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Formation Stabilization with Cone-like Sensing Field

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Abstract: While most existing literature assumes agents in formation control are with no sensing limitations or with 360-degree sensing fields, we consider each agent in formation stabilization problem is with a cone-like sensing filed for the leader-follower formation in three-dimensional space. We discuss the strategies for the agent with cone-like sensing filed to follow one leader, two leaders and three leaders, respectively. And correspondingly the related control laws are proposed to achieve the formation. Finally, simulation results, including an animation, about four-agent formation with cone-like sensing field demonstrate the effectiveness of the strategies and the proposed control laws.

Key Words: Multiple Agents, Formation Stabilization, Cone-like Sensing Field, Leader-Follower

1 Introduction

Formation control is an important issue in coordinated control for a group of agents and has attracted considerable attention in recent literature due to its broad range of applications, for example, in space missions, security patrols, search and rescue in hazardous environments. Different aspects of multi-agent formation systems have been intensively studied over the last decade, see [1–7]. Among different approaches to formation control, leader-follower method, for which each agent except one takes one or more neighboring agents as reference(s) to determine its motion, has been adopted by many researchers [1–3]. The referenced agents are called *leader(s)*, the following agent is called a *follower* and the one without leader is called a *lead agent* which directly or indirectly leads all other agents in the formation. If the lead agent is still, then a formation control problem is specialized as a “formation stabilization” problem [3].

The agents considered in most of existing work are assumed to be without sensing limitations or with 360-degree sensing fields. For example, the robots in [1, 2] are equipped with omni-directional cameras, which have 360-degree views of surroundings. Robots in [3, 4] can measure the relative positions of any object (robots or environment boundary) if the object is within a given distance and there are no obstacles in between. The control laws or formation architectures discussed in [5, 6] are all assumed the agents can obtain in prior the relative information of their neighboring agents regardless of where the neighbors are. However, in many applications where GPS or other global localization techniques are not available such as in outer space or indoor environment, robots have to obtain information through the onboard sensors. At these cases it is inevitable that there are some limitations on sensing fields for robots. For example, camera, which is a frequently used onboard sensor, has the cone-like sensing capability (after all the omni-directional camera in [1] is much more sophisticated and expensive, and is not widely used in robots); Lidar (typically Hokuyo Lidar productions), another useful onboard sensor, can only scan a sector area in a specified plane. Therefore in this study, we focus on one typical limitation in multi-agent formation stabilization with the leader-follower structure in three-dimensional (3D) space, that is the agents considered have cone-like fields of view, or each agent in formation can only obtain the relative information of their neighbors which locate in a given cone area centered at the agent itself, just as each agent is equipped with an onboard camera.

Two key points are separately considered because of the cone-like sensing field. First, the follower should keep the leader(s) always in the cone-like field during the whole evolution of motion if the leader(s) are in its sensing field at the initial time instant. Second, the orientation of the follower can and should be adjusted together with its motion to keep the leader(s) in appropriate relative positions such that the follower can require more accurate information or/and the leader(s) have more redundancy of motion. Cone-like field of view is also considered in [7], but it is quite different from our focus without considerations of above two points. In our study, the problem is decomposed into two aspects: “where should (it) go” and “how to go”. As pointed out in [7], formation with cone-like view field agents can be modeled by a directed graph. Further each follower is at most able to follow three independent leaders for persistent formation in 3D space (refer to [5, 6] for the related notions about ‘directed graph’, ‘persistent formation’). Therefore, in “where should go” aspect we discuss the strategies for an agent with cone-like sensing filed to follow one leader, two leaders and three leaders, respectively. The strategies are to obtain an appropriate position which satisfies all constraints and an appropriate orientation point where follower should heading to. Afterwards, in “ how to go” aspect our work is to control the follower to achieve the strategies. In this paper we recall a new control law by means of unit dual quaternion to deal with this task, viz. to make the followers converge asymptotically to the designed positions and orientations simultaneously.

A dual quaternion is a natural extension of a quaternion.

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It provides an efficient global representation for transformation (translation and rotation simultaneously) without singularities. It is more compact and computationally efficient to represent transformation than other mathematical tools, such as homogeneous transformation matrix, quaternion/vector pairs, Lie algebra and alike [8, 9]. Readers, who are not familiar with the notions of quaternion and dual quaternion, can browse [10] first to obtain some preliminary knowledge. In many applications related to transformation, such as Motion Design [11], Hand-Eye Calibration [12] and Navigation [13], the unit dual quaternion is an elegant and useful tool. Especially, in [14, 15], the logarithmic feedback of unit dual quaternion is utilized to derive controllers, which can control rotation and translation simultaneously. In this paper, we recall the control law in [14] to make the follower achieve the designed position and orientation simultaneously.

The rest of this paper is organized as follows. The problem studied is stated in Section 2. The strategies, including for an agent to follow one leader, two leaders and three leaders, are discussed in Section 3. Section 4 recalls and discusses the control law based on the unit dual quaternion to achieve the strategies. Simulation results, including an animation, about a formation composing of four agents with cone-like sensing fields are presented in Section 5 to show the effectiveness of the strategies and control law. Finally, the last section draws the conclusions and proposes future work.

2 Problem Statement

Consider a group of agents with cone-like sensing fields in 3D space. We assume each cone-like sensing field is a *right circular cone* (shown in Fig. 1), where *right* means that the *axis* passes through the center of the *base* at right angle to its plane, and *circular* means that the *base* is a *circle*. The *perimeter* of the base of a cone is called the *directrix*, and each of the line segments between the directrix and apex is a *generatrix* of the lateral surface. The *aperture* of a right circular cone is the maximum angle between two generatrix lines; if the generatrix makes an angle θ to the axis, the aperture is 2θ. The base radius of a circular cone is the *radius* of its base; often this is simply called the *radius* of the cone. The *height* of the right circular cone is the distance between the apex and the center of the base.

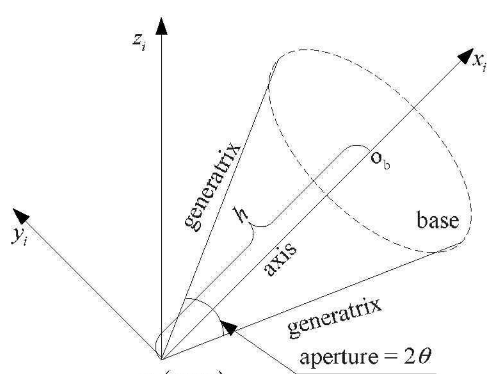

Fig. 1: Sensing field of agent: right circular cone

The right-handed coordinate system is established for each agent, where the origin is the position of agent and the x-axis is the axis of the right circular cone apart from agent, as shown in Fig. 1. Such coordinate system is called *body-frame i*. Denote the sensing field of agent i by $cone_i$. We assume the height of $cone_i$ is h_i and the aperture of $cone_i$ is $2\theta_i$, namely the angle between generatrix and the axis is θ_i in this study. And further we assume agent i can only sense the relative positions of the agents which are in $cone_i$. If agent j is not in $cone_i$, then agent i can not obtain any information about agent j.

The objective of the paper is to study the formation stabilization problem for multiple agents with leader-follower structure and cone-like sensing fields. Assume that at initial time instant each agent starts at a different location, and the formation shape is specified by different desired distance constraints. Further, the lead agent in formation is still. Then we aim to design an appropriate strategy together with the related control laws to make agents achieve the specified shape by solving "where should go" and "how to go" problems with the local information that agent i can sense.

In the sequel, we denote the relative positions of agent j, k and m in body-frame i by p_j^i, p_k^i and p_m^i, respectively, denote the specified distances between agent i and j, k, m by d_{ij}^*, d_{ik}^* and d_{im}^*, respectively. The "where should go" aspect and the "how to go" aspect are discussed in Section 3 and 4, respectively.

3 Strategies

We focus on "where should go" aspect in this section. We assume the distance constraints are all meaningful and each agent can move and rotate simultaneously. To solve "where should go" problem, it is required for the follower not only to obtain an appropriate position (call the *desired position*) which satisfies all constraints but also to obtain an appropriate orientation point (call the *desired orientation point*) where the follower should heading to. The rotation of agent i is to keep its leader(s) in sound position(s) in body-frame i. Generally speaking, more closer to the axis of $cone_i$, more accurate the locations of leaders will be obtained by follower i, and more redundant space will hold for the motions of leaders. Note that the desired orientation point is not considered in the formation problem without sensing limitation.

In general, formation with cone-like sensing field should be persistent (after modeled by a directed graph) and each follower is at most able to follow three independent leaders in 3D space. Though there may be many pairs of leader and follower in leader-follower method, there are just three basic cases for a persistent formation, which are agent i follows one leader j with specified distance d_{ij}^*, agent i follows two leaders j and k with specified distances d_{ij}^* and d_{ik}^*, and agent i follows three leaders j, k and m with specified distances d_{ij}^*, d_{im}^* and d_{im}^*. Complex persistent formations in 3D space can be achieved easily by fitting together these three basic cases. Hence in the sequel, we will discuss how to obtain the desired positions and the desired orientation points for three basic cases. If there are more than one desired positions satisfied all distance constraints, then the one with shortest distance to agent i will be chosen. We denote further the desired position and the desired orientation point in body-frame i by $p_{i'}^i$ and p_h^i, respectively, and the desired orientation of agent i by $q_{i'}^i$ referring to body-frame i.

3.1 Case 1: Following one leader

This case is simple. If agent i only follows one leader j with specified distance $d_{ij}^* < h_i$, then $p_{i'}^i$, the position met d_{ij}^* constraint, should be chosen on the line of agent i and agent j with correct distance. That is

$$p_{i'}^i = (1 - \frac{d_{ij}^*}{d_{ij}})p_j^i, \tag{1}$$

where $d_{ij} = |p_j^i|$ is the distance between agent i and j. The orientation of agent i is required to keep the leader j on the *axis* of $cone_i$, thus the desired orientation point is simply considered as $p_h^i = p_j^i$.

3.2 Case 2: Following two leaders

In this case, agent i follows two leaders j and k with specified distances d_{ij}^* and d_{ik}^*, respectively. Note that leaders j and k must be in the sensing field of agent i and $d_{ij}^* + d_{ik}^* > d_{jk}$, where d_{jk} is the distance between leader j and k. Denote the positions of leaders j and k by $p_j^i = (x_j^i, y_j^i, z_j^i)^T$ and $p_k^i = (x_k^i, y_k^i, z_k^i)^T$ in body-frame i, respectively. Then the best heading of agent i is the bisection of angle β_i subtended at agent i by two agents j and k (as Fig. 2). Denote the intersection of the bisector and $\overrightarrow{p_j^i p_k^i}$ by p_h^i. According to the property of bisector, equation $|\overrightarrow{p_j^i p_h^i}|/|\overrightarrow{p_h^i p_k^i}| = d_{ij}/d_{ik}$ holds where d_{ij} and d_{ik} are the distances between agent i and leader j and k, respectively. Then

$$p_h^i = \frac{d_{ik}}{d_{ij} + d_{ik}} p_j^i + \frac{d_{ij}}{d_{ij} + d_{ik}} p_k^i. \tag{2}$$

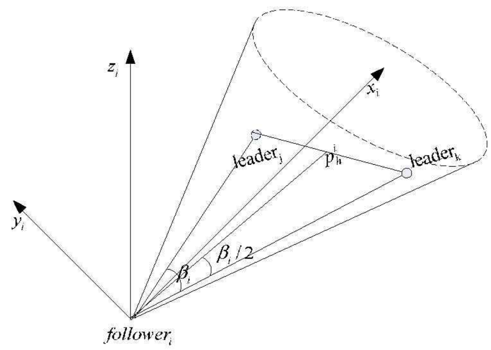

Fig. 2: Illustration for agent i following two leaders j and k

Obviously, the positions met d_{ij}^* and d_{ik}^* constraints are located on a circle in 3D space and it is easy to know the closest point $p_{i'}^i$ in this circle is also in the plane constructed by agent i, j and k (denoted by face ijk). The normal of face ijk is $\overrightarrow{ij} \times \overrightarrow{ik} = (y_j^i z_k^i - y_k^i z_j^i, -(x_j^i z_k^i - x_k^i z_j^i), x_j^i y_k^i - x_k^i y_j^i)^T$, correspondingly face ijk is represented as

$$(y_j^i z_k^i - y_k^i z_j^i)x - (x_j^i z_k^i - x_k^i z_j^i)y + (x_j^i y_k^i - x_k^i y_j^i)z = 0. \tag{3}$$

Considering d_{ij}^* and d_{ik}^* constraints, we obtain

$$(x - x_j^i)^2 + (y - y_j^i)^2 + (z - z_j^i)^2 = {d_{ij}^*}^2, \tag{4}$$

$$(x - x_k^i)^2 + (y - y_k^i)^2 + (z - z_k^i)^2 = {d_{ik}^*}^2. \tag{5}$$

From (3), (4) and (5), $p_{i'}^i$ is solved, that is [1]

$$p_{i'}^i = (\frac{b_1c_2 - b_2c_1}{a_1b_2}z - \frac{b_1 d}{b_2}, -\frac{c_2}{b_2}z + \frac{a_1 d}{b_2}, z)^T, \tag{6}$$

with $z = \frac{-b_3 \pm \sqrt{b_3^2 - 4a_3c_3}}{2a_3}$, where $a_3 = (\frac{b_1c_2 - b_2c_1}{a_1b_2})^2 + (\frac{c_2}{b_2})^2 + 1$, $b_3 = -2\frac{b_1c_2 + b_2c_1}{a_1b_2}(\frac{b_1}{b_2}d + x_j^i) - 2\frac{c_2}{b_2}(\frac{a_1}{b_2}d - y_j^i) - 2z_j^i$, $c_3 = (\frac{b_1}{b_2}d - x_j^i)^2 + (\frac{a_1}{b_2}d - y_j^i)^2 + {z_j^i}^2 - {d_{ij}^*}^2$, $b_2 = 2(y_k^i - y_j^i)a_1 - 2(x_k^i - x_j^i)b_1$, $c_2 = 2(z_k^i - z_j^i)a_1 - 2(x_k^i - x_j^i)c_1$, $a_1 = y_j^i z_k^i - y_k^i z_j^i$, $b_1 = -(x_j^i z_k^i - x_k^i z_j^i)$, $c_1 = x_j^i y_k^i - x_k^i y_j^i$ and $d = {d_{ij}^*}^2 - {d_{ik}^*}^2 + {x_k^i}^2 + {y_k^i}^2 + {z_k^i}^2 - {x_j^i}^2 - {y_j^i}^2 - {z_j^i}^2$. It should be noted that there are two solutions, and the closer one, viz. the one with smaller $|p_{i'}^i|$, is our expectation.

3.3 Case 3: Following three leaders

In this case, agent i follows three leaders j, k and m with specified distances d_{ij}^*, d_{ik}^* and d_{im}^*, respectively. If agent i and three leaders j, k and m are coplanar, then the heading of the follower i is the bisection of max angle subtended at agent i by any two agents among j, k and m, which is similar to case 2. If agent i and three leaders are not coplanar, then the follower i is best heading to the circumcenter of triangle $j'k'm'$, where j', k' and m' are the projections of leaders j, k and m on any plane perpendicular to x-axis, respectively (shown in Fig. 3).

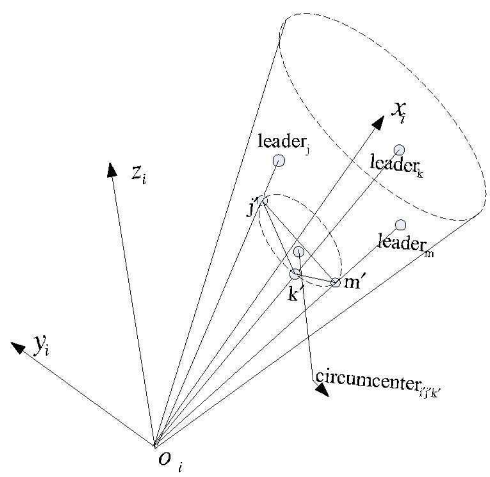

Fig. 3: Illustration for agent i following three leaders.

Assume the positions of leaders j, k and m are $p_j^i = (x_j^i, y_j^i, z_j^i)^T$, $p_k^i = (x_k^i, y_k^i, z_k^i)^T$ and $p_m^i = (x_m^i, y_m^i, z_m^i)^T$, respectively. Without loss of generality, we make j' coincide with j, which means $p_{j'}^i = p_j^i$. Then the coordinations of k' and m' are $p_{k'}^i = \frac{x_j^i}{x_k^i}p_k^i$ and $p_{m'}^i = \frac{x_j^i}{x_m^i}p_m^i$ as the projective plane $j'k'm'$ is perpendicular to the x-axis. After some operations, the circumcenter of triangle $jk'm'$ is $c_{jk'm'} = (D_1/D, D_2/D, D_3/D)$, where

$$D = \begin{vmatrix} n_1 & n_2 & n_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}, \quad D_1 = \begin{vmatrix} -d & n_2 & n_3 \\ -d_1 & b_1 & c_1 \\ -d_2 & b_2 & c_2 \end{vmatrix},$$

[1]This solution is the general solution without considering $b_2 = 0$, $a_1 = 0$ or $a_3 = 0$. Due to space limitation, the details are not discussed herein for simplification, but considered in simulations.

$$D_2 = \begin{vmatrix} n_1 & -d & n_3 \\ a_1 & -d_1 & c_1 \\ a_2 & -d_2 & c_2 \end{vmatrix}, \quad D_3 = \begin{vmatrix} n_1 & n_2 & -d \\ a_1 & b_1 & -d_1 \\ a_2 & b_2 & -d_2 \end{vmatrix},$$

in which $n_1 = (y^i_{k'} - y^i_j)(z^i_{m'} - z^i_j) - (y^i_{m'} - y^i_j)(z^i_{k'} - z^i_j)$, $n_2 = -(x^i_{k'} - x^i_j)(z^i_{m'} - z^i_j) + (x^i_{m'} - x^i_j)(z^i_{k'} - z^i_j)$, $n_3 = (x^i_{k'} - x^i_j)(y^i_{m'} - y^i_j) - (x^i_{m'} - x^i_j)(y^i_{k'} - y^i_j)$, $a_1 = 2(x^i_{k'} - x^i_j)$, $b_1 = 2(y^i_{k'} - y^i_j)$, $c_1 = 2(z^i_{k'} - z^i_j)$, $a_2 = 2(x^i_{m'} - x^i_j)$, $b_2 = 2(y^i_{m'} - y^i_j)$, $c_2 = 2(z^i_{m'} - z^i_j)$, $d_1 = ({x^i_j}^2 + {y^i_j}^2 + {z^i_j}^2) - ({x^i_{k'}}^2 + {y^i_{k'}}^2 + {z^i_{k'}}^2)$, $d_2 = ({x^i_j}^2 + {y^i_j}^2 + {z^i_j}^2) - ({x^i_{m'}}^2 + {y^i_{m'}}^2 + {z^i_{m'}}^2)$ and $d = -n_1 x^i_j - n_2 y^i_j - n_3 z^i_j$.

Obviously, the position $p^i_{i'}$, which met d^*_{ij}, d^*_{ik} and d^*_{im} constraints and closest to agent i, can be solved from the following equations

$$\begin{cases} (x - x^i_j)^2 + (y - y^i_j)^2 + (z - z^i_j)^2 = {d^*_{ij}}^2 \\ (x - x^i_k)^2 + (y - y^i_k)^2 + (z - z^i_k)^2 = {d^*_{ik}}^2 \\ (x - x^i_m)^2 + (y - y^i_m)^2 + (z - z^i_m)^2 = {d^*_{im}}^2 \end{cases}.$$

That is [2]

$$p^i_{i'} = \left(\frac{c_2 b_{jm} - b_2 c_{jm}}{b_2 a_{jm}} z + \frac{b_2 d_{jm} - d_2 b_{jm}}{b_2 a_{jm}}, -\frac{c_2}{b_2} z + \frac{d_2}{b_2}, z\right)^T,$$

where $z = \frac{-b_3 \pm \sqrt{b_3^2 - 4a_3 c_3}}{2a_3}$, $a_3 = (\frac{c_2 b_{jm} - b_2 c_{jm}}{b_2 c_{jm}})^2 + (\frac{c_2}{b_2})^2 + 1$, $b_3 = 2\frac{c_2 b_{jm} - b_2 c_{jm}}{b_2 c_{jm}}(\frac{b_2 d_{jm} - d_2 b_{jm}}{b_2 c_{jm}} - x^i_j) - 2\frac{c_2}{b_2}(\frac{d_2}{b_2 c_{jm}} - y^i_j) - 2z^i_j$, $c_3 = (\frac{b_2 d_{jm} - d_2 b_{jm}}{b_2} - x^i_j)^2 + (\frac{d_2}{b_2} - y^i_j)^2 + {z^i_j}^2 - {d^*_{ij}}^2$, $b_2 = b_{jk} a_{jm} - b_{jm} a_{jk}$, $c_2 = c_{jk} a_{jm} - c_{jm} a_{jk}$, $d_2 = d_{jk} a_{jm} - d_{jm} a_{jk}$, $a_{jk} = 2(x^i_k - x^i_j)$, $b_{jk} = 2(y^i_k - y^i_j)$, $c_{jk} = 2(z^i_k - z^i_j)$, $a_{jm} = 2(x^i_m - x^i_j)$, $b_{jm} = 2(y^i_m - y^i_j)$, $c_{jm} = 2(z^i_m - z^i_j)$, $d_{jk} = {d^*_{ij}}^2 - {d^*_{ik}}^2 + {x^i_k}^2 + {y^i_k}^2 + {z^i_k}^2 - {x^i_j}^2 - {y^i_j}^2 - {z^i_j}^2$, and $d_{jm} = {d^*_{ij}}^2 - {d^*_{im}}^2 + {x^i_m}^2 + {y^i_m}^2 + {z^i_m}^2 - {x^i_j}^2 - {y^i_j}^2 - {z^i_j}^2$.

Remark: If there are more than three agents in the sensing field of agent i, then for a persistent formation, agent i has to choose no more than three (virtual) agents to be leaders. One possible selection is: projecting all the agents in a projective plane which is perpendicular to the x-axis first. If all agents project in the same point, then this point can be selected as the virtual leader and case 1 can be adopted. Similarly, if there are two or three projective points, case 2 or 3 can be adopted too. If there are more than three projective points, then we can circumscribe all the projective points firstly and then select any three points on the circumscribed circle as the virtual leaders.

4 Control Methods

In the previous section, the desired positions and the desired orientation points have been obtained. In the sequel, "how to go" aspect, which is to design a control law for agent i to achieve the desired positions and the desired orientation points simultaneously, will be discussed.

The composition of position and orientation in 3D space can be termed by 'configuration' and be represented by a unit dual quaternion. Denote the desired configuration (composition of the desired position and orientation) by a unit dual quaternion $\hat{q}^i_{i'}$ referred to body-frame i. Then $\hat{q}^i_{i'}$ can be represented as $\hat{q}^i_{i'} = q^i_{i'} + \frac{\epsilon}{2} q^i_{i'} \circ p^i_{i'}$, where $q^i_{i'}$ is the desired orientation and will be obtained from p^i_h latter. The kinematic equation of $\hat{q}^i_{i'}$ can be expressed with unit dual quaternion language as follows:

$$\dot{\hat{q}}^i_{i'} = \frac{1}{2} \hat{q}^i_{i'} \circ \xi^i_{i'}, \quad (7)$$

where $\xi^i_{i'} = \omega^i_{i'} + \epsilon(\dot{p}^i_{i'} + \omega^i_{i'} \times p^i_{i'})$ is the *twist in body-frame* i (control input) of agent i including the angular velocity $\omega^i_{i'}$ and the linear velocity $\dot{p}^i_{i'}$[13]. Correspondingly, our control objective: to design a control law for agent i to achieve the desired position and orientation simultaneously is then equivalent to make $\hat{q}^i_{i'}$ converge into $\hat{Q}_I = (1,0,0,0) + \epsilon(0,0,0,0)$ when $t \to \infty$ by designing appropriate $\xi^i_{i'}$.

Investigate $\hat{q}^i_{i'}$ in (7), which requires $q^i_{i'}$ and $p^i_{i'}$. The $p^i_{i'}$ has been gotten directly as well as the desired orientation point p^i_h. To obtain $q^i_{i'}$ from p^i_h, we require the following lemma, whose proof is in Appendix A.

Lemma 1. *Supposed coordination of* target T *is* (x_t, y_t, z_t) *in a local frame* $O - xyz$ *as shown in Fig. 4, the orientation of vector* $\overrightarrow{OT}$ *in the local frame can be represented by a unit quaternion as* $q_t = (\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\vec{n})$, *where* $\alpha = \arccos\frac{x_t}{d}$ *with* $d = \sqrt{x_t^2 + y_t^2 + z_t^2}$ *and* $\vec{n} = (0, -z_t/d_1, y_t/d_1)^T$ *with* $d_1 = \sqrt{z_t^2 + y_t^2}$.

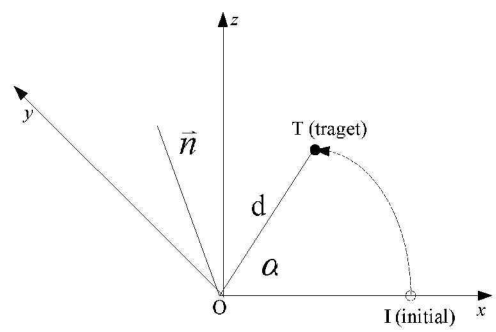

Fig. 4: Illustration for obtaining the quaternion of a vector

Denote $p^i_h = (x^i_h, y^i_h, z^i_h)^T$. According to Lemma 1, we obtain

$$q^i_{i'} = (\cos\frac{\alpha_i}{2}, \sin\frac{\alpha_i}{2}\vec{n}_i),$$

where $\alpha_i = \arccos\frac{x^i_h}{d_{ih}}$ with $d_{ih} = \sqrt{{x^i_h}^2 + {y^i_h}^2 + {z^i_h}^2}$, $\vec{n}_i = (0, -z^i_h/d_{1ih}, y^i_h/d_{1ih})^T$ with $d_{1ih} = \sqrt{{z^i_h}^2 + {y^i_h}^2}$. By composition of the desired position $p^i_{i'}$ and the desired orientation $q^i_{i'}$, the desired configuration $\hat{q}^i_{i'}$ required in (7) is obtained.

A generalized proportional control law on unit dual quaternion is then recalled from [14] to adopt our task, which is

$$\xi^i_{i'} = 2\hat{k}_i \cdot \ln \hat{q}^i_{i'}, \quad (8)$$

where $\hat{k}_i = k_{ri} + \epsilon k_{di}$ is a dual vector with each component greater than 0. Denote $\ln \hat{q}^i_{i'} = \frac{1}{2}(\alpha^i_{i'} + \epsilon p^i_{i'})$. In [14], it has proven that control law (8) ensure $|\alpha^i_{i'}|$ and $|p^i_{i'}|$ asymptotically converge to 0. That means when $t \to \infty$, $\hat{q}^i_{i'}(t) \to \hat{Q}_I$,

[2]Again we assume the solution always exists, and here we just give the general solution for simplification. More details are considered in simulations.

and $|\alpha^i_{i'}|$ and $|p^i_{i'}|$ will become smaller and smaller. Thus when the leader(s) are still, once the leader(s) are in $cone_i$, they will always be in $cone_i$ under control law (8). The lead agent in formation stabilization problem is still, thus for the agents taking the lead agent as the only leader, the lead agent is always in their sensing fields. It should be noted that when the leader(s) are moving, this control law can not ensure the leaders always in $cone_i$ as it does not restrict the behaviors of leaders. Of course, we can plan the motions of leaders to ensure them always in $cone_i$. In the formation stabilization problem with specified formation shape, the leader(s) can be guaranteed always in $cone_i$ by designing appropriate $\hat{k}_i$ in (8), but this work is beyond the scope of this paper.

5 Simulation Results

We consider four-agent formation problem with cone-like sensing fields in simulations, where agent 1 is the lead agent and in the sensing fields of agent 2, 3 and 4, agent 2 is in the sensing fields of agent 3 and 4, and agent 3 is in the sensing field of agent 4. Agent 2 is required to follow agent 1 with specified distance d^*_{12}; agent 3 is required to follow agent 1 and 2 with specified distances d^*_{13} and d^*_{23}, respectively; and agent 4 is required to follow agent 1, 2 and 3 with specified distances d^*_{14}, d^*_{24} and d^*_{34}, respectively. The control relationships can be modeled by a directed graph as Fig. 5.

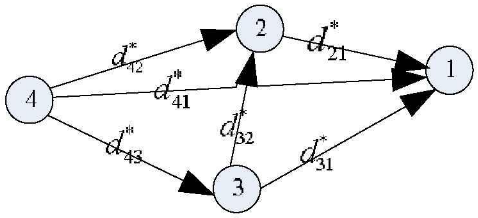

Fig. 5: Directed graph of four-agent formation in simulation

The sensing field of agent i ($i = 2, 3, 4$) is right circular cone with aperture 0.5π and height 200m, namely $\theta_i = \frac{\pi}{4}$ and $h_i = 200$m for each $cone_i$. For convenience and clarity, the position p_i and the orientation q_i are all expressed in spatial-frame in this study. In simulations, these positions and orientations are translated into body-frame in real time. The initial orientation q_i and position p_i are set in Table 1. All the leaders are in the sensing fields of the followers at initial instant. The specified distances $d^*_{21} = 30$m, $d^*_{31} = 40$m, $d^*_{32} = 40$m, $d^*_{41} = 160$m, $d^*_{42} = 160$m and $d^*_{43} = 140$m. The parameters in (8) are $\hat{k}_i = (1, 1, 1)^T + \epsilon(2, 2, 2)^T$ for $i = 2, 3, 4$.

Table 1: Initial orientations and positions of 4 agents

$agent$	q_i	p_i
1		$(200; 30; 30)$
2	$0.8660 + 0i - 0.3535j + 0.3535k$	$(200; -20; -10)$
3	$1 + 0i + 0j + 0k$	$(160; 10; 10)$
4	$0.9848 + 0i - 0.1228j + 0.1228k$	$(60; 15; 15)$

The simulations are performed with Simulink on Matlab for a time span of 10s with the sample time 0.01s. An animation about four agents with cone-like sensing fields has been built for this scenario to show the motions of the agents and the rotations of their sensing fields under control law (8). The video of the animation is available at `http://cecs.anu.edu.au/~bradyu/ccc2011.rar`. It is shown clearly that in the video, agent 2, 3 and 4 move and rotate to the desired positions and orientations with keeping their leaders always in their sensing fields, respectively. The evolutions of the distances between followers and leaders, and the angles between the axes of the sensing fields and the leaders are shown in Fig. 6 and Fig. 7. In Fig. 6, all the distances between followers and their leaders, viz. the distances from agent 2 to 1, from agent 3 to 1 and 2, from agent 4 to 1, 2, and 3, all converge to the specified distances asymptotically. In Fig. 7, the angle between the axis of $cone_2$ and agent 1 converges to 0 asymptotically, the angles from the axis of $cone_3$ to agent 1 and 2 converge asymptotically to a same value as well as the angles from the axis of $cone_4$ to agent 1, 2 and 3. Further, all the distances in the evolutions are smaller than $h_i = 200$m and the angles are smaller than $\theta_i = \frac{\pi}{4}$, which implies the leaders are always in the followers' sensing fields.

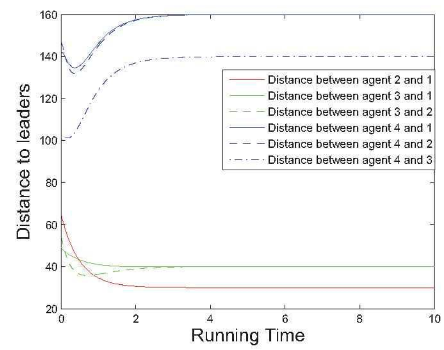

Fig. 6: Evolutions of distances between followers and leaders

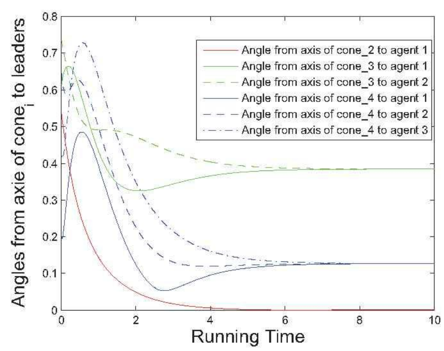

Fig. 7: Evolutions of angles between axis of $cone_i$ and leaders

6 Concluding Remarks

In this paper, we consider a special class of multi-agent formation stabilization problem in 3D space with leader-follower structure, where each agent is with a cone-like field of view. The problem is decomposed into "where should (it) go" and "how to go" aspects. In "where should go" aspect the strategies for an agent with cone-like sensing filed to follow one leader, two leaders and three leaders are discussed, respectively. In "how to go" aspect, a unit-dual-quaternion-based control law is recalled to ensure the agents asymptotically converge to the designed positions and orientations simultaneously. Simulation results, including an animation, about four-agent formation with cone-like sensing field, are presented to show the effectiveness.

Beyond the study of this paper, some future work can be envisaged. First, all agents except the lead agent are moving, accordingly the twist of the desired configuration $\hat{q}^{i}_{i'}$ can be obtained from the leaders' velocities. If extending control law (8) by considering the desired twist (similar to the further discussion in [14]), more rapid convergence rate can be expected than that with control law (8). In addition, when the lead agent is moving (which relates to the formation tracking problem), the control law (8) can not ensure the leaders always in $cone_i$. To guarantee this, the lead agent should check their desired positions real time and communicate with the followers, which means some checking rules must be given and the motions of the lead agent must be designed carefully.

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A Proof of Lemma 1

Consider a new point I (*initial*) with coordination $(d, 0, 0)$ where $d = \sqrt{x_t^2 + y_t^2 + z_t^2}$ in local frame $O - xyz$ (refer to Fig. 4). Obviously, vector $\overrightarrow{OT}$ can be obtained by rotating vector $\overrightarrow{OI}$ on the unit Normal $\vec{n}$ of Face TOI through angle α. Considering $\vec{n} \perp \overrightarrow{OI}$, $\vec{n} \perp \overrightarrow{OT}$ and $\overrightarrow{OI} \times \overrightarrow{OT} = (d, 0, 0)^T \times (x_t, y_t, z_t)^T = (0, -dz_t, dy_t)$, the unit normal $\vec{n}$ is $\vec{n} = (0, -z_t/d_1, y_t/d_1)$ with $d_1 = \sqrt{z_t^2 + y_t^2}$. Note that α is the angle from vector $\overrightarrow{OI}$ to $\overrightarrow{OT}$, therefore it satisfies

$$\cos\alpha = \frac{< \overrightarrow{OI}, \overrightarrow{OT} >}{|\overrightarrow{OI}||\overrightarrow{OT}|} = \frac{x_t}{d},$$

which implies

$$\alpha = \arccos\frac{x_t}{d}.$$

The orientation of $\overrightarrow{OI}$ is $Q_I = (1, 0, 0, 0)$ in local frame $O - xyz$, so the orientation of $\overrightarrow{OT}$ is

$$q_t = (\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\vec{n}).$$

We complete the proof.