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Multiple UAVs Configuration Formation Control via the Dual Quaternion Method

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Abstract: The considered multiple Unmanned Aerial Vehicles (UAVs) formation control in this paper aims to design a control law to keep fixed-wing UAV flight with specified relative configurations (attitude and position) under predefined topology, of which each node is an underactuated dynamic system. In order to realize the stable formation flight, we proposed a control method, composing of the characteristics of fixed-wing UAVs with the advantages of dual quaternions to achieve and preserve formation configurations. The proposed formation control law is distributed by only using the neighbors' information, and without requiring to decouple the rotation and translation. Combining the consensus protocol and the property called Pairwise Asymptotic Stability (PAS), the overall system can be certified asymptotically stable. Finally, simulation results are presented to show the effectiveness of this approach.

Key Words: Multiple UAVs Formation, Dual Quaternion, Feedback Linearization, Consensus Protocol

1 Introduction

Multiple Unmanned Aerial Vehicles (UAVs) cooperation in formation is the main mode for the fixed-wing UAV carrying out diversified tasks. Formation control mainly includes formation configuration design, formation reconfiguration strategy and obstacle avoidance algorithm and so on. Tae Soo No, Youdan Kim et al. put forward a hierarchical overlapping structure of UAVs formation [1]. In their paper, the formation is divided into several branches, as shown in Fig. 1. and each UAV, except for the global leader, takes a role of

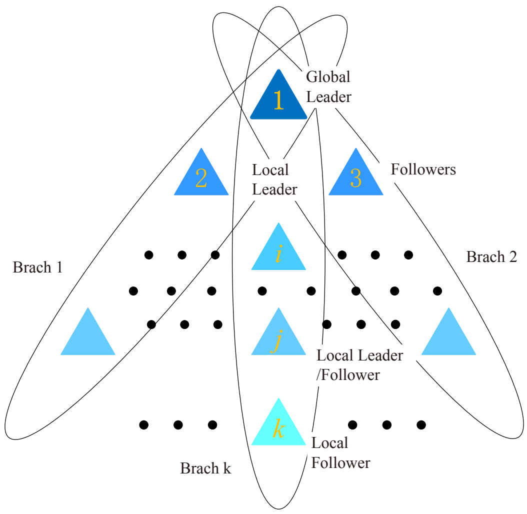

Fig. 1: A hierarchical overlapping structure for UAVs formation

local leader, local follower or both depending on its status in the branch. Then a cascade type guidance law is designed to control the entire system. [2] studies multiple UAVs formation coordination problem under a unified optimal control framework. Under the specific communication topology, the UAV is able to obtain an analytical form of the optimum control law of the distributed system depending on its adjacent aircrafts, so that the formation can fly in the minimum cost. Ajorlou Amir and Moezzi Kaveh, et al. develop the formation reconfiguration strategy [3]. They obtain the desired UAV position in formation according to the current position and a weighted possible position, then the optimal formation configuration during transformation process is designed, regarding the energy consumption of a formation reconfiguration as optimization index. In further researches, the time optimal formation reconfiguration algorithm is presented with acceleration and velocity constraints [4].

Whatever, the existing UAVs formation control methods usually regard the UAV as a point agent, and investigate the formation in two-dimensional space, less considering the formation control problem in three-dimensional space (including three rotational and three translational degrees of freedom). Whereas the applications of UAVs formation, such as multi-angle reconnaissance, joint striking, battlefield assessment, and electronic countermeasures are required to consider the position and attitude simultaneously. Therefore, it is necessary to further analyse the characteristics of fixed-wing UAVs formation model in three-dimensional space, in order to design effective guidance and control law to solve UAVs formation control problem in the 6 degrees of freedom space. With excellent properties, the dual quaternion and its related theory are more and more applied to formation movement modeling and control in three-dimensional space.

In recent years, researchers establish a Li-group structure of the unit dual quaternion to study the general motion in three-dimensional space [5, 6]. They attempt to introduce unit dual quaternion logarithm into general rigid motion control to realise the position and attitude synchronization control. Further, this work has been extended to deal with the formation of multiple quad-rotor [7], in which property called pairwise asymptotic stability of the overall system is analyzed and validated. In orbit and attitude coupling control of spacecraft, especially for satellite formation, the dual quaternion is also applicable [8]. In view of the asymmetric underactuated spacecraft [9], Wu and Liu, et al. set up the dual quaternion form of dynamic equation, and split the

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dual quaternion into real and dual parts, corresponding to the spacecraft's attitude and orbit motion. Then a sliding mode variable structure controller is proposed to track the desired trajectory. [10] discusses spacecraft relative position and attitude synchronous control problem under the limited input condition. The paper use the dual quaternion to establish the relative motion of the spacecraft formation model, and the adaptive sliding mode control method to realize the formation global asymptotically stable control. Besides, Li and Wang propose a novel algorithm for formation flying satellites combining attitude with position[11]. The trajectory of principal satellite is obtained by dual quaternion interpolation, then the relative position and attitude of the deputy satellite are obtained by dual quaternion modeling on the principal satellite. The successful use of dual quaternion in these complex formation systems provides a possible way for dual quaternion-based UAV formation control.

In this paper, we focus on the multiple UAVs formation control problem. The main contribution of this paper lies in

- building the neighbored UAVs formation dynamics model based on dual quaternion.
- using feedback linearization design a UAVs formation controller, combining the consensus protocol and pairwise asymptotic stability property

Finally, the formation is shown to be asymptotically stable.

The outline of this paper is as follows: In Section 2 the UAV relative motion model and the configuration of formation are re-depicted by using dual quaternion describers. Section 3 presents a feedback linearization controller of the formation. In order to validate the theoretical results, a simulation of the proposed control law is shown and discussed in Section 4.

2 Dual Quaternion Description of UAV Formation

Multiple UAVs formation model describes the geometry relationship of each UAV's position and attitude in the formation. With the aid of the mathematical model of fixed-wing UAVs formation, the relative dynamic model can be established. It is natural to design a control law to realize the motion control of the whole formation based on the model. It means that the control law can make the followers and leaders converge to a specific configuration and keep the formation flight.

2.1 Relative Dynamics Based on Dual Quaternion

First of all, we discribe the relative dynamics equation of two neighbored UAVs in the formation by using dual quaternion.

Suppose one is the leader and another is the follower, which tracks the leader's flight, shown as Fig. 2. Denote the leader and the follower's configurations by $\hat{q}_j$ and $\hat{q}_i$, respectively. Based on the multiplicative error, the follower pose relative to the leader $\hat{q}_{ij}$ can be shown as follows

$$\hat{q}_{ij} = \hat{q}_j^* \circ \hat{q}_i = q_{ij} + \frac{\epsilon}{2} q_{ij} \circ p_{ij}^i. \tag{1}$$

In equation (1), $q_{ij} = q_j^* \circ q_i$ is the relative attitude, $p_{ij}^i = p_i^i - q_{ij}^* \circ p_j^j \circ q_{ij}$ is the relative position's projection in the body frame system of leader, and ϵ is nilpotent with $\epsilon^2 = 0$, but $\epsilon \neq 0$.

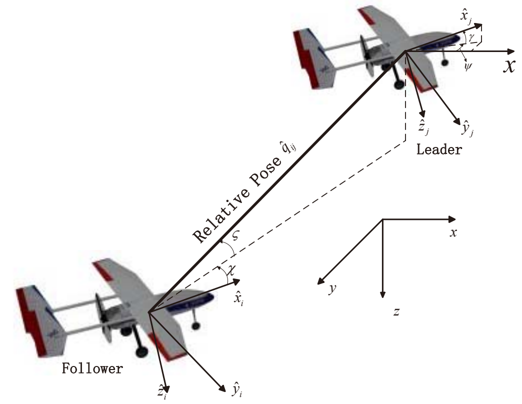

Fig. 2: Relative model of two neighbored UAVs

In the sequel, the relative dynamics model is established by differentiating the kinematics equation (1), that is

$$\dot{\hat{q}}_{ij} = \dot{\hat{q}}_j^* \circ \hat{q}_i + \hat{q}_j^* \circ \dot{\hat{q}}_i. \tag{2}$$

Considering $\dot{\hat{q}}_j = \frac{1}{2}\hat{q}_j \circ \xi_j^j$ and $\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i \circ \xi_i^i$, yields

$$\begin{aligned}\dot{\hat{q}}_{ij} &= \frac{1}{2}\xi_j^{j*} \circ \hat{q}_j^* \circ \hat{q}_i + \frac{1}{2}\hat{q}_j^* \circ \hat{q}_i \circ \xi_i^i \\ &= \frac{1}{2}\hat{q}_{ij} \circ \left(\xi_i^i - \hat{q}_{ij}^* \circ \xi_j^j \circ \hat{q}_{ij}\right).\end{aligned} \tag{3}$$

Letting

$$\hat{q}_{ij}^* \circ \xi_j^j \circ \hat{q}_{ij} = Ad_{\hat{q}_{ij}^*}\xi_j^j \tag{4}$$

and

$$\xi_i^i - Ad_{\hat{q}_{ij}^*}\xi_j^j = \xi_{ij}^i, \tag{5}$$

the relative kinematics becomes

$$\dot{\hat{q}}_{ij} = \frac{1}{2}\hat{q}_{ij} \circ \xi_i^i. \tag{6}$$

Further, let us derive the relative dynamic equations of two neighbored UAVs. Differentiating formula (5), we get the relative dynamics model

$$\begin{aligned}\dot{\xi}_{ij}^i &= \dot{\xi}_i^i - \dot{\hat{q}}_{ij}^* \circ \xi_j^j \circ \hat{q}_{ij} - \hat{q}_{ij}^* \circ \dot{\xi}_j^j \circ \hat{q}_{ij} - \hat{q}_{ij}^* \circ \xi_j^j \circ \dot{\hat{q}}_{ij} \\ &= \dot{\xi}_i^i - Ad_{\hat{q}_{ij}^*}\dot{\xi}_j^j - Ad_{\hat{q}_{ij}^*}\xi_j^j \times \xi_{ij}^i.\end{aligned}$$

In the process of UAVs formation flight, the leader can be actual or virtual. The follower and the leader need to keep expected position and attitude relationship,and the relative pose is unchanged over a period of time after the formation is achieved.

The relative configuration can be expressed as follows

$$\hat{q}_{dj} = \hat{q}_j^* \circ \hat{q}_d = q_{dj} + \frac{\epsilon}{2} p_{dj}^j \circ q_{dj}, \tag{7}$$

where $\hat{q}_{dj}$ is the desired pose related to the leader and it is a constant, namely

$$\dot{\hat{q}}_{dj} = \hat{0}. \tag{8}$$

Then, (7) can be differentiated as

$$\dot{\hat{q}}_{dj} = \dot{\hat{q}}_j^* \circ \hat{q}_d + \hat{q}_j^* \circ \dot{\hat{q}}_d = \frac{1}{2}\left(Ad_{\hat{q}_{dj}}\xi_d^d - \xi_j^j\right) \circ \hat{q}_{dj}. \tag{9}$$

Because of $\hat{q}_{dj} \neq \hat{0}$, we have

$$\xi^j_{dj} = Ad_{\hat{q}_{dj}}\xi^d_d - \xi^j_j = \hat{0}. \tag{10}$$

For the formation control law designing, we further derive the follower tracking error kinematics model relative to the desired pose, which is shown as

$$\hat{q}_{id} = \hat{q}^*_d \circ \hat{q}_i. \tag{11}$$

where $\hat{q}_{id}$ is the follower's error pose. Employing the left invariance of multiplicative error, the error dynamics can be expressed as

$$\begin{cases} \dot{\hat{q}}_{id} = \frac{1}{2}\hat{q}^*_{dj} \circ \hat{q}_{ij} \circ \xi^i_{ij} \\ \dot{\xi}^i_{ij} = \dot{\xi}^i_i - Ad_{\hat{q}^*_{ij}}\dot{\xi}^j_j - Ad_{\hat{q}^*_{ij}}\xi^j_j \times \xi^i_{ij}, \end{cases} \tag{12}$$

where

$$\dot{\xi}^i_i = -\mathbf{J}^{-1}(\omega^i \times \mathbf{J}\omega^i) + \mathbf{J}^{-1}\boldsymbol{\tau}^i + \epsilon(-\omega^i \times \dot{p}^i + m^{-1}\mathbf{F}^i).$$

In the famula, $\boldsymbol{\tau}^i$ is the resultant moment of follower in its body frame system, and $\mathbf{F}^i$ is the resultant force, including the gravity, the aerodynamic forces and the thrust.

2.2 Formation Topology and Dynamics

It is general to use "graph", which includes edges and nodes, to represent and analyze the topology among each UAV in formation. After multiple UAVs formation is modeled as a "graph", it is necessary to design the specific control (or communication) contact structure, naming control (or communication) graph.

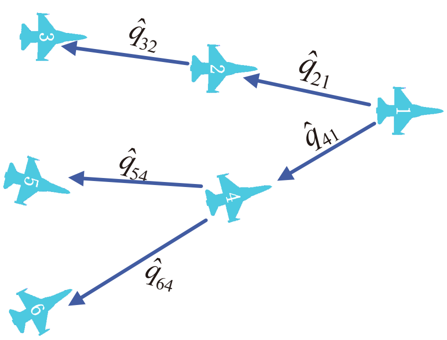

Fig. 3: Control graph of multiple UAVs formation

According to the consensus protocol [12], if and only if the directed communication topology contains a rooted directed spanning tree or the undirected communication topology is connected, the state of each node can reach consistency, achieving consensus. Thus, this paper only focuses on the formation, whose control (communication) topology is a directed spanning tree. An example of the considered control graph is illustrated, as shown in Fig. 3. In the formation, UAV-2 and UAV-4 follow UAV-1. At the same time, UAV-3 follows UAV-2 while UAV-4 leads UAV-5 and UAV-6. Refer to the topology, the entire formation can be regarded as a number of combination of neighbored formation in pair. Immediately, based on the pairwise asymptotic stability [7], if the formation control law can make two UAVs, which have communication relationship, convergence to the desired relative configuration, the whole formation is asymptotically stable.

When the formation keeps stability in a desired configuration, the spinor $\xi^i_{ij}(t)$ is $\hat{0}$, and the relative pose $\hat{q}_{ij}(t)$ keeps $\hat{q}_{d_{ij}}$ which is a constant.

3 Formation Controller Design

In this section, we design a multiple UAVs formation controller based on feedback linearization firstly. Then, its stability is analyzed.

The basic principle of feedback linearization is that the nonlinear system is transformed into a linear system through appropriate feedback transformation. In this way, linear control law can be adopt for this system. Finally, using dynamic inversion theory, control variables of the original nonlinear system are obtained by pseudo-inverse transformation.

Before presenting our main result, a definition about stability is provided, from [7].

Definition(PAS): For any pair of rigid bodies i and j in the overall system, if the actual relative configuration $\hat{q}_{ij}$ converges to the desired relative configuration $\hat{q}_{d_{ij}}$ asymptotically, viz., when $t \to \infty$, $\hat{q}_{ij}(t) \to \hat{q}_{d_{ij}}$, then we say that the overall system is pairwise asymptotically stable.

Now it is in the right position to provide our main result.

Theorem: In multiple UAVs formation, suppose that its control graph has a spanning tree. Denote UAV i and j as arbitrary two neighbored UAVs in formation, and their expected relative pose by $\hat{q}^*_{d_{ij}}$.

Letting $\hat{k}_{p_{ij}} = k_{pr_{ij}} + \epsilon k_{pd_{ij}} = (k_{pr1_{ij}}, k_{pr2_{ij}}, k_{pr3_{ij}})^T + \epsilon(k_{pd1_{ij}}, k_{pd2_{ij}}, k_{pd3_{ij}})^T$ and $\hat{k}_{\nu_{ij}} = k_{\nu r_{ij}} + \epsilon k_{\nu d_{ij}} = (k_{\nu r1_{ij}}, k_{\nu r2_{ij}}, k_{\nu r3_{ij}})^T + \epsilon(k_{\nu d1_{ij}}, k_{\nu d2_{ij}}, k_{\nu d3_{ij}})^T$, the control law

$$\begin{aligned} \hat{U}_i = &- 2\hat{k}_{p_{ij}} \cdot \ln(\hat{q}^*_{d_{ij}} \circ \hat{q}_{ij}) - \hat{k}_{\nu_{ij}} \cdot \xi^i_{ij} - \hat{F}_i + Ad_{\hat{q}^*_{ij}}\dot{\xi}^j_j \\ &+ Ad_{\hat{q}^*_{ij}}\xi^j_j \times \xi^i_{ij} \end{aligned} \tag{13}$$

where $\hat{k}_{\nu_{ij}} > \hat{0}, \hat{k}_{p_{ij}} > \hat{0}$, $k_{pd1_{ij}} = k_{pd2_{ij}} = k_{pd3_{ij}}$ can make the spinor $\xi^i_{ij}(t)$ converge to $\hat{0}$, and $\hat{q}_{ij}(t)$ converge to $\hat{q}_{d_{ij}}$. And further, overall system is PAS.

Proof. Inserting (13) into (12), yields

$$\dot{\xi}^i_{ij} = -2\hat{k}_{p_{ij}} \cdot \ln \hat{q}_{ij} - \hat{k}_{\nu_{ij}} \cdot \xi^i_{ij} \tag{14}$$

and using $2\ln \hat{q}_{ij} = \theta^i_{ij} + \epsilon p^i_{ij}$ it can become

$$\begin{aligned} \dot{\xi}^i_{ij} = &- \hat{k}_{p_{ij}} \cdot (\theta^i_{ij} + \epsilon p^i_{ij}) \\ &- \hat{k}_{\nu_{ij}} \cdot (\omega^i_{ij} + \epsilon(\dot{p}^i_{ij} + \omega^i_{ij} \times p^i_{ij})) \end{aligned} \tag{15}$$

Decomposing its real and dual parts, one can write

$$\dot{\omega}^i_{ij} = -k_{pr_{ij}} \cdot \theta^i_{ij} - k_{\nu r_{ij}} \cdot \omega^i_{ij} \tag{16}$$

$$\begin{aligned} (\dot{p}^i_{ij} + \omega^i_{ij} \times p^i_{ij})' = &- k_{\nu d_{ij}} \cdot (\dot{p}^i_{ij} + \omega^i_{ij} \times p^i_{ij}) \\ &- k_{pd} \cdot p^i_{ij} \end{aligned} \tag{17}$$

A Lyapunov function can be defined as

$$V = \frac{1}{2}|\xi^i_{ij}|^2 + 2|\hat{\gamma} \cdot \ln \hat{q}_{ij}|^2 \tag{18}$$

where $\hat{\gamma} = \gamma_r + \epsilon\gamma_d$ is a dual vector with every element nonzero. Apparently, it is positive definite and radially unbounded. Using (16) and (17), its derivative becomes

$$\begin{aligned}\dot{V} =& -(\omega_{ij}^i)^T(k_{\nu r_{ij}} \cdot \omega_{ij}^i) \\ & - (\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)^T(k_{\nu d_{ij}} \cdot (\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)) \\ & - (\omega_{ij}^i)^T(k_{pr_{ij}} \cdot \theta_{ij}^i) + (\gamma_r \cdot \theta_{ij}^i)^T(\gamma_r \cdot \dot{\theta}_{ij}^i) \\ & - (\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)^T(k_{pd_{ij}} \cdot p_{ij}^i) \\ & + (\gamma_d \cdot p_{ij}^i)^T(\gamma_d \cdot \dot{p}_{ij}^i)\end{aligned} \tag{19}$$

Choose $k_{pr_{ij}}$ and $k_{pd_{ij}}$, which is satisfied

$$(\omega_{ij}^i)^T(k_{pr_{ij}} \cdot \theta_{ij}^i) = (\gamma_r \cdot \theta_{ij}^i)^T(\gamma_r \cdot \dot{\theta}_{ij}^i) \tag{20}$$

$$\begin{aligned}&(\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)^T(k_{pd_{ij}} \cdot p_{ij}^i) \\ &= (\gamma_d \cdot p_{ij}^i)^T(\gamma_d \cdot \dot{p}_{ij}^i),\end{aligned} \tag{21}$$

we have

$$\begin{aligned}\dot{V} =& -(\omega_{ij}^i)^T(k_{\nu r_{ij}} \cdot \omega_{ij}^i) \\ & - (\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)^T(k_{\nu d_{ij}} \cdot (\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i)).\end{aligned} \tag{22}$$

Thus the Lyapunov function V satisfies $\dot{V} \leq 0$ with $\hat{k}_{\nu_{ij}} > \hat{0}$. From (22) , we have $\omega_{ij}^b = 0$ and $\dot{p}_{ij}^i + \omega_{ij}^i \times p_{ij}^i = 0$ when $\dot{V} = 0$. Inserting (14), yield $\dot{\hat{\xi}}_{ij}^i = -2\hat{k}_p \cdot \ln \lambda\hat{q}_{ij}$. Because every element of $\hat{k}_p$ is nonzero, if and only if $\ln \hat{q}_{ij} = \hat{0}$, we have $\dot{\hat{\xi}}_{ij}^b = \hat{0}$. According to LaSalle invariance theorem, when the origin $\hat{q}_{ij}(0) \neq -\hat{I}$, the system will globally converge to $\hat{q}_{ij} = \hat{I}$ and $\hat{\xi}_{ij}^i = \hat{0}$. Noting $\dot{\theta}_{ij}^i = \omega_{ij}^i$, from (20) and (21), yield

$$k_{pr_{ij}} = \gamma_r \cdot \gamma_r \tag{23}$$

$$(k_{pd_{ij}} - \gamma_d \cdot \gamma_d)(\dot{p}_{ij}^i)^T(k_{pd_{ij}} \cdot p_{ij}^i) = 0. \tag{24}$$

Thus (24) is satisfied if choose $k_{pd1_{ij}} = k_{pd2_{ij}} = k_{pd3_{ij}}$ and $k_{pd_{ij}} = \gamma_d \cdot \gamma_d$. To sum up, if there exists $\hat{k}_{p_{ij}}$ and $\hat{k}_{\nu_{ij}}$ such that $\hat{k}_{p_{ij}} > \hat{0}$, $k_{pd1_{ij}} = k_{pd2_{ij}} = k_{pd3_{ij}}$ and $k_{\nu_{ij}} > \hat{0}$, the control law is globally stable.

From *Theorem 2* in [7], it is known that the overall system under a spanning tree is stable, if $\hat{q}_{ij}$ converges to the desired relative configuration $\hat{q}_{d_{ij}}$ asymptotically, for any pair of UAVs i and j.

Thus, the theorem is concluded. □

4 Simulation Experiment and Result Analysis

4.1 Simulation Scenario

Suppose the leader and followers are exactly the same fixed-wing UAVs. Considering flight at a level situation, the UAV maintains a flight angle of attack to generate lift, in order to balance the gravity.

Simlation scenario is that five UAVs convergence to a desired formation in the space. Before the formation beginning to form, the leader maintain level flight at a speed of 17 m/s along the X-axis in ground coordinate system, meanwhile the followers are all flying at the speed of 16 m/s, the initial position and attitude are listed in Table 1. When the UAVs are instructed, they follow their leaders, and eventually form a linear formation. The six UAVs fly at the same attitude. The control contact graph is defined as Fig. 3. Considering the contact graph, we can design the control system block diagram of six UAVs formation, shown as Fig. 4.

Table 1: The initial position and attitude of the UAVs

	Position (X,Y,Z)	Attitude (Rotation angle, Axis)
Leader	$(40, 0, -100)\ m$	$(2.15°, [0, 1, 0])$
UAV2	$(20, -50, -90)\ m$	$(45°, [0, 0, -1])$
UAV3	$(20, 60, -110)\ m$	$(45°, [0, 0, 1])$
UAV4	$(10, -150, -100)\ m$	$(45°, [0, 0, 1])$
UAV5	$(0, 30, -100)\ m$	$(45°, [0, 0, 1])$
UAV6	$(0, 90, -100)\ m$	$(45°, [0, 0, 1])$

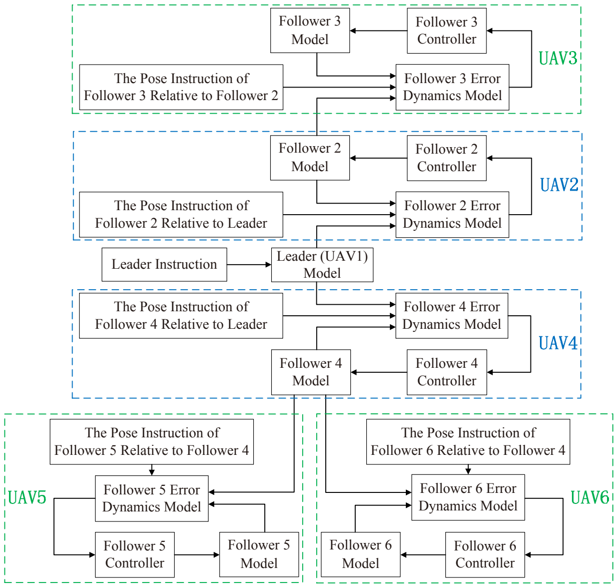

Fig. 4: Control system structure diagram

According to the control block diagram, we can construct simulation system by using Simulink, verifying the effectiveness of the proposed formation control law.

4.2 Simulation Result and Analysis

The time span of the simulation is set to be 160 s. The evolutions of these six UAVs in the formation are shown in Fig. 5. It can be seen that the leader maintains level flight, while other followers change the flight path by adjusting the angular and linear velocity, after receiving the formation formed instruction. Eventually the six UAVs converge to the specified formation configurations. In the figure the corresponding nodes on the path are UAVs' position at the same time. It is worth to note that although their aircraft flight path exist overlapping, it doesn't cause the collision because of the time difference.

Fig. 6 shows evolutions of the position over time along X-axis, Y-axis, and Z-axis in the ground coordinate system, respectively. It can be seen from the figure that after about 100 s, the relative position no longer changes; Fig. 7 shows the evolutions of attitudes. After 80 s, their attitudes tend to be consistent and stable.

This simulation demonstrates the effectiveness of multiple UAVs formation control law and verify that the formation

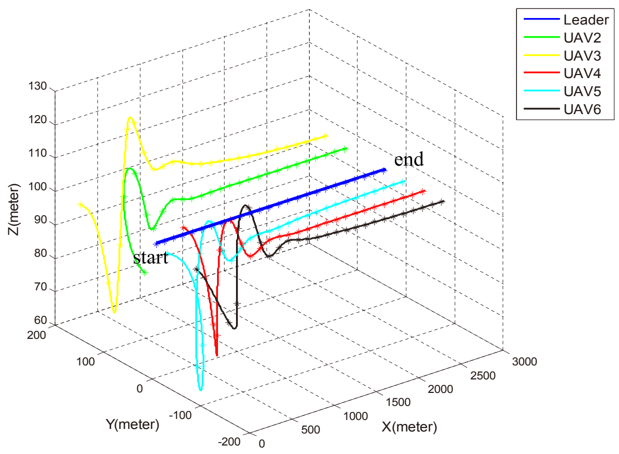

Fig. 5: The evolutions of six UAVs formation flight

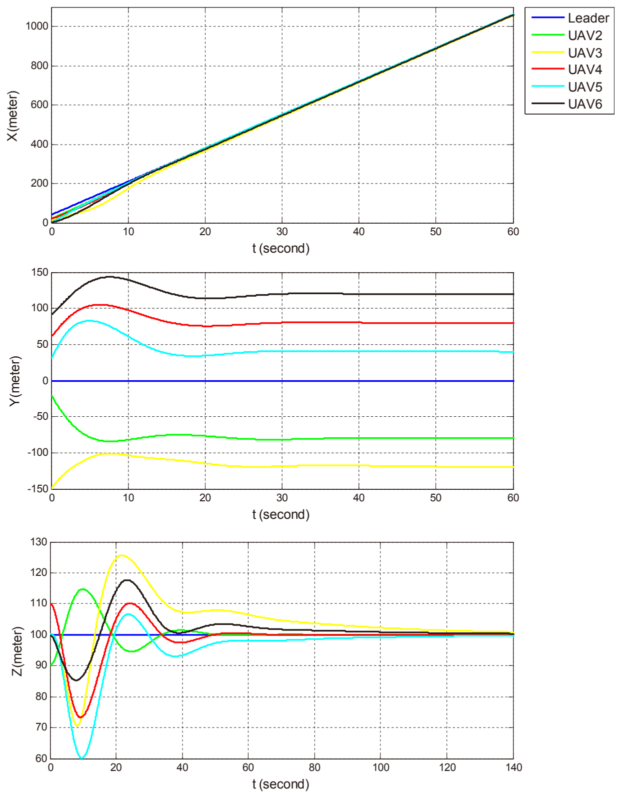

Fig. 6: Positions of each UAV in formation

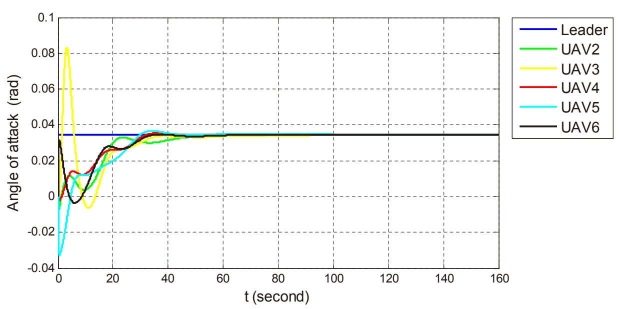

Fig. 7: Attitudes of each UAV in formation

can converge to the expected configuration.

5 Conclusion

In this paper, the dual quaternion algebra is used to study UAV formation control problem in the three-dimensional s-pace. Firstly, we study the neighbored UAVs formation mod-el and design a formation control law based on the dual quaternion to ensure the formation to form and maintain. In the formation, excepting the formation leader, every UAV follows its leader or virtual leader, so that their position and attitude can asymptotically converge to expected stable s-tates synchronously. In the further research, it is necessary to explore the formation control strategy, which is more close to the actual situation.

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