder what condition is needed for the non-singularity of the matrix $\sum_{j \in \tilde{\mathscr{N}}_i}(P_{g_{jt}}/d_{jt})$. In order to answer this question, we define a special matrix as follows:

$$[\mathscr{B}(\mathscr{G})]_{ij} = \begin{cases}\mathbf{0}_{d \times d} & i \neq j, (i,j) \notin \mathscr{E}, \\ -\dfrac{P_{g_{ij}}}{d_{ij}} & (i,j) \in \mathscr{E}, \\ \displaystyle\sum_{k \in \mathscr{N}_i}\frac{P_{g_{ik}}}{d_{ik}} & i = j, i \in \mathscr{V}.\end{cases} \tag{18}$$

Comparing (18) with (3), we easily observe that $\mathscr{B} = R_{\mathrm{b}}^{\mathrm{T}}R_{\mathrm{b}}$. Therefore, the matrix $\mathscr{B}$ has the same rank and kernel space as the matrix R_{b}. Let δp represent an infinitesimal bearing motion under the framework $\mathscr{G}_p$. According to the definition of infinitesimal bearing motion, we have $R_{\mathrm{b}}(p)\delta p = 0$ and further obtain $\mathscr{B}(\mathscr{G})\delta p = 0$, i.e. $\delta p \in \mathrm{Null}(\mathscr{B}(\mathscr{G}))$. It is not difficult to observe the fact that the matrix $\mathscr{B}$ has a structure and properties similar to those of the traditional Laplacian matrix. In addition, the matrix $\mathscr{B}$ contains the bearing information among agents and therefore is named *Bearing Laplacian Matrix*. As the circumnavigating agents bear known sensor information, they are termed as *anchor nodes*.

According to the partition of the anchor nodes and the target node, the matrix $\mathscr{B}(\mathscr{G})$ can be partitioned into the following form:

$$\mathscr{B}(\mathscr{G}) = \begin{bmatrix} \mathscr{B}_{aa} & \mathscr{B}_{aT} \\ \mathscr{B}_{Ta} & \mathscr{B}_{TT} \end{bmatrix},$$

where $\mathscr{B}_{aa} \in \mathbb{R}^{n_a \times n_a}$, $\mathscr{B}_{aT} = \mathscr{B}_{Ta} \in \mathbb{R}^{n_a \times n_T}$, $\mathscr{B}_{TT} \in \mathbb{R}^{n_T \times n_T}$, and n_a and n_T are the numbers of anchor nodes and target nodes, respectively.

Let p' denote the vector composed of target T, agent i and its connected agents, namely,

$$p' \triangleq [(p_{j_1})^\mathrm{T}, (p_{j_2})^\mathrm{T}, \ldots, (p_{j_s})^\mathrm{T}, (p_i)^\mathrm{T}, (p_T)^\mathrm{T}]^\mathrm{T},$$

where $j_s \in \mathscr{N}_i$ and $\mathscr{G}(p')$ denote the subnetwork configured by p' as well as its connection relationship. According to the definition of the bearing Laplacian matrix, matrix $\sum_{j \in \bar{\mathscr{N}}_i}(P_{g_{jT}}/d_{jT})$ is actually the submatrix $\mathscr{B}_{TT}$ in $\mathscr{B}(\mathscr{G}(p'))$. Therefore, the proof of the singularity of the matrix $\sum_{j \in \bar{\mathscr{N}}_i}(P_{g_{jT}}/d_{jT})$ equals the proof of the singularity of the submatrix $\mathscr{B}_{TT}$. Let $\delta p'$ denote an infinitesimal bearing motion under the framework $\mathscr{G}(p')$, and then we have the following theorem:

Theorem 2: In the subnetwork $\mathscr{G}(p')$ composed of target T, agent i and its neighbour agents, the target T's velocity is observable for agent i if and only if an arbitrary infinitesimal bearing motion $\delta p'$ in $\mathscr{G}(p')$ involves at least one anchor node. That is, for any non-zero infinitesimal bearing motion

$$\delta p' = \begin{bmatrix} \delta p'_a \\ \delta p'_T \end{bmatrix} \in \mathrm{Null}(\mathscr{B}(\mathscr{G}')), \tag{19}$$

the vector $\delta p'_a$ corresponding to anchor nodes must be non-zero.

Proof: We only need to show that $\mathscr{B}_{TT}$ is singular if and only if there exists non-zero vector $\delta p' \in \mathrm{Null}(\mathscr{B}(\mathscr{G}'))$ with $\delta p'_a = 0$.

(Sufficiency): Suppose that there exists $\delta p' \in \mathrm{Null}(\mathscr{B}(\mathscr{G}))$ satisfying $\delta p'_a = 0$ and $\delta p'_T \neq 0$. Then $(\delta p'_T)^\mathrm{T} \mathscr{B}_{TT} \delta p'_T = (\delta p')^\mathrm{T} \mathscr{B}(\mathscr{G}') \delta p' = 0$, which implies that $\mathscr{B}_{TT}$ is singular.

(Necessity:) Suppose that $\mathscr{B}_{TT}$ is singular. Then there exists non-zero vector $\vec{v} \in \mathbb{R}^{dn_T}$ such that $B_{TT}\vec{v} = 0$. Let $\delta p' = [0, \vec{v}^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{dn}$. Then $(\delta p')^\mathrm{T} \mathscr{B}(\mathscr{G}') \delta p' = \vec{v}^\mathrm{T} \mathscr{B}_{TT} \vec{v} = 0$. Hence $\delta p' \in \mathrm{Null}(\mathscr{B}(\mathscr{G}'))$ and $\delta p'_a = 0$. □

From Theorem 2, it can be concluded that the matrix $\sum_{j \in \bar{\mathscr{N}}_i}(P_{g_{jt}}/d_{jt})$ is non-singular if and only if an arbitrary infinitesimal bearing motion $\delta p'$ in $\mathscr{G}(p')$ involves at least one agent node. Then, on the basis of Theorem 2, we can further obtain the lower bound of the agent's number for the observability of the target's velocity:

Lemma 1: In a subnetwork $\mathscr{G}(p')$ composed of target T, agent i and its neighbour agents, the target's velocity is observable for agent i if and only if agent i has at least one neighbouring agent.

Proof (Sufficiency): From Theorem 2, for any non-zero vector infinitesimal bearing motion $\delta p'$ defined as in (19), the target's velocity is observable if and only if $\delta p'_a$ is non-zero. It is not difficult to verify that the subnetwork $\mathscr{G}(p')$ is infinitesimally bearing rigid, which implies $\mathrm{Null}(\mathscr{B}(\mathscr{G}')) = \mathrm{span}\{\mathbf{1}_{(n_a+1)} \otimes I_\mathrm{d}, p'\}$. Define the coefficient vector $K \in \mathbb{R}^d$ as

$$K \triangleq [k_1, k_2, \ldots, k_d]^\mathrm{T},$$

and δ'_p can be represented as

$$\delta'_p = \mathbf{1}_{(n_a+1)} \otimes K + k_{d+1} p',$$

where $k_1, k_2, \ldots, k_d, k_{d+1} \in \mathbb{R}$. Then define

$$p'' \triangleq [(p_{j_1})^\mathrm{T}, (p_{j_2})^\mathrm{T}, \ldots, (p_{j_s})^\mathrm{T}, (p_i)^\mathrm{T}]^\mathrm{T},$$

where $j_s \in \mathscr{N}_i$. According to the definition in (19), we have $\delta p'_a = \mathbf{1}_{n_a} \otimes K + k_{d+1} p''$. As $\delta p'$ is non-zero, $k_1, k_2, \ldots, k_{d+1}$ cannot be all non-zero. Therefore, $\delta p'_a$ is a zero vector only if $p'' = (\mathbf{1}_{n_a} \otimes K)/k_{d+1}$. This condition implies that the positions of all anchor nodes are the same, which is impossible if there are at least two different anchor nodes. Therefore, $\delta p'_a$ must be non-zero if there are more than one anchor nodes.

(Necessity): Let $l = \dim(\mathrm{Null}(\mathscr{B}(\mathscr{G}')))$ and $N \in \mathbb{R}^{dn \times l}$ be a basis matrix of $\mathrm{Null}(\mathscr{B})$ which means $\mathrm{Range}(N) = \mathrm{Null}(\mathscr{B}(\mathscr{G}'))$. Then any non-zero $\delta p \in \mathrm{Null}(\mathscr{B}(\mathscr{G}'))$ can be expressed as $\delta p = Ny$, where $y \in \mathbb{R}^l$ and $y \neq 0$. Partition N and express Ny as

$$\delta p = Ny = \begin{bmatrix} N_a y \\ N_T y \end{bmatrix},$$

where $N_a \in \mathbb{R}^{dn_a \times l}$. The target T's velocity is observable if and only if $N_a y \neq 0, \forall y \in \mathbb{R}^l, y \neq 0$. As a result, the matrix N_a must have full column rank, which requires N_a to be a matrix with $d \times n_a > l = \dim(\mathrm{Null}(\mathscr{B}))$. Since $\dim(\mathrm{Null}(\mathscr{B})) = \dim(\mathrm{Null}(R_\mathrm{b})) \geq (d+1)$, we have $n_a \geq \dim(\mathrm{Null}(\mathscr{B}))/d \geq (d+1)/d > 1$. □

Remark 1: Lemma 1 reveals that the target T is observable for any agent aligned with one or more neighbouring agents. Obviously, any agent in our designed communication topology has at least one neighbours. That is, the target T's velocity is observable for every agent.

4 Circumnavigation control law design and stability analysis

4.1 Control law design

In [28], the authors propose a bearing-only formation control law as follows:

$$\mu_i = -\sum_{j \in \mathscr{N}_i} P_{g_{ij}(t)} g^*_{ij}. \tag{20}$$

The agents controlled by law (20) will be rendered to achieve desired relative bearing while keeping their formation scale and centroid unvaried [28]. Inspired by [28], we modify the control law (20) as follows:

$$\mu_i = -k_\alpha \sum_{j \in \mathscr{N}_i} P_{g_{ij}}(t) g^*_{ij} - k_\beta P_{g_{iT}}(t) g^*_{iT}. \tag{21}$$

Compared with the control law (20), the control law (21) adds an item $-k_\beta P_{it}(t) g^*_{it}$ in order to ensure that the centroid of the formation converges to the target. In fact, the control law (21) has a clear geometric interpretation as illustrated in Fig 3. The two components in (21) correspond to the bearing errors with the neighbouring agents and the target T, respectively. In addition, the control components (the red arrows in Fig. 3) are actually the projection of the actual bearing in the perpendicular direction of the desired bearing. Under the control law (21), the bearing errors will be reduced gradually.

It seems that the control law (21) can satisfy all the requirements of circumnavigation control. However, the control law (21) has two main problems. First, the control law (21) is mainly aimed at the static target. When the target is moving, the control performance will not be as good as expected. Second, the formation's scale will enlarge when the control law (21) is directly applied in a discrete-time system. In a continuous-time system, the formation's scale is unvaried theoretically under the control law

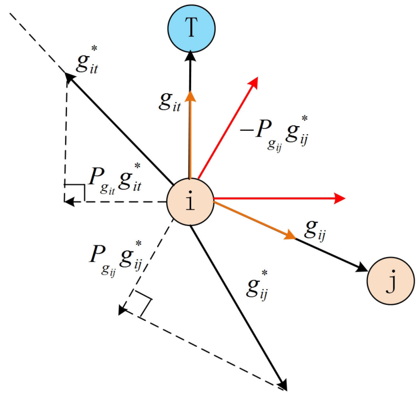

Fig. 3 *Geometric interpretation of the control law (21)*

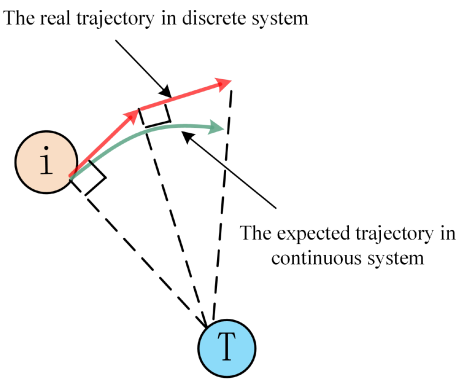

Fig. 4 *Geometric explanation of the variations of the formation's scale*

(3). However, when we apply this control law on a digital computer, it can be observed that the formation expands its scale slowly. The expanding velocity is inversely proportional to the control frequency. The lower the control frequency is, the larger the expanding velocity of the formation's scale is. Fig. 4 illustrates an agent's partially enlarged trajectories both in continuous- and discrete-time domain. In the continuous-time case, if an agent is controlled by the second term in (21), the agent will move on an arc trajectory perpendicular to the bearing vector g_{iT} all the time. Therefore, the formation scale keeps unchanged. However, in the discrete-time case, the controller's output is changed only at the sampling time. As a result, the trajectory of agents between two sampling time is a short line. Therefore, the agent's trajectory in a discrete-time case is actually not an expected circle, but a polygon whose radius is increasing.

In order to solve the above problems, the circumnavigation control law is redesigned as follows:

$$\begin{aligned}\mu_i = & -k_\alpha \sum_{j \in N_i} P_{g_{ij}}(t) g_{ij}^* - k_\beta P_{g_{iT}}(t) g_{iT}^* \\ & + k_\gamma(\hat{r}_i - r_\mathrm{d}) g_{iT} + \hat{\boldsymbol{v}}_{Ti},\end{aligned} \tag{22}$$

where k_α, k_β, and k_γ are coefficients, r_d is the given distance constraint, $\hat{r}_i$ and $\hat{v}_{Ti}$ are the agent i's estimation of the distance to the target T and target T's velocity, respectively. Specifically, $\hat{r}_i$ and $\hat{v}_{Ti}$ can be obtained using Theorem 1.

Remark 2: It can be observed that two terms $k_\gamma(\hat{r}_i - r_\mathrm{d}) g_{iT}$ and $\hat{\boldsymbol{v}}_{Ti}$ are added into (22), compared to the control law (21). These two terms are mainly used to solve the problem of formation's scale constraint and target tracking. Compared with the control law (20) proposed in [28], the control law (22) can not only drive the multi-agent system to form the bearing rigidity shape, but also make the system achieve the desired scale and centre (i.e. the target).

Up to this point, the overall system can be summarised as: In a multi-agent system in which the agents' dynamics obey $\dot{p}_i = \mu_i$, the agents are driven by the control law (22) to circumnavigate around a target whose dynamics and positions are totally unknown. The goal is to achieve three constraints, namely, $g_{ij}//g_{ij}^*, g_{iT}//g_{iT}^*$, $\lim_{t \to \infty} c(t) \to p_T(t)$, and $\lim_{t \to \infty} s(t) = r_\mathrm{d}$. In the following section, we will present the proof for the stability and the convergence analysis of the multi-agent system controlled by (22).

4.2 Stability analysis

Firstly, define the error of the distances between agent i and target T as follows:

$$\sigma_i = \| p_i - p_T \| - r_\mathrm{d}, \quad \sigma = [\sigma_1, \ldots, \sigma_2]^\mathrm{T}.$$

Through the discussion in Section 3, the agents' estimations of the target's distance and velocity are unbiased. For the convenience of analysis, the control law (22) can be expressed in the following matrix form:

$$\dot{p} = k_\alpha (\tilde{R}_b)^\mathrm{T} g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t + \mathbf{1}_n \otimes \dot{p}_T, \tag{23}$$

where $\Delta = \mathrm{diag}\{\sigma_i\} \otimes I_\mathrm{d}$ is the diagonal matrix of the distance error.

Then, let $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ denote the vector of the distance error between agents and target T and we will prove the multi-agent system will satisfy the constraint of target distance in the following lemma.

Lemma 2: Considering the multi-agent system $\dot{p} = \mu$ under the control law (23), then the error of the distance between each agent and the target $\sigma_i, i = 1, \ldots, n$ will converge to 0 at an exponential rate and the convergence rate is estimated as follows:

$$\| \sigma(t) \|^2 = e^{-2k_\gamma t} \| \sigma(0) \|^2 .$$

Proof: Define a Lyapunov function candidate in terms of the distance error σ as follows:

$$F = \| \sigma \|^2 = \sum_{i=1}^{n} \sigma_i^2 .$$

The derivative of F is given by

$$\begin{aligned}\dot{F} &= 2\sum_{i=1}^{n} \sigma_i \dot{\sigma}_i = 2\sigma^\mathrm{T} \mathrm{diag}\left\{ \frac{(p_i - p_T)^\mathrm{T}}{\| p_i - p_T \|} \right\} (\dot{p} - \mathbf{1}_n \otimes \dot{p}_T) \\ &= 2\sigma^\mathrm{T} \mathrm{diag}\left\{ \frac{(p_i - p_T)^\mathrm{T}}{\| p_i - p_T \|} \right\} (k_\alpha (\tilde{R}_b)^\mathrm{T} g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t) .\end{aligned}$$

Since $\{p, \mathbf{1}_n\} \subseteq \mathrm{Null}(\tilde{R}_b), P_{g_{iT}}(p_i - p_T) = 0$, we have $\mathrm{diag}\{(p_i - p_T)^\mathrm{T}\}\tilde{R}_b = 0$, $\mathrm{diag}\{(p_i - p_T)^\mathrm{T}\} D_b = 0$. Then we can obtain that

$$\dot{F} = 2k_\gamma \sigma^\mathrm{T} \mathrm{diag}\left\{ \frac{(p_i - p_T)^\mathrm{T}}{\| p_i - p_T \|} \right\} \Delta g_t = -\sum_{i=1}^{n} k_\gamma \sigma_i^2 \le 0. \tag{24}$$

From (24), we obtain the time derivative of $\| \sigma \|^2$ as

$$\frac{\mathrm{d}(\| \sigma \|^2)}{\mathrm{d}t} = \dot{F} = -2k_\gamma \sum_{i=1}^{n} \sigma_i^2 = -2k_\gamma \| \sigma \|^2 .$$

Therefore, the convergence rate of $\| \sigma \|^2$ can be described by

$$\| \sigma(t) \|^2 = e^{-2k_\gamma t} \| \sigma(0) \|^2 .$$

Obviously, the distance error σ will converge asymptotically to zero, i.e. $\sigma_i \to 0, i = 1, \ldots, n$. This completes the proof. □

From Lemma 2, we can further obtain some properties about the behaviour of the formation trajectories.

Corollary 1: The system's trajectory under the control law (22) will satisfy the following inequalities:

(a) $\| p_i(t) - p_T(t) \| \le \sqrt{2 \| \sigma(0) \|^2 + 2nr_d^2}, \forall i \in \mathscr{V}, \forall t \ge 0.$

(b) $\| p_i(t) - p_j(t) \| \le 2\sqrt{2 \| \sigma(0) \|^2 + 2nr_d^2}, \forall i, j \in \mathscr{V}, \forall t \ge 0.$

Proof: We first prove that inequality (a) holds. The following inequality can be verified.

$$
\begin{aligned}
\sum_{i=1}^{n} \| p_i(t) - p_T(t) \|^2 &= \sum_{i=1}^{n} (\sigma_i(t) + r_d)^2 \\
&= \sum_{i=1}^{n} \sigma_i^2(t) + 2\sum_{i=1}^{n} \sigma_i r_d + nr_d^2 \\
&\le \sum_{i=1}^{n} \sigma_i^2(t) + \sum_{i=1}^{n} (\sigma_i^2(t) + r_d^2) + nr_d^2 \\
&= 2 \| \sigma(t) \|^2 + 2nr_d^2 .
\end{aligned}
$$

From Lemma 2, it is concluded that $\| \sigma(t) \|$ asymptotically converge to 0, implies that $\| \sigma(t) \| \le \| \sigma(0) \|$. Further, we have

$$
\begin{aligned}
\| p_i(t) - p_j(t) \|^2 &\le \sum_{i=1}^{n} \| p_i(t) - p_T(t) \|^2 \\
&\le 2 \| \sigma(0) \|^2 + 2nr_d^2 .
\end{aligned}
$$

Therefore, the inequality $\| p_i(t) - p_T(t) \| \le \sqrt{2 \| \sigma(0) \|^2 + 2nr_d^2}$ is true. The inequality (b) is obtained from the fact that $\| p_i(t) - p_j(t) \| = \| (p_i(t) - p_T(t)) - (p_j(t) - p_T(t)) \| \le \| p_i(t). - p_T(t) \| + \| p_j(t) - p_T(t) \| \le 2\sqrt{2 \| \sigma(0) \|^2 + 2nr_d^2}$
This completes the proof. □

Next, we will show that the bearing constraint $g_{ij}//g_{ij}^*, g_{iT}//g_{iT}^*$ will be satisfied under the control law (23). In order to prove that, a coordinate transformation is firstly performed. Based on the original world coordinate, we construct a new coordinate called the target coordinate by attaching the coordinate-origin on the target and keep the three axes unchanged. After the coordinate transformation, it is obvious the relative bearings between any two objects just stay the same. Therefore, the bearing constraint will be satisfied in the world coordinate as long as it is satisfied in the target coordinate. Since the target is always located in the origin point in the target coordinate, the target is static in the target coordinate. As a result, the control law is transformed into

$$
\dot{p} = k_\alpha(\tilde{R}_b)^{\mathrm{T}} g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t . \tag{25}
$$

Before presenting the proof, we first give a useful lemma [30].

Lemma 3: For a positive semi-definite matrix A, let $\mathrm{Null}(A)^{\perp}$ be the orthogonal complement space of A's kernel space $\mathrm{Null}(A)$. Then for any vector $\vec{v} \in \mathrm{Null}(A)^{\perp}$, the inequality $\vec{v}^{\mathrm{T}} A \vec{v} \ge \lambda_2(A) \vec{v}^{\mathrm{T}} \vec{v}$ always holds.

Then, the following lemma shows that the bearing constraint $g_{ij}//g_{ij}^*, g_{iT}//g_{iT}^*$ will be satisfied.

Lemma 4: Considering the multi-agent system $\dot{p} = \mu$ with the control law (23), the bearing constraint $g_{ij}//g_{ij}^*, g_{iT}//g_{iT}^*$ will be satisfied when the system converges to its stable point.

Proof: Obviously, in the target coordinate, it is satisfied that $\dot{p} = k_\alpha(\tilde{R}_b)^{\mathrm{T}} g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t = 0$ when the system is stable. This further implies that

$$
\begin{aligned}
0 = {} & ((p^* - \mathbf{1}_n \otimes p_T) - (p - \mathbf{1}_n \otimes p_T))^{\mathrm{T}} \\
& \times (k_\alpha(\tilde{R}_b)^{\mathrm{T}} g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t) .
\end{aligned} \tag{26}
$$

where p^* denotes the position of agents when the desired formation is achieved.

As stated in Section 2, a framework $\mathscr{G}(p)$ in $\mathbb{R}^d$ always satisfies $\mathrm{span}\{\mathbf{1}_n \otimes I_d, p)\} \subseteq \mathrm{Null}(R_b(p))$. Therefore, $p \subseteq \mathrm{Null}(R_b(p))$, which implies that $\tilde{R}_b p = 0$. In light of $0 = \mathrm{diag}(P_{g_{T\hat{k}}}) g_{T\hat{k}} \| e_{T\hat{k}} \| = D_b(p - \mathbf{1}_n \otimes p_T)$, (26) can be further simplified as follows:

$$
\begin{aligned}
0 &= k_\alpha(p^* - \mathbf{1}_n \otimes p_T)^{\mathrm{T}} (\tilde{R}_b)^{\mathrm{T}} g^* + k_\beta (p^* - \mathbf{1}_n \otimes p_T)^{\mathrm{T}} D_b g_t^* \\
&\quad - k_\alpha (\mathbf{1}_n \otimes p_T)^{\mathrm{T}} (\tilde{R}_b)^{\mathrm{T}} g^* + k_\gamma (p^* - p)^{\mathrm{T}} \Delta g_t \\
&= k_\alpha (p^*)^{\mathrm{T}} (\tilde{R}_b)^{\mathrm{T}} g^* + k_\beta (p^* - \mathbf{1}_n \otimes p_T)^{\mathrm{T}} D_b g^* \\
&\quad + k_\gamma (p^* - p)^{\mathrm{T}} \Delta g_t \\
&= k_\alpha (\bar{H} p^*)^{\mathrm{T}} \mathrm{diag}(P_{g_k}) g^* + k_\beta (p^* - \mathbf{1}_n \otimes p_T)^{\mathrm{T}} \mathrm{diag}(P_{g_{T\hat{k}}}) g^* \\
&\quad + k_\gamma (p^* - p)^{\mathrm{T}} \Delta g_t \\
&= k_\alpha \sum_{k=1}^{m} \| e_k^* \| g_k^{\mathrm{T}} P_{g_k} g_k^* + k_\beta \sum_{\hat{k}=1}^{n} \| e_{T\hat{k}}^* \| g_{T\hat{k}}^{\mathrm{T}} P_{g_{T\hat{k}}} g_{T\hat{k}}^* \\
&\quad + k_\gamma \sum_{k=1}^{n} \frac{\sigma_i}{\sigma_i + r_d} (p_i - p_i^*)^{\mathrm{T}} (p_i - p_T) .
\end{aligned} \tag{27}
$$

As Lemma 2 reveals, $\sigma_i = 0$ when the system is stable. Consequently, the term $k_\gamma \sum_{k=1}^{n} (\sigma_i/(\sigma_i + r_d))(p_i - p_i^*)^{\mathrm{T}} (p_i - p_T)$ will also converge to zero when the system is stable. On the other hand, since both the matrices $P_{g_k}^*$ and $P_{g_{T\hat{k}}}^*$ are positive semi-definite, the inequalities $g_k^{\mathrm{T}} P_{g_k} g_k^* \ge 0$ and $g_{T\hat{k}}^{\mathrm{T}} P_{g_{T\hat{k}}} g_{T\hat{k}}^* \ge 0$ are always valid. Therefore, (27) is valid if and only if $g_k^{\mathrm{T}} P_{g_k} g_k^* = 0$ and $g_{T\hat{k}}^{\mathrm{T}} P_{g_{T\hat{k}}} g_{T\hat{k}}^* = 0$, i.e. $g_{ij}//g_{ij}^*, g_{iT}//g_{iT}^*$ in the target coordinate. On the other hand, since the relative bearing relationship between any two agents is just the same in the target coordinate and the origin coordinate, the conclusion is also valid in the origin coordinate. The proof is completed. □

Next, let $\delta(t) = p(t) - p^*(t)$ denote the error between the system's current position and desired position when the desired formation is achieved. We will prove that the $\delta(t)$ will asymptotically converge to zero.

Lemma 5: Consider the multi-agent system $\dot{p} = \mu$ with the control law (23), then the error between the system's current position and desired position $\delta(t)$ will asymptotically converge to zero at a exponential-like rate, described by

$$
\| \delta(t) \| \le e^{-(k_\alpha C_1 + k_\beta C_2)t} \| \delta(0) \|^2 ,
$$

where the constants C_1 and C_2 are given by

$$
\begin{aligned}
C_1 &= \frac{r_d}{4\sigma^2(0) + 4nr_d^2} \lambda_2(H^{\mathrm{T}} \mathrm{diag}(P_{g_k}^*) H), \\
C_2 &= \frac{2r_d}{\sigma^2(0) + nr_d^2} \sin\frac{\pi}{n} \lambda_2(\mathrm{diag}(P_{g_{T\hat{k}}}^*)) .
\end{aligned}
$$

Proof: First define a Lyapunov function $V = \frac{1}{2} \| \delta \|^2$. The derivative of V is obtained as

$$
\begin{aligned}
\dot{V} &= \delta^{\mathrm{T}}\dot{\delta} = \delta^{\mathrm{T}}\dot{p} = (p - p^*)^{\mathrm{T}}(k_\alpha(\tilde{R}_b)^{\mathrm{T}}g^* + k_\beta D_b g_t^* + k_\gamma \Delta g_t) \\
&= [(p - \mathbf{1}_n \otimes p_T) - (p^* - \mathbf{1}_n \otimes p_T)]^{\mathrm{T}} \times (k_\alpha(\tilde{R}_b)^{\mathrm{T}}g^* \\
&\quad + k_\beta D_b g_t^* k_\gamma \Delta g_t) \\
&= \underbrace{-k_\alpha \sum_{k=1}^{m} \frac{1}{\| e_k^* \|}(e_k^*)^{\mathrm{T}} P_{g_k} e_k^*}_{S_1} \underbrace{- k_\beta \sum_{\hat{k}=1}^{n} \frac{1}{\| e_{T\hat{k}}^* \|}(e_{T\hat{k}}^*)^{\mathrm{T}} P_{g_{T\hat{k}}} e_{T\hat{k}}^*}_{S_2} \\
&\quad \underbrace{+ k_\gamma \sum_{k=1}^{n} \frac{\sigma_i}{\sigma_i + r_{\mathrm{d}}}(p_i - p_i^*)^{\mathrm{T}}(p_i - p_T)}_{S_3}.
\end{aligned}
$$

According to Lemma 2, $\sigma_i \to 0$, which further implies that the item $S_3 \to 0$. Therefore, the derivative of V is mainly determined by items S_1 and S_2. That is,

$$
\begin{aligned}
\dot{V} &= -k_\alpha \sum_{k=1}^{m} \frac{(e_k^*)^{\mathrm{T}} P_{g_k} e_k^*}{\| e_k^* \|} - k_\beta \sum_{\hat{k}=1}^{n} \frac{(e_{T\hat{k}}^*)^{\mathrm{T}} P_{gT\hat{k}} \| e_{Tk}^* \|}{\| e_{T\hat{k}}^* \|} \\
&= -k_\alpha \sum_{k=1}^{m} \| e_k^* \| (g_k^*)^{\mathrm{T}} P_{g_k} g_k^* - k_\beta \sum_{\hat{k}=1}^{n} \| e_{T\hat{k}}^* \| (g_{T\hat{k}}^*)^{\mathrm{T}} P_{gT\hat{k}} g_{T\hat{k}}^*
\end{aligned}
$$

As the goal configuration of the agents is a polygon with radius r_{d}, it can be easily obtained that $\| e_{T\hat{k}}^* \| = r_{\mathrm{d}}$ and $\| e_{T\hat{k}}^* \| = 2r_{\mathrm{d}}\sin(\pi/n)$. For any two unit vectors $g_1, g_2 \in \mathscr{R}^d$, there always exists that $g_1^{\mathrm{T}} P_{g2} g_1 = g_2^{\mathrm{T}} P_{g1} g_2$ [28]. Thus we have

$$
\begin{aligned}
\dot{V} &= -k_\alpha r \sum_{k=1}^{m} g_k^{\mathrm{T}} P_{g_k}^* g_k - 2k_\beta r \sin\frac{\pi}{n} \sum_{\hat{k}=1}^{n} g_{T\hat{k}}^{\mathrm{T}} P_{gT\hat{k}}^* g_{T\hat{k}} \\
&= -k_\alpha \frac{r_{\mathrm{d}}}{\| e_k \|^2} \sum_{k=1}^{m} e_k^{\mathrm{T}} P_{g_k}^* e_k - \frac{2k_\beta r_{\mathrm{d}}}{\| e_{T\hat{k}} \|^2} \sin\frac{\pi}{n} \sum_{\hat{k}=1}^{n} e_{T\hat{k}}^{\mathrm{T}} P_{gT\hat{k}}^* e_{T\hat{k}}.
\end{aligned} \tag{28}
$$

From Lemma 2, it has been obtained that

$$
\begin{aligned}
\| e_k \| &\le 2\sqrt{2 \| \sigma(0) \|^2 + 2nr_{\mathrm{d}}^2}, \\
\| e_{T\hat{k}} \| &\le \sqrt{2 \| \sigma(0) \|^2 + 2nr_{\mathrm{d}}^2}.
\end{aligned}
$$

Then, combining with (28), we can derive the following inequality:

$$
\begin{aligned}
\dot{V} \le &-\frac{k_\alpha r_{\mathrm{d}}}{8\sigma^2(0) + 8nr_{\mathrm{d}}^2} \sum_{k=1}^{m} e_k^{\mathrm{T}} P_{g_k}^* e_k \\
&-\frac{k_\beta r_{\mathrm{d}}}{\sigma^2(0) + nr_{\mathrm{d}}^2} sin\frac{\pi}{n} \sum_{\hat{k}=1}^{n} e_{T\hat{k}}^{\mathrm{T}} P_{gT\hat{k}}^* e_{T\hat{k}}.
\end{aligned} \tag{29}
$$

Since $e = HP, (Hp^*)^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*) = 0$ and $(p^* - p_T)^{\mathrm{T}}\mathrm{diag}(P_{gT\hat{k}}^*) = 0$, the inequality (29) can be rewritten as

$$
\begin{aligned}
\dot{V} \le &-\frac{k_\alpha r_{\mathrm{d}}}{8\sigma^2(0) + 8nr_{\mathrm{d}}^2}(H\delta)^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*)H\delta \\
&-\frac{k_\beta r_{\mathrm{d}}}{\sigma^2(0) + nr_{\mathrm{d}}^2} sin\frac{\pi}{n}\delta^{\mathrm{T}}\mathrm{diag}(P_{gT\hat{k}}^*)\delta
\end{aligned}
$$

Since $\mathrm{diag}(P_{g_k}^*) = (\mathrm{diag}(P_{g_k}^*))^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*)$, we have $H^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*)H = (\mathrm{diag}(P_{g_k}^*)H)^{\mathrm{T}}(\mathrm{diag}(P_{g_k}^*)H)$. So both the matrices $H^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*)H$ and $\mathrm{diag}(P_{gT\hat{k}}^*)$ are positive semi-definite. Then, according to Lemma 3, we obtain that

$$
\begin{aligned}
\dot{V} \le &-\frac{k_\alpha r_{\mathrm{d}}}{8\sigma^2(0) + 8nr_{\mathrm{d}}^2}\lambda_2(H^{\mathrm{T}}\mathrm{diag}(P_{g_k}^*)H) \| \delta \|^2 \\
&-\frac{k_\beta r_{\mathrm{d}}}{\sigma^2(0) + nr_{\mathrm{d}}^2}\sin\frac{\pi}{n}\lambda_2(\mathrm{diag}(P_{gT\hat{k}}^*)) \| \delta \|^2
\end{aligned}
$$

Thus we have

$$
\frac{1}{2}\frac{\mathrm{d}(\| \delta \|^2)}{\mathrm{d}t} \le -\frac{1}{2}(k_\alpha C_1 + k_\beta C_2) \| \delta \|^2 .
$$

Finally, we obtain that the convergence rate of $\delta(t)^2 \|$ can be described as follows:

$$
\| \delta(t) \| \le e^{-(k_\alpha C_1 + k_\beta C_2)t} \| \delta(0) \|^2 .
$$

The proof is completed. □

The following lemma proves that the centre of the multi-agent system converges to the position of the target.

Lemma 6: Considering the multi-agent system $\dot{p} = \mu$ with the control law (23), the centre of the multi-agent system defined in (6) will converge to the position of the target p_T, i.e. $\lim_{t \to \infty} c(t) = p_T$.

Proof: Define the error between the centre of the multi-agent system and the target's position as $\delta_c = c(t) - p_T$. Since the centre of the expected formation is actually the position of the target, i.e. $1/n(\mathbf{1}_n \otimes I_{\mathrm{d}})^{\mathrm{T}}p^* = p_T$, we have

$$
\begin{aligned}
\| \delta_c \|^2 &= \frac{1}{n^2} \| (\mathbf{1}_n \otimes I_{\mathrm{d}})^{\mathrm{T}}(p - p^*) \|^2 \\
&= \frac{1}{n^2} \| \sum_{i=1}^{n}(p_i - p_i^*) \|^2 \le \frac{1}{n^2}\left(\sum_{i=1}^{n} \| p_i - p_i^* \|\right)^2 \\
&\le \frac{1}{n^2} \times n \times \sum_{i=1}^{n} \| p_i - p_i^* \|^2 \\
&\le \frac{1}{n}\sum_{i=1}^{n} \| p_i - p_i^* \|^2 = \frac{1}{n} \| \delta \|^2
\end{aligned}
$$

It has been proven that $\| \delta \|$ converges to zero asymptotically. As $\| \delta_c \|^2 = \frac{1}{n^2} \| \delta \|^2$, evidently $\| \delta \|$ will also converge to zero and consequently $\lim_{t \to \infty} c(t) = p_T$. This completes the proof. □

Remark 3 ((Parameter Selection)): In the control law (23), the three parameters k_α, k_β, and k_γ need to be determined. It has been proven that the convergence rate of the system is in proportion to k_α, k_β, and k_γ as in Lemmas 2, 5 and 6. On one hand, the system will converge to its stable point at a larger speed if the parameters k_α, k_β, and k_γ become larger. On the other hand, the overshoot of the system will also get larger along with the increase of the parameters. Therefore, a trade-off has to be made when selecting these parameters.

Finally, combining Lemmas 2, 4 and 6, we can obtain the main result of this paper.

Theorem 3: For the multi-agent system $\dot{p} = \mu$ with the control law (23), then the system will achieve the circumnavigation of a moving target, i.e. all the constraints $g_{ij}//g_{ij}^*, g_{it}//g_{it}^*, \forall(i,j) \in \mathscr{E}$, $\lim_{t \to \infty} c(t) \to p_T(t)$, and $\| p_i - p_T \| = r, i = 1, \ldots, n$ will be satisfied.

5 Simulations

To verify the effectiveness of the proposed control law, we present two Gazebo simulations in which five UAVs circumnavigate around a moving target. Gazebo is a powerful 3D simulator which has a powerful physical simulation engine and delicate simulation views.

In the first simulation scenario, the initial position of the target UAV is set as $p_T = [0, 0]^{\mathrm{T}}$m and those of the five circumnavigating UAVs are set as $p_1 = [0, -1.5]^{\mathrm{T}}$m, $p_2 = [-2, -1]^{\mathrm{T}}$m, $p_3 = [-6, 0]^{\mathrm{T}}$m, $p_4 = [-4.5, 1]^{\mathrm{T}}$m, and $p_5 = [-1, 4.5]^{\mathrm{T}}$m. The initial position setup in Gazebo is illustrated in Fig. 5. The desired

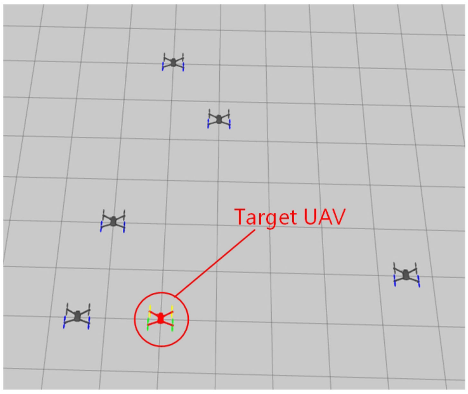

Fig. 5 *Initial position setup of the target UAV and circumnavigating UAVs in Gazebo*

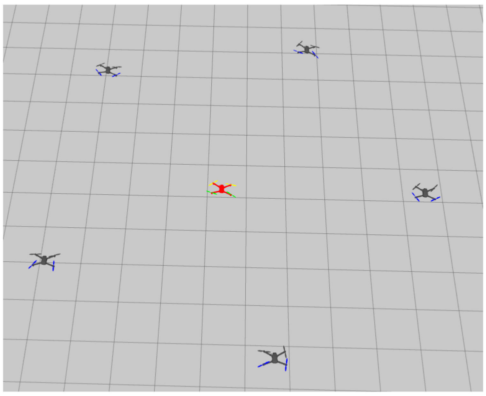

Fig. 6 *Five UAVs circumnavigate around a target UAV in Gazebo*

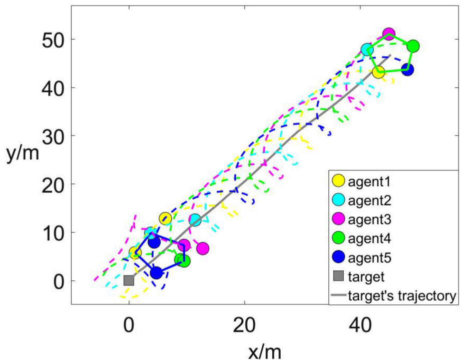

Fig. 7 *Trajectories of the circumnavigating UAVs and the target*

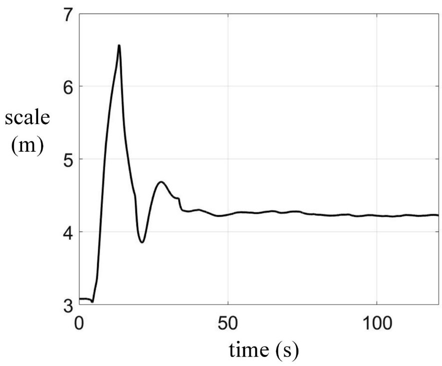

Fig. 8 *Variation of the formation's scale*

formation radius is 4 m and the velocity of the target UAV is set as $[0.3, 0.3]^{\mathrm{T}}\mathrm{m/s}$. The control gain is set as $k_{\alpha} = k_{\beta} = 5$, $k_{\gamma} = 1$. The simulation result is illustrated in Figs. 6–11.

After the simulation starts, five UAVs adjust their positions under the control law (22) and form the predefined circle formation

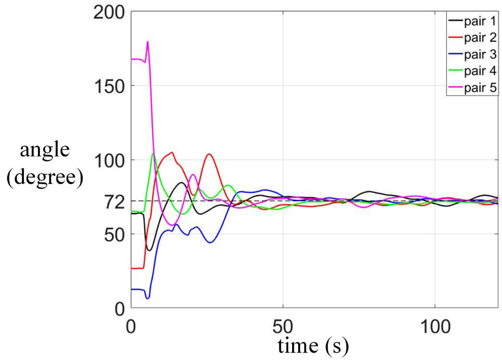

Fig. 9 *Angles between neighbouring UAVs*

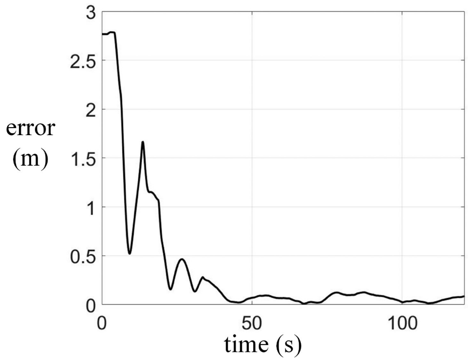

Fig. 10 *Distance error between the formation's centre and the target*

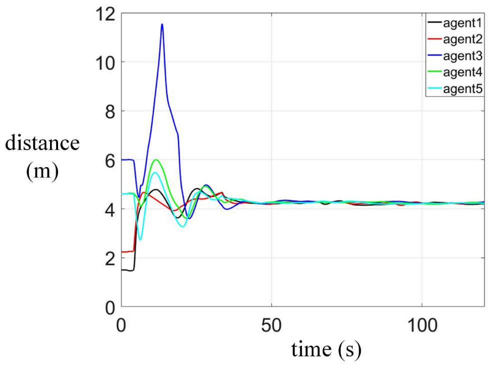

Fig. 11 *Distances between each UAV and the target*

as illustrated in Fig. 6. Then, the formation will maintain the desired scale radius and circumnavigate around the target. The trajectories of the circumnavigating UAVs and the target UAV are illustrated in Fig 7. In Fig 7, the line connection represents the shape of the formation in a moment during the circumnavigation, which is obviously a pentagon. The variation of the formation's scale is illustrated in Fig. 8, which shows that the formation's scale will reach the desired value. Fig. 9 demonstrates the variation of the angle between two neighbouring UAVs, which converges to the same (72°) gradually as time goes on. The variations of the distance errors between the formation's centre and the target and the distances between each UAV and the target are illustrated in Figs. 10 and 11, respectively. It can be observed that each UAV will gradually keep a desired distance with the target and the distance error between the formation's centre and the target will converge to zero.

In the second scenario, we consider a more complex situation. The expected and actual initial positions of the UAVs are illustrated in Fig 12. It can be found that the positions of UAV 2 and UAV 4, UAV 3, and UAV 5 are exchanged from their expected position deliberately. To form the desired goal, the corresponding UAVs need to exchange their positions, which is obviously more complex than the situation in the first scenario. Meanwhile, the target will move on a circular curve, as illustrated in Fig. 13. The experiment results are illustrated in Figs. 14–17. It can be seen that the control

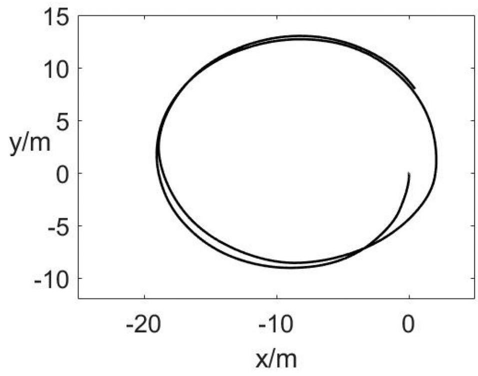

Fig. 12 *Trajectory of the target UAV in the second scenario*

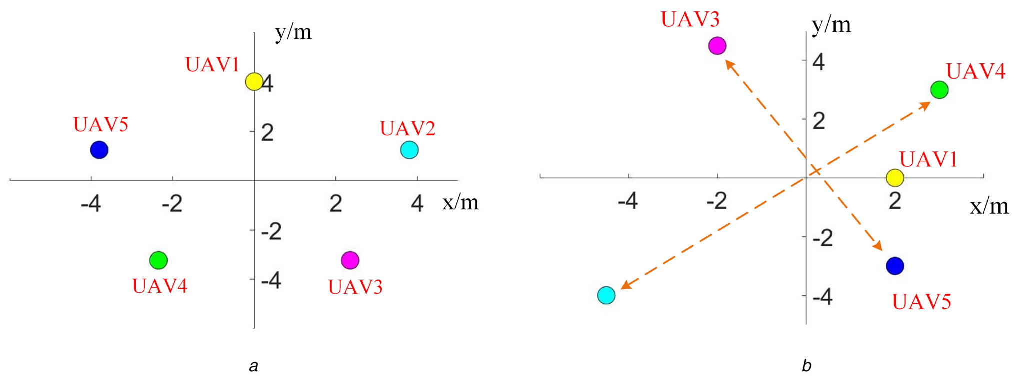

Fig. 13 *Expected and actual initial positions of the UAVs in the second scenario*
(a) Expected initial UAV positions, ***(b)*** Actual initial UAV positions in the second scenario

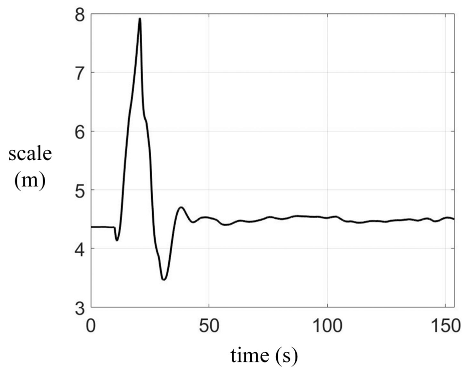

Fig. 14 *Variation of formation's scale in the second scenario*

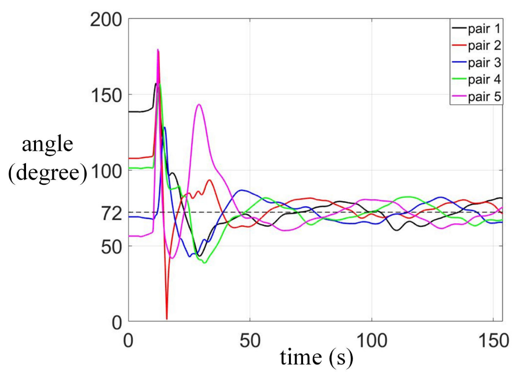

Fig. 15 *Variation of the angles between neighbouring UAVs in the second scenario*

performance in the second scenario is a little worse than in the first scenario. This is due to the fact that the trajectory of the target in the second scenario is more complex and there exist some overshoots in the control of the UAVs when tracking the target.

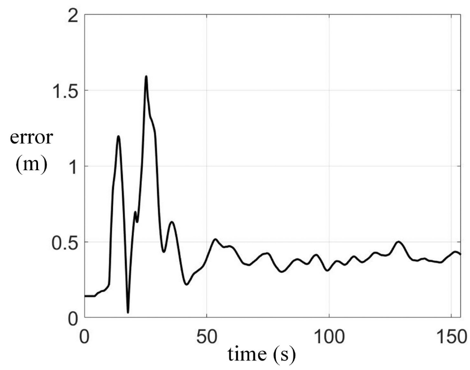

Fig. 16 *Distance error between the formation's centre and the target in the second scenario*

The control performance is acceptable and we will try to improve the control performance in our future work.

6 Conclusion and future work

In this paper, we have investigated the multi-agent circumnavigation control problem by employing the newly developed bearing rigidity theory. A distributed algorithm has been designed for the circumnavigating agent to estimate its distance to the target as well as the target's velocity, based on the local bearing and velocity information. A distributed circumnavigation control law has been further designed to drive a team of agents to circumnavigate around a moving target and the stability of the control law has been also proved. The experimental results conducted on the Gazebo platform indicate that the designed algorithm in this paper works well and can be applied to the physical robot.

The dynamical model of the agent is considered as a single integrator in this paper. In our future work, we may investigate the circumnavigation control problem for the case with more complex agent dynamics. Besides, the environment we considered is an ideal environment without any external disturbance. In reality,

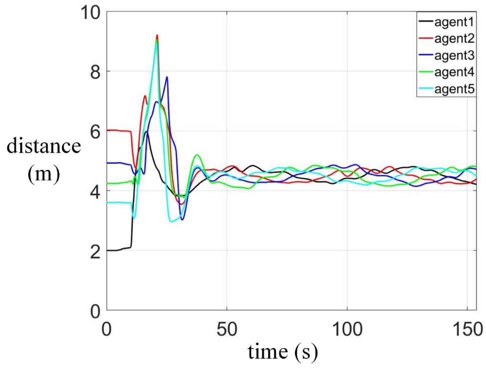

Fig. 17 *Distances between each UAV and the target in the second scenario*

external disturbances such as wind disturbance to UAVs cannot be neglected and affect the performance of the algorithm. Solving the circumnavigation control problem in the presence of imperfect communication or external disturbances will be another interesting research topic.

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