The Unit Dual Quaternion Based Flight Control for a Fixed-wing Unmanned Aerial Vehicle

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***Abstract*—In this paper the problem of controlling a fixed-wing UAV (Unmanned Aerial Vehicle) based on unit dual quaternion is studied. The unit dual quaternion is the most concise and the highest computing efficiency globally mathematical presentation to describe the attitude and position simultaneously. The dynamics of a fixed-wing UAV described by the unit dual quaternion is developed. Then a feedback linearization controller is devised according to the error dynamics of this underactuated and nonholonomic constrained system, which combined attitude and position control law to a single one. Finally, Simulation results are presented to demonstrate the effectiveness of the proposed approaches.**

I. INTRODUCTION

Unmanned Aerial Vehicle (UAV) is one of unmanned aerial system which has the fastest development, the highest level and the widest application in unmanned system areas. Since the 1990s, research on UAV has received unprecedented attention in the world. Currently, there are over 40 countries are developing the use of UAV [1]. Specially, a typical fixed-wing UAV platform is a nonholonomic constrai- ned system on the kinematic and a underactuated nonlinear system on the dynamics. As a high-speed moving vehicle, it is challenging to achieve control of the attitude and position simultaneously [2]. In this paper we exploit UAV's feature and use the unit dual quaternion to solve the problem.

The unit dual quaternion is an effective tool to describe rotation and translation in three-dimensional. It is widely used in the configuration control, navigation, computer graphics and motion design etc.. Early in 1873, based on Charles Theorem, Clifford proposed using a dual quaternion to describe a general rigid body motion. And then Studies had shown that the dual quaternion is the most simple and most efficient tool to describe rigid motion [3]. It can obviously describe general rigid motion without singularity, so as to be widely used in various research related to the general rigid motion. For example, Ge designed attitude and position trajectory without decoupling using unit dual quaternion [4]. Sariyildiz et al [5] applied spiral movement and dual quaternion theory to the control of multi joint arm, and introduced kinematics modeling method of complex system based on dual quaternion. D. P. Han et al [6], studied synchronization control problem of relative position and attitude of spacecraft under input limited conditions. They established the spacecraft motion model using dual quaternion, and developed an adaptive sliding mode control method to achieve global asymptotic stability control. With the characteristics of dual quaternion, which can describe the relative position and attitude at the same time, its application [7] in the system of satellite formation flying system can realize attitude control of satellite formation. In recent years, D. P. Han and X. K. Wang [8~10], from National University of Defense Technology, founded the Li Group structure of unit dual quaternion, and imported the unit dual quaternion logarithmic mapping into the general rigid motion to solve the problem of attitude and position synchronization control of the rigid body. Compared to the traditional attitude and position control law, the unit dual quaternion logarithm based control, which uses simple, non-singular description language, provides a unified, not decoupled solution framework for rotation and translation simultaneously control. Obviously, it has a wide potential application [11]. Using the dual quaternion logarithm feedback, researchers have designed dual quaternion algebra control method for the all-wheel-drive mobile platform, such as the omnidirectional mobile robot and the micro quadrotor. Besides, the effectiveness of the control strategies are verified on the soccer robot platform [12~13]. However, for the underactuated and nonholonomic constrained system, such as the fixed-wing UAV, there are no related work till now. The purpose of this paper is to develop a control strategy based on unit dual quaternion that guarantees global stability to the desired position and attitude simultaneously in three-dimensional space, for a fixed-wing UAV.

The outline of this paper is as follows: Section 2 provide some preliminaries including some definitions and useful model are presented. Based on unit dual quaternion, underactuated nonlinear dynamic model of a fixed-wing UAV is derived in Section 3. In section 4, a feedback linearization control strategy is proposed, also the convergence analysis of the controller is developed. Finally, simulation results for the proposed control strategy is shown and discussed in section 5.

II. PRELIMINARIES

A. Rigid Dynamics Model Based On Unit Dual Quaternion

We use the unit dual quaternion to represent the dynamic model of the general rigid motion.

A unit dual quaternion $\hat{\boldsymbol{q}} = \boldsymbol{q} + \epsilon \frac{1}{2} \boldsymbol{q} \circ \boldsymbol{p}^b$ can be used to describe the general rigid motion, where $\boldsymbol{q}$ is a unit quaternion representing rotation and $\boldsymbol{p}^b$ is a vector quaternion in the body frame representing translation.

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We employ $\boldsymbol{\omega}^b$ and $\boldsymbol{p}^b$ to describe respectively the angular and linear velocity of the movement; $\boldsymbol{J} \in \boldsymbol{R}^{3\times 3}$ represents the inertia matrix in body coordinates. $\boldsymbol{T} \in \boldsymbol{R}$ is the control moment vector and $\boldsymbol{F} \in \boldsymbol{R}$ is the control force along body frame axes. $m \in R$ is the mess of rigid. The rigid dynamics model based on unit dual quaternion can be described as [9]:

$$\begin{cases} \dot{\hat{q}} = \dfrac{1}{2}\hat{q} \circ \xi^b \\ \xi^b = \omega^b + \epsilon(\dot{p}^b + \omega^b \times p^b) \\ \dot{\xi}^b = \hat{F} + \hat{U} \end{cases} \tag{1}$$

where $a = -\boldsymbol{J}^{-1}(\omega^b \times \boldsymbol{J}\omega^b)$ and

$$\begin{cases} \hat{F} = a + \epsilon(a \times p^b + \omega^b \times \dot{p}^b) \\ \hat{U} = \boldsymbol{J}^{-1}T + \epsilon(m^{-1}F + \boldsymbol{J}^{-1}T \times p^b) \end{cases} \tag{2}$$

B. Dynamics Model of UAV in SE(3) Space

The model of fixed-wing UAV is shown in Fig. 1.

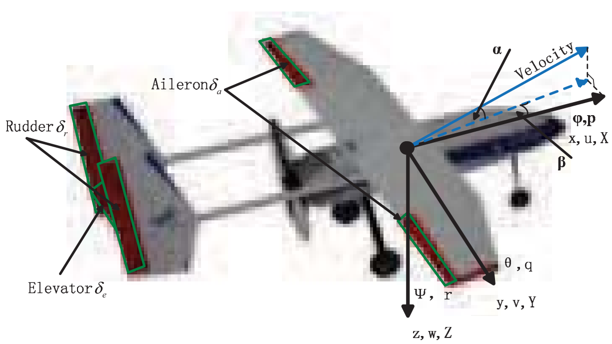

Figure 1. Geometrical Relationship in Body Frame

The moving dynamic equation of UAV in the body coordinate is [14]:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} r\cdot v - q\cdot w \\ p\cdot w - r\cdot u \\ q\cdot u - p\cdot w \end{bmatrix} + \begin{bmatrix} -\sin\theta \\ \sin\phi\cdot\cos\theta \\ \cos\phi\cdot\cos\theta \end{bmatrix} g + \begin{bmatrix} \dfrac{\overline{q}\cdot s}{m} C_{X,t} \\ \dfrac{\overline{q}\cdot s}{m} C_{Y,t} \\ \dfrac{\overline{q}\cdot s}{m} C_{Z,t} \end{bmatrix} + \begin{bmatrix} \dfrac{T}{m} \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

where u、v、w are respectively the acceleration projection along X-axis, Y-axis and Z-axis in UAV body coordinate; p、q、r are the projection of the UAV angular rate $\boldsymbol{\omega}$ along UAV frame axes; $\overline{q} = \rho V^2/2$ is the UAV dynamic pressure ($V = \sqrt{u^2+v^2+w^2}$); s is the wing area; T is the engine thrust force; ϕ、θ、ψ are the attitude angles; $\mathbf{g}$ is the gravitational acceleration; and $C_{X,t}$、$C_{Y,t}$、$C_{Z,t}$ are the dimensionless coefficients of the aerodynamic force. The expressions of them are:

$$\begin{cases} C_{X,t} = C_L \sin\alpha - C_D \cos\alpha \\ C_{Y,t} = C_{Y_\beta}\beta + C_{Y_{\delta_r}}\delta_r + \dfrac{b}{2V}\left[C_{Y_r} r + C_{Y_p} p\right] \\ C_{Z,t} = -C_D \sin\alpha - C_L \cos\alpha \end{cases} \tag{4}$$

where α and β are the attack angle and the sideslip angle, respectively; δ_r is the rudder deflection; b is the wing span, and C_L and C_D are the lift and the drag coefficient, respectively. They have the following forms [14]:

$$\begin{cases} C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e + \dfrac{\overline{c}}{2V}(C_{L_{\dot\alpha}}\dot{\alpha} + C_{L_q} q) \\ C_D = C_{D_0} + C_{D_{\delta_e}}\delta_e + C_{D_{\delta_r}}\delta_r + \dfrac{C_L - C_{L_{\min}}}{\pi\cdot e\cdot AR} \end{cases} \tag{5}$$

where δ_e is the elevator deflection and $\overline{c}$ is the aerodynamic mean chord.

The equations describing the projections of the moments along the body frame axes are as follows:

$$\begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{zx} & 0 & J_z \end{bmatrix}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = -\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{zx} & 0 & J_z \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} \overline{q}sbC_{l,t} \\ \overline{q}s\overline{c}C_{m,t} \\ \overline{q}sbC_{n,t} \end{bmatrix} \tag{6}$$

with J_x、J_y、J_z as the axial inertia moments, J_{xz} the planar inertia moment, $C_{l,t}$、$C_{m,t}$、$C_{n,t}$ the aerodynamic coefficients with the expressions:

$$\begin{cases} C_{l,t} = C_{l_\beta}\beta + C_{l_{\delta_r}}\delta_r + C_{l_{\delta_a}}\delta_a + \dfrac{b}{2V}\left(C_{l_r} r + C_{l_p} p\right) \\ C_{m,t} = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + \dfrac{\overline{c}}{2V}\left(C_{m_{\dot\alpha}}\dot{\alpha} + C_{m_q} q\right) \\ C_{n,t} = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r + C_{n_{\delta_a}}\delta_a + \dfrac{b}{2V}\left(C_{n_r} r + C_{n_p} p\right) \end{cases} \tag{7}$$

To sum up, (3) ~ (7) describes UAV translational and rotational dynamics equation. Further, we have:

$$\begin{cases} m\dfrac{\delta v^b}{\delta t}(t) = -m\omega^b(t)\times v^b(t) + \boldsymbol{R}_{ab}^T(t)\boldsymbol{e}_3(mg) + F_A^b(t) + \mathrm{F}_T^b(t) \\ \boldsymbol{J}\dot{\omega}^b(t) = -\omega^b(t)\times\left(\boldsymbol{J}\omega^b(t)\right) + \tau^b(t) \end{cases} \tag{8}$$

where $\left[\overline{q}sC_{X,t}, \overline{q}sC_{Y,t}, \overline{q}sC_{Z,t}\right]^T$ is $\boldsymbol{F}_A^b(t)$ (aerodynamic force), $\left[\overline{q}sbC_{l,t}, \overline{q}s\overline{c}C_{m,t}, \overline{q}sbC_{n,t}\right]^T$ is $\boldsymbol{\tau}^b(t)$ (moment of force), and $[T,0,0]^T$ is $\boldsymbol{F}_T^b(t)$ (thrust force). They are all in body frame. The inertia matrix of UAV is

$$
\boldsymbol{D}=\begin{bmatrix} \bar{q}s((C_{L_0}+C_{L_\alpha}\alpha+\frac{\bar{c}}{2V}(C_{L_{\dot{\alpha}}}\dot{\alpha}+C_{L_q}q))(\sin\alpha-\frac{\cos\alpha}{\pi\cdot e\cdot AR})-(C_{D_0}-\frac{C_{L_{\min}}}{\pi\cdot e\cdot AR})\cos\alpha) \\ \bar{q}s(C_{Y_\beta}\beta+\frac{b}{2V}(C_{Y_r}r+C_{Y_p}p)) \\ -\bar{q}s((C_{D_0}-\frac{C_{L_{\min}}}{\pi\cdot e\cdot AR})\sin\alpha+(C_{L_0}+C_{L_\alpha}\alpha+\frac{\bar{c}}{2V}(C_{L_{\dot{\alpha}}}\dot{\alpha}+C_{L_q}q))(\cos\alpha+\frac{\sin\alpha}{\pi\cdot e\cdot AR})) \end{bmatrix} \tag{12}
$$

$$
\boldsymbol{J}=\begin{bmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{zx} & 0 & J_z \end{bmatrix}
$$

In UAV nonlinear dynamics equations, the controllable input is $\boldsymbol{u}=[T,\delta_e,\delta_r,\delta_a]^T$, namely thrust, and deflection angle of elevator, rudder and aileron. Relatively speaking, the UAV in three-dimensional space has six degrees of freedom, so the dynamics model (8) has the typical characteristics of underactuated and nonlinear. Designing UAV control law aims to design a control input for dynamics (8) so that UAV can track the given flight trajectory stability in the three dimensional space.

III. UAV Dynamics Based On Unit Dual Quaternion

Based on these preliminary knowledge, in this section we restated UAV dynamics model. In order to get the control law, the UAV error dynamics model in the form of unit dual quaternion is derived.

A. The Unit Dual Quaternion Form of UAV Dynamics

Model 1: Based on unit dual quaternion form of general rigid dynamics model and the dynamics equations of UAV in SE(3) space, we can get the unit dual quaternion description of UAV dynamics:

$$
\begin{cases} \dot{\hat{q}}=\frac{1}{2}\hat{q}\circ\xi^b \\ \xi^b=\omega^b+\epsilon(\dot{p}^b+\omega^b\times p^b) \\ \dot{\xi}^b=\hat{F}+\hat{U} \end{cases} \tag{9}
$$

where $a=-\boldsymbol{J}^{-1}(\omega^b\times\boldsymbol{J}\omega^b)$, and

$$
\begin{cases} \hat{F}=a+\epsilon(a\times p^b) \\ \hat{U}=\boldsymbol{J}^{-1}\tau+\epsilon(m^{-1}F+\boldsymbol{J}^{-1}\tau\times p^b) \end{cases} \tag{10}
$$

and

$$
F=q^*\circ\boldsymbol{e_3}(mg)\circ q+\boldsymbol{C}\begin{bmatrix}\delta_e\\ \delta_r\\ \delta_a\end{bmatrix}+\begin{bmatrix}T\\0\\0\end{bmatrix}+\boldsymbol{D}
$$

$$
\tau=\boldsymbol{M}\begin{bmatrix}\delta_e\\ \delta_r\\ \delta_a\end{bmatrix}+\boldsymbol{N}
$$

where

$$
\boldsymbol{C}=\begin{bmatrix} \bar{q}s(C_{L_{\delta_e}}(\sin\alpha-\frac{\cos\alpha}{\pi\cdot e\cdot AR})-C_{D_{\delta_e}}\cos\alpha) & -\bar{q}sC_{D_{\delta_r}}\cos\alpha & 0 \\ 0 & \bar{q}sC_{Y_{\delta_r}} & 0 \\ -\bar{q}s(C_{L_{\delta_e}}(\cos\alpha+\frac{\sin\alpha}{\pi\cdot e\cdot AR})+C_{D_{\delta_e}}\sin\alpha) & -\bar{q}sC_{D_{\delta_r}}\sin\alpha & 0 \end{bmatrix} \tag{11}
$$

$$
\boldsymbol{M}=\begin{bmatrix} 0 & \bar{q}sbC_{l_{\delta_r}} & \bar{q}sbC_{l_{\delta_a}} \\ \bar{q}s\bar{c}C_{m_{\delta_e}} & 0 & 0 \\ 0 & \bar{q}sbC_{n_{\delta_r}} & \bar{q}sbC_{n_{\delta_a}} \end{bmatrix} \tag{13}
$$

$$
\boldsymbol{N}=\begin{bmatrix} \bar{q}sb\left(C_{l_\beta}\beta+\frac{b}{2V}\left(C_{l_r}r+C_{l_p}p\right)\right) \\ \bar{q}s\bar{c}\left(C_{m_0}+C_{m_\alpha}\alpha+\frac{\bar{c}}{2V}\left(C_{m_{\dot{\alpha}}}\dot{\alpha}+C_{m_q}q\right)\right) \\ \bar{q}sb\left(C_{n_\beta}\beta+\frac{b}{2V}\left(C_{n_r}r+C_{n_p}p\right)\right) \end{bmatrix} \tag{14}.
$$

It is clear that only $\hat{\boldsymbol{U}}$ is correlative directly with $\boldsymbol{\tau}^b$ and $\boldsymbol{F}$, so $\hat{\boldsymbol{U}}$ can be generalized into the external input. Then the actual input of the system can be obtained by the pseudo inverse method.

Remark: Considering $\upsilon^b=\dot{p}^b+\omega^b\times p^b, \frac{\delta\upsilon^b}{\delta t}=\ddot{p}^b$, inset (8) into (1) and (2), so that we can get the unit dual quaternion form of UAV dynamics. Subject to space limitations, we will not go specifically derivation.

B. Error Dynamics Model Based On the Unit Dual Quaternion

In order to solve the tracking problem, based on model 1, we further derived the UAV error dynamics model described by dual quaternion from the left invariant error.

Model 2: (UAV error dynamics model based on dual quaternion) consider current configuration $\hat{\boldsymbol{q}}=\boldsymbol{q}+\frac{\epsilon}{2}\boldsymbol{q}\circ\boldsymbol{p}^b$ and target configuration $\hat{\boldsymbol{q}}_d=\boldsymbol{q}_d+\frac{\epsilon}{2}\boldsymbol{q}_d\circ\boldsymbol{p}_d^d$, error dynamics model is derived from left invariant error:

$$\begin{cases} \dot{\hat{q}}_e = \frac{1}{2}\hat{q}_e \circ \xi_e^b \\ \xi_e^b = \omega_e^b + \epsilon(\dot{p}_e^b + \omega_e^b \times p_e^b) = \xi^b - Ad_{\hat{q}_e^*}\xi_d^d \\ \dot{\xi}_e^b = \hat{F} - Ad_{\hat{q}_e^*}\dot{\xi}_d^d - Ad_{\hat{q}_e^*}\xi_d^d \times \xi_e^b + \hat{U} \end{cases} \quad (15)$$

where $\hat{q}_e = \hat{q}_d^* \circ \hat{q}$, $q_e = q_d^* \circ q$, $\omega_e^b = \omega^b - Ad_{q_e^*}\omega_d^d$, and $p_e^b = p^b - Ad_{q_e^*}p_d^d$; ξ^b and ξ_d^d are the spinor of $\hat{q}_d$ and $\hat{q}$ respectively. $\hat{F}$ and $\hat{U}$ are represented by equations (10) ~ (14) . We also have:

$$\begin{cases} Ad_{\hat{q}_e^*}\dot{\xi}_d^b = Ad_{q_e^*}\dot{\omega}_d^b + \epsilon v1_d \\ Ad_{\hat{q}_e^*}\xi_d^b \times \xi_e^b = Ad_{q_e^*}\omega_d^b \times \omega^b + \epsilon\left(v2_d \times \omega^b + Ad_{q_e^*}\omega_d^b \right. \\ \qquad \left. \times\left(\dot{p}^b + \omega^b \times p^b\right)\right) \end{cases} \quad (16)$$

where

$$\begin{cases} v1_d = Ad_{q_e^*}(\dot{p}_d^d + \omega_d^d \times p_d^d)' + Ad_{q_e^*}\dot{\omega}_d^d \times \dot{p}_e^b \\ v2_d = Ad_{q_e^*}(\dot{p}_d^d + \omega_d^d \times p_d^d) + Ad_{q_e^*}\omega_d^d \times p_e^b \end{cases} \quad (17)$$

IV. FEEDBACK LINEARIZATION CONTROL DESIGN

According to the model 2, design a feedback linearization tracking law, as described below:

Theorem (Feedback Linear Control Law)**:** We write $\hat{\boldsymbol{k}}_p = \boldsymbol{k}_{pr} + \epsilon\boldsymbol{k}_{pd} = (k_{pr1}, k_{pr2}, k_{pr3})^T + \epsilon(k_{pd1}, k_{pd2}, k_{pd3})^T$ and $\hat{\boldsymbol{k}}_v = \boldsymbol{k}_{vr} + \epsilon\boldsymbol{k}_{vd} = (k_{vr1}, k_{vr2}, k_{vr3})^T + \epsilon(k_{vd1}, k_{vd2}, k_{vd3})^T$, Design of tracking law:

$$\hat{U} = -2\hat{\boldsymbol{k}}_p \ln \lambda\hat{q}_e - \hat{\boldsymbol{k}}_v\xi_e^b - \hat{F} + Ad_{\hat{\boldsymbol{q}}_e^*}\dot{\boldsymbol{\xi}}_d^b + Ad_{\hat{\boldsymbol{q}}_e^*}\boldsymbol{\xi}_d^b \times \boldsymbol{\xi}_e^b \quad (18)$$

and

$$\lambda = \begin{cases} 1, & if \quad q_{e1} \geq 0 \\ -1, & others \end{cases}$$

where q_{e1} is the first element of $\hat{\boldsymbol{q}}_e$. When $\hat{\boldsymbol{k}}_p > \hat{\boldsymbol{0}}$, $\hat{\boldsymbol{k}}_v > \hat{\boldsymbol{0}}$ and $k_{pd1} = k_{pd2} = k_{pd3}$, the tracking law makes $\hat{q}_e(t)$ and $\xi_e^b(t)$ in the model 2 converge to $(\hat{\boldsymbol{I}}, \hat{\boldsymbol{0}})$ or $(-\hat{\boldsymbol{I}}, \hat{\boldsymbol{0}})$ along the shortest path.

Proof: Inseting (20) into (15), yields

$$\dot{\xi}_e^b = -2\hat{\boldsymbol{k}}_p \ln \lambda\hat{q}_e - \hat{\boldsymbol{k}}_v\xi_e^b \quad (19)$$

and using that $2\ln\hat{q}_e = \theta_e^b + \epsilon p_e^b$, it can become

$$\dot{\xi}_e^b = -\hat{\boldsymbol{k}}_p(\theta_e^b + \epsilon p_e^b) - \hat{\boldsymbol{k}}_v\left(\omega_e^b + \epsilon(\dot{p}_e^b + \omega_e^b \times p_e^b)\right) . \quad (20)$$

Decomposing its real and dual parts, one can write

$$\dot{\omega}_e^b = -\boldsymbol{k}_{pr}\theta_e^b - \boldsymbol{k}_{vr}\omega_e^b \quad (21)$$

$$(\dot{p}_e^b + \omega_e^b \times p_e^b)' = -\boldsymbol{k}_{pd}p_e^b - \boldsymbol{k}_{vd}(\dot{p}_e^b + \omega_e^b \times p_e^b) . \quad (22)$$

A Lyapunov candidate function can be defined as

$$V = \frac{1}{2}\left|\xi_e^b\right|^2 + 2\left|\hat{\gamma} \cdot \ln\hat{q}_e\right|^2 \quad (23)$$

where $\hat{\boldsymbol{\gamma}} = \boldsymbol{\gamma}_r + \epsilon\boldsymbol{\gamma}_d$ is a dual vector with every element nonzero. Apparently, it is positive definite and radially unbounded.

Using (21) and (22) its derivative becomes

$$\begin{aligned} \dot{V} = &-\left(\omega_e^b\right)^T\left(\boldsymbol{k}_{vr} \cdot \omega_e^b\right) - \left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)^T\left(\boldsymbol{k}_{vd} \cdot \left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)\right) \\ &-\left(\omega_e^b\right)^T\left(\boldsymbol{k}_{pr} \cdot \theta_e^b\right) + \left(\boldsymbol{\gamma}_r \cdot \theta_e^b\right)^T\left(\boldsymbol{\gamma}_r \cdot \dot{\theta}_e^b\right) \\ &-\left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)^T\left(\boldsymbol{k}_{pd} \cdot p_e^b\right) + \left(\boldsymbol{\gamma}_d \cdot p_e^b\right)^T\left(\boldsymbol{\gamma}_d \cdot \dot{p}_e^b\right). \end{aligned} \quad (24)$$

Choose $\boldsymbol{k}_{pr}$ and $\boldsymbol{k}_{pd}$, letting

$$\left(\omega_e^b\right)^T\left(\boldsymbol{k}_{pr} \cdot \theta_e^b\right) - \left(\boldsymbol{\gamma}_r \cdot \theta_e^b\right)^T\left(\boldsymbol{\gamma}_r \cdot \dot{\theta}_e^b\right) = 0 \quad (25)$$

$$\left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)^T\left(\boldsymbol{k}_{pd} \cdot p_e^b\right) - \left(\boldsymbol{\gamma}_d \cdot p_e^b\right)^T\left(\boldsymbol{\gamma}_d \cdot \dot{p}_e^b\right) = 0 \quad (26)$$

we have

$$\dot{V} = -\left(\omega_e^b\right)^T\left(\boldsymbol{k}_{vr} \cdot \omega_e^b\right) - \left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)^T\left(\boldsymbol{k}_{vd} \cdot \left(\dot{p}_e^b + \omega_e^b \times p_e^b\right)\right). \quad (27)$$

Thus the Lyapunov function V satisfies $\dot{V} \leq 0$ with $\hat{\boldsymbol{k}}_v > \hat{\boldsymbol{0}}$.

From (27)， it is seen that $\omega_e^b = 0$ and $\dot{p}_e^b + \omega_e^b \times p_e^b = 0$ when $\dot{V} = 0$. Inserting (19), yield $\dot{\xi}_e^b = -2\hat{\boldsymbol{k}}_p \ln\lambda\hat{q}_e$. Because every element of $\hat{\boldsymbol{k}}_p$ is nonzero, if and only if $\ln\hat{q}_e = \hat{\boldsymbol{0}}$, we have $\dot{\xi}_e^b = \hat{\boldsymbol{0}}$. According to LaSalle invariance theorem, when the origin $\hat{q}_e(0) \neq -\hat{\boldsymbol{I}}$, the system will globally converge to $\hat{q}_e = \hat{\boldsymbol{I}}$ and $\xi_e^b = \hat{\boldsymbol{0}}$.

Noting $\dot{\theta}_e^b = \omega_e^b$, from (25) and (26), yield

$$\boldsymbol{k}_{pr} = \boldsymbol{\gamma}_r \cdot \boldsymbol{\gamma}_r \quad (28)$$

$$\left(\boldsymbol{k}_{pd} - \boldsymbol{\gamma}_d \cdot \boldsymbol{\gamma}_d\right)\left(\dot{p}_e^b\right)^T p_e^b + \left(\omega_e^b \times p_e^b\right)^T\left(\boldsymbol{k}_{pd} \cdot p_e^b\right) = 0 \ . \quad (29)$$

Thus (29) is satisfied if choose $k_{pd1} = k_{pd2} = k_{pd3}$ and $\boldsymbol{k}_{pd} = \boldsymbol{\gamma}_d \cdot \boldsymbol{\gamma}_d$.

To sum up, if there exists $\hat{\boldsymbol{k}}_p$ and $\hat{\boldsymbol{k}}_v$ such that $\hat{\boldsymbol{k}}_p > \hat{\boldsymbol{0}}$, $k_{pd1} = k_{pd2} = k_{pd3}$ and $\hat{\boldsymbol{k}}_v > \hat{\boldsymbol{0}}$, the control law is globally stable.

Since $\hat{U} = f(F, \tau) = g(T, \delta_e, \delta_r, \delta_a)$, as long as the actual input can be solve by this function, the control law can be achieved on the UAV stability control.

If $\hat{U} = u_r + \epsilon u_d$ then

$$\begin{cases} u_r = Ad_{q_e^*}\dot{\omega}_d^b + Ad_{q_e^*}\omega_d^b \times \omega^b - a - \boldsymbol{k}_{pr}\cdot\theta_e^b - \boldsymbol{k}_{vr}\cdot\omega_e^b \\ u_d = v1_d + v2_d \times \omega^b + Ad_{q_e^*}\omega_d^b \times (\dot{p}^b + \omega^b \times p^b) - a \times p^b \\ \quad +\omega^b \times \dot{p}^b - \boldsymbol{k}_{pd}\cdot p_e^b - \boldsymbol{k}_{vd}\cdot(\dot{p}_e^b + \omega_e^b \times p_e^b). \end{cases} \tag{30}$$

Since the corresponding terms are equal, it is obtained that

$$\begin{cases} \tau^b = \boldsymbol{J}u_r \\ F^b = m(u_d - \tau^b \times p^b). \end{cases} \tag{31}$$

Besides,

$$\begin{bmatrix} F - q^* \circ \boldsymbol{e}_3(mg) \circ q - \boldsymbol{D} \\ \tau - \boldsymbol{N} \end{bmatrix} = \boldsymbol{A}_{6\times4}[T, \delta_e, \delta_r, \delta_a]^T$$

where $\boldsymbol{A}_{6\times4} = \begin{bmatrix} \boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3 & \boldsymbol{C} \\ \boldsymbol{0} & \boldsymbol{M} \end{bmatrix}$, so the control inputs can be obtained through

$$[T, \delta_e, \delta_r, \delta_a]^T = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T \begin{bmatrix} F - q^* \circ \boldsymbol{e}_3(mg) \circ q - \boldsymbol{D} \\ \tau - \boldsymbol{N} \end{bmatrix}. \tag{32}$$

V. Simulations On Matlab / Simulink

The proposed control strategy was simulated in Matlab /Simulink on a fixed-wing UAV model referenced in [14]. The relative control block diagram is constructed as shown in Fig. 2. The simulation time is set to be 25s.

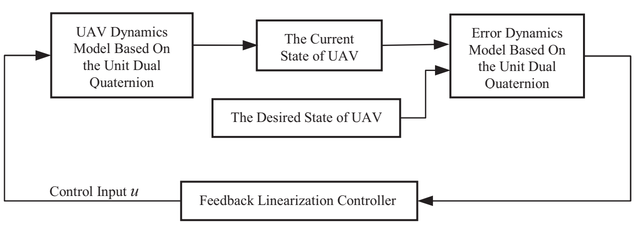

Figure 2. Fixed-wing UAV Control Block

In the simulation, the initial attitude and position are set as $\boldsymbol{q}(0) = (1,0,0,0)^T$ and $\boldsymbol{p}^s(0) = (0,0,0)^T$. The desired rotation and translation are assumed to be $\boldsymbol{p}_d^s = (5,5,5)^T$ and $\boldsymbol{q}_d = (-0.707, 0, 0.707, 0)^T$, and the desired spinor is $\boldsymbol{\xi}_d^s = \hat{\boldsymbol{0}}$.

The simulation of the UAV parameters are gathered from [14] and the gains are chosen as

$$\hat{\boldsymbol{k}}_p = (0.5, 0.5, 0.5)^T + \epsilon(1.3, 1.3, 1.3)^T ,$$

and

$$\hat{\boldsymbol{k}}_v = (1,1,1)^T + \epsilon(2,2,2)^T .$$

The simulation results are shown in Figs. 3-6. The translation and rotation of the UAV are shown in Fig. 3 and Fig. 4. Fig. 5 and Fig. 6 show the position and attitude tracking errors.

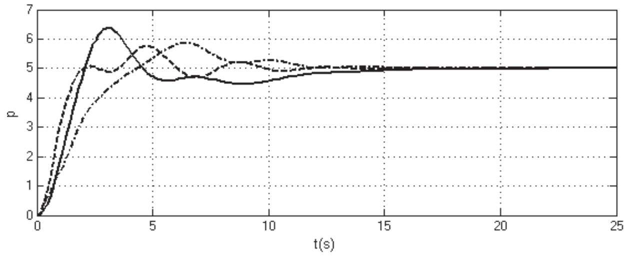

Figure 3. Three Component of Translation (p^s)

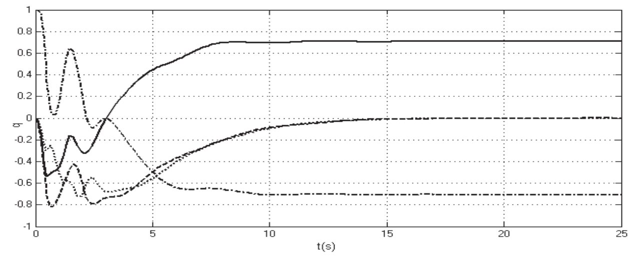

Figure 4. Four Component of Rotation (q)

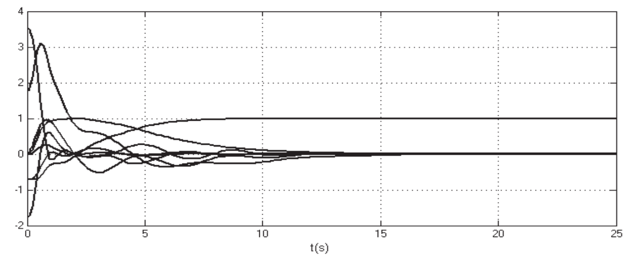

Figure 5. Each Component of State Error ($\hat{q}_e$)

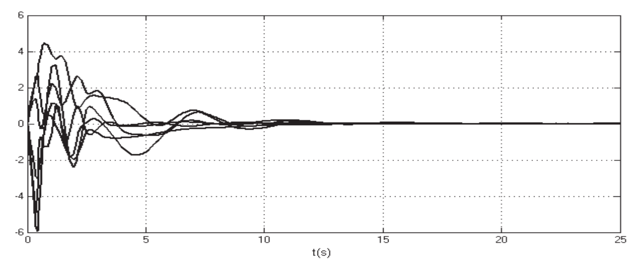

Figure 6. Each Component of Error Spinor (ξ_e^b)

As shown, the translation and rotation tracking errors converge before 15s, which validate the stability and rapid convergence of the proposed controller.

VI. Conclusion And Future Works

In this paper, the unit dual quaternion based control law is extended to be application for the nonholonomic constraints /underactuated system, and basic control framework of a fixed wing UAV based on dual quaternion is constructed. As a result, to control the UAV model is preliminary implemented.

In the following research, we will further optimize the control law, and verify in simulation system. In addition, considering difficulties of UAV modeling, such as imprecision of the inertia matrix and quality, designing the adaptive control law based on the needs of the actual system application will be more practical. Besides, it is necessary to consider the UAV nonholonomic constraint characteristics when to design the desired tracking path.

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