Robust H_2 Consensus for Multi-Agent Systems with Parametric Uncertainties

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***Abstract*—This paper is concerned with the robust H_2 consensus problem of linear multi-agent systems in the presence of parametric uncertainties and noise signals. The robust H_2 consensus problem is defined and the design procedure for the distributed state feedback controller is developed to synchronize the network as well as guarantee a prescribed H_2 performance. As s special case, the robust consensus problem with only parametric uncertainties is also studied and is converted into the simultaneous H_∞ stabilization problems for a set of subsystems. The controller parameters are determined by solving some linear matrix inequalities.**

***Index Terms*—Multi-agent systems, parametric uncertainties, consensus, robust control, H_2 control.**

I. Introduction

Consensus control of multi-agent systems has become an emerging and active research domain in the last two decades, whose importance manifests in formation control, average tracking, resource allocation, distributed optimization, etc [1], [2], [3], [4], [5], [6], [7], [8], [9]. In practice, the multi-agent systems to be controlled usually suffer from various kinds of uncertainties, disturbances and noises, resulting in the heterogeneity of the network dynamics. This makes consensus control problem more challenging.

Till now, quite a large amount of progress has been made in the realm of robust consensus control of uncertain multi-agent systems. For instance, [10] considered the robust consensus problem for scalar systems. Synchronization problems for linear multi-agent systems subject to additive dynamic uncertainties and coprime factor uncertainties were considered in [11] and [12], respectively. The authors in [13] generalized the results in [11] to the directed graph case. Multi-agent systems communicating via uncertain networks were considered in [14], [15]. In [16], the authors tackled the stochastic uncertain network case. For more generalizations of robust consensus problems, please refer to [17], [18], [19], [20]. Essentially speaking, the aforementioned works mainly focused on robust consensus problems under various uncertainties in the frequency domain, which can be seen as perturbations on the transfer functions of the nominal agents. Actually, there is another typical class of system uncertainties in the time domain, called parametric uncertainties, which are widely encountered when the system matrices of the plants are located within certain bounded intervals [21]. Nevertheless, to the knowledge of the authors, there are few works addressing the robust consensus of multi-agent systems with time-domain parametric uncertainties. Although [22] incorporated time-varying norm-bounded uncertainties into multi-agent systems, the design objective therein was limited to achieving distributed quadratic stabilization, which severely restricted its applications in many aspects. The robust consensus problem for multi-agent systems under parametric uncertainties remains to be dealt with.

In addition to uncertainties, measurement noises will also bring considerable difficulties to the consensus problems of multi-agent systems [23]. Under measurement noises, the consensus errors generally do not converge to zero. Letting the consensus error be the performance output variable, it is known from [25], [23] that the H_2 norm of a multi-agent system characterizes the coherence/centrality of consensus. Specifically, a smaller H_2 norm represents a smaller consensus error, and thus a better consensus performance. Therefore, it is also important to study the H_2 performance of consensus networks. Note that many literatures considered H_2 performances for nominal linear multi-agent systems; see, e.g., [24], [25], [26], [27], [28], [29]. The H_2 performance for multi-agent systems with parametric uncertainties, however, remains an open problem and has not been well studied so far.

Motivated by the above discussions, in this paper we intend to tackle the robust H_2 consensus control problem for multi-agent systems whose input matrices are subject to time-varying norm-bounded parametric uncertainties. We first give the definition of the robust H_2 consensus problem. A sufficient condition as well as the corresponding design method in terms of linear matrix inequalities (LMIs) are then provided to guarantee a prescribed H_2 consensus performance level. We also study the special case when there are no noises, under which the robust H_2 consensus problem reduces to the robust consensus problem. We transform the robust consensus problem of the uncertain network to the simultaneous H_∞ quadratical stabilization problems of a set of subsystems. The robust consensus controller is further designed.

Compared to [22], this paper makes several significant improvements. Firstly, the current paper considers the consensus problem, which has more generic applications in, e.g., formation control, distributed tracking and distributed optimization, compared to the distributed stabilization problem in [22]. Secondly, by taking measurement noises into consideration,

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this paper further considers the robust H_2 performance of consensus. Thirdly, we consider multi-agent systems whose input matrices are subject to parametric uncertainties while [22] considered the state matrix uncertainty case. Besides, this work also forms crucial extensions of and complements the results in [11], [12], [13], where only frequency-domain uncertainties were considered.

The remaining part of this paper is unfolded as follows: In Section II, the robust H_2 consensus problem is defined and studied. Section III considers the robust consensus problem as a special case. Section V concludes the paper.

II. Distributed Robust H_2 Control Under Parametric Uncertainties

Consider a group of N agents. Suppose that each agent has a general linear system dynamics subject to parametric uncertainties, described as

$$\dot{x}_i = Ax_i + (B + \Delta B_i)u_i + B_2\omega_i,\ i = 1, \cdots, N, \tag{1}$$

where $x_i \in \mathbf{R}^n$ represents the state variable, $u_i \in \mathbf{R}^m$ is the control input and $\omega_i \in \mathbf{R}^q$ denotes the noise signal of the i-th agent, respectively, $A \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$ is the state matrix, $B \in \mathbf{R}^{n\times m}$ is the nominal input matrix, $B_2 \in \mathbf{R}^{n\times q}$ is the noise input matrix. In (1), ΔB_i is an unknown matrix that represents the uncertainty associated with the input matrix of agent i. Assume that ΔB_i has the form of $\Delta B_i = D\Delta_i E$, where D and E are known constant matrices that characterize the structure of uncertainties and $\Delta_i \in \mathbf{R}^{r\times k}$ being Lebesgue measurable [22]. Here, the uncertain matrix Δ_i is time-varying and bounded in the sense that

$$\Delta_i^T\Delta_i \leq \delta^2 I_r, \tag{2}$$

where $\delta > 0$ is a positive scalar and I_r denotes the r-dimensional identity matrix.

Each agent only communicates with its neighbors. An undirected graph $\mathcal{G} \triangleq \{\mathcal{V}, \mathcal{E}\}$ is used to describe the network topology among the agents, where $\mathcal{V} = \{1, 2, \ldots, N\}$ denotes the set of vertices and $\mathcal{E}$ denotes the set of edges. Throughout this paper, we assume that the following assumption holds.

Assumption 1: The graph $\mathcal{G}$ is connected, i.e., there exists an undirected path between every pair of distinct agents.

Since we want to measure the effect of the noises ω_i on the consensus error, we define a performance variable as follows [24]:

$$z_i = \frac{1}{N}\sum_{j=1}^{N} C_1(x_i - x_j), \tag{3}$$

where C_1 is a given constant matrix. Let $z = [z_1, \cdots, z_N]^T$ and $w = [w_1, \cdots, w_N]^T$.

In this section, we aim to design distributed controllers such that the states of the agents reach consensus in the sense of $x_i - x_j \to 0$ as $t \to \infty$, $\forall i = 1, \cdots, N$, for all admissible uncertainties ΔB_i satisfying (2) when $\omega_i = 0$. When noises exist, we want to minimize the consensus error index z.

It is well known that the H_2 norm of a system characterizes the variance of the output signals when the system input is Gaussian white noise with unit variance [32]. Therefore, the smaller the H_2 norm of the network is, the smaller the weighted consensus error z will be, implying that the multi-agent network has a better consensus performance. It is rather imperative and meanwhile hard to optimize the H_2 performance of consensus under parameter uncertainties. We consider the robust H_2 suboptimal consensus problem, defined as follows.

Definition 1: If the network achieves consensus when $\omega = 0$ and the H_2 norm of its transfer function $T_{\omega z}$ from ω to z is less than γ under parameter uncertainties satisfying (2), i.e., $\|T_{\omega z}\|_2 < \gamma$, then the robust H_2 suboptimal consensus problem is solved.

Assuming that the relative state information of neighboring agents can be obtained directly, we can use the following distributed state feedback controller:

$$u_i = \sum_{j=1}^{N} a_{ij}K(x_i - x_j), \tag{4}$$

where K is the feedback gain matrix to be determined and a_{ij} denotes the (i, j)-th element of the adjacency matrix of $\mathcal{G}$, defined as $a_{ii} = 0$, $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise.

The closed-loop network dynamics resulted from (1), (3) and (4) can be written as

$$\begin{aligned}\dot{x} &= (I_N \otimes A + L \otimes BK + (I_N \otimes D)\Delta(L \otimes EK))x \\ &\quad + (I_N \otimes B_2)\omega, \\ z &= (M \otimes C_1)x,\end{aligned} \tag{5}$$

where $\Delta = \mathrm{diag}\{\Delta_1, \cdots, \Delta_N\}$ and L is the Laplacian matrix of $\mathcal{G}$, defined as $L_{ii} = \sum_{j=1}^{N} a_{ij}$ and $L_{ij} = -a_{ij}$ for $i \neq j$.

Before moving forward, we introduce two lemmas.

Lemma 1: [32] Let $G(s) = C_1(sI - A)^{-1}B$ and $\gamma > 0$. The following two statements are equivalent.

1) A is stable and $\|G(s)\|_2 < \gamma$.
2) There exists a $P > 0$ such that

$$AP + PA^T + BB^T < 0, \quad \mathrm{tr}(C_1PC_1^T) < \gamma^2.$$

Lemma 2: [31] Given matrices V, H, G of appropriate dimensions and V symmetric,

$$V + HFG + G^TF^TH^T < 0$$

for all F satisfying $F^TF \leq I$ if and only if there exists a scalar $\epsilon > 0$ such that

$$V + \epsilon HH^T + \epsilon^{-1}G^TG < 0.$$

Now we are ready to present the main result of this section.

Theorem 1: Suppose that there exist matrices $P \in \mathbf{R}^{n\times n} > 0$, $Q \in \mathbf{R}^{m\times n}$, and a scalar $\epsilon > 0$ satisfying

$$\begin{bmatrix} A_c & \lambda_i Q^TE^T \\ \lambda_i EQ & -\epsilon I \end{bmatrix} < 0, \quad i = 2, N, \tag{6}$$

and

$$\mathrm{tr}(C_1PC_1^T) < \frac{\gamma^2}{N}, \tag{7}$$

where $A_c = AP + PA^T + \lambda_i BQ + \lambda_i Q^TB^T + B_2B_2^T + \epsilon\delta^2DD^T$ and λ_i, $i = 2, \cdots, N$, are the nonzero eigenvalues of L. Then the controller (4) with $K = QP^{-1}$ solves the robust H_2 consensus problem.

Proof 1: Denote by $\xi = (M \otimes I_n)x$ the consensus error, where $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Then it follows from (5) that

$$\begin{aligned}\dot{\xi} &= (I_N \otimes A + L \otimes BK + (M \otimes D)\Delta(L \otimes EK))\xi \\ &\quad + (M \otimes B_2)\omega, \\ z &= (M \otimes C_1)\xi.\end{aligned} \tag{8}$$

It is easy to see that the robust H_2 consensus problem is solved if and only if for all admissible uncertainty Δ satisfying (2), ξ is quadratically stable when $\omega = 0$ and $\|T_{\omega z}\|_2 < \gamma$ when $\omega \neq 0$. Since Assumption 1 holds, it is known from [24] that there exists a unitary matrix $U = [\frac{1}{\sqrt{N}}1_N, V]$, where $V \in \mathbf{R}^{n\times(n-1)}$, such that $U^T L U = \Lambda = \mathrm{diag}\{0, \lambda_2, \cdots, \lambda_N\}$. Letting $\tilde{\xi} = (U^T \otimes I_n)\xi$, $\tilde{z} = (U^T \otimes I_n)z$, $\tilde{\omega} = (U^T \otimes I_n)\omega$ and noting also that $\tilde{\xi}_1 \equiv 0$, it is not difficult to derive that

$$\begin{aligned}\dot{\tilde{\xi}} &= (I_N \otimes A + \Lambda \otimes BK + (T \otimes D)\tilde{\Delta}(\Lambda \otimes EK))\tilde{\xi} \\ &\quad + (I_N \otimes B_2)\tilde{\omega} \\ \tilde{z} &= (T \otimes C_1)\tilde{\xi},\end{aligned} \tag{9}$$

where $T = \mathrm{diag}\{0, 1, \cdots, 1\}$, $\tilde{\Delta} = (U^T \otimes I)\Delta(U \otimes I)$ and $\tilde{\Delta}^T\tilde{\Delta} \leq \delta^2 I$. Denoting $\zeta = [\tilde{\xi}_2^T, \cdots, \tilde{\xi}_N^T]^T$, $\bar{z} = [\tilde{z}_2^T, \cdots, \tilde{z}_N^T]^T$ and $\bar{\omega} = [\tilde{\omega}_2^T, \cdots, \tilde{\omega}_N^T]^T$, we have

$$\begin{aligned}\dot{\zeta} &= (I_{N-1} \otimes A + \bar{\Lambda} \otimes BK + (I_{N-1} \otimes D)\bar{\Delta}(\bar{\Lambda} \otimes EK))\zeta \\ &\quad + (I_{N-1} \otimes B_2)\bar{\omega} \\ \bar{z} &= (I_{N-1} \otimes C_1)\zeta,\end{aligned} \tag{10}$$

where $\bar{\Delta} = \tilde{\Delta}_{(m+1):Nm;(k+1):Nk}$, representing the submatrix of $\tilde{\Delta}$ which takes the $m+1$ to Nm rows and $k+1$ to Nk columns of the matrix $\tilde{\Delta}$. By Lemma 3, $\bar{\Delta}^T\bar{\Delta} \leq \delta^2 I_{(N-1)m}$. Then, we can obtain that the robust H_2 consensus problem is solved, if ζ is quadratically stable and $\|T_{\bar{\omega}\bar{z}}\|_2 < \gamma$ for all admissible uncertainty $\bar{\Delta}$. According to Lemma 1, this holds, if there exists a matrix $\mathcal{P} > 0$ such that

$$\begin{aligned}&(I_{N-1} \otimes A + \bar{\Lambda} \otimes BK + (I_{N-1} \otimes D)\bar{\Delta}(\bar{\Lambda} \otimes EK))\mathcal{P} \\ &+ \mathcal{P}(I_{N-1} \otimes A + \bar{\Lambda} \otimes BK + (I_{N-1} \otimes D)\bar{\Delta}(\bar{\Lambda} \otimes EK))^T \\ &+ (I_{N-1} \otimes B_2 B_2^T) < 0,\end{aligned} \tag{11}$$

and

$$\mathrm{tr}((I_{N-1} \otimes C_1)\mathcal{P}(I_{N-1} \otimes C_1^T)) < \gamma^2. \tag{12}$$

In light of Lemma 2, we obtain that (11) holds if and only if there exists a scalar $\epsilon > 0$ such that

$$\begin{aligned}&(I_{N-1} \otimes A + \bar{\Lambda} \otimes BK)\mathcal{P} + \mathcal{P}(I_{N-1} \otimes A + \bar{\Lambda} \otimes BK)^T \\ &+ (I_{N-1} \otimes B_2 B_2^T) + \epsilon\delta^2(I_{N-1} \otimes DD^T) \\ &+ \epsilon^{-1}\mathcal{P}(\bar{\Lambda}^2 \otimes K^T E^T E K)\mathcal{P} < 0,\end{aligned} \tag{13}$$

Notice that all the terms except $\mathcal{P}$ in (12) and (13) are block diagonal. Therefore, if we choose $\mathcal{P} = I_N \otimes P$, then (12) and (13) become

$$\begin{aligned}&I_{N-1} \otimes AP + I_{N-1} \otimes PA^T + \bar{\Lambda} \otimes BKP + \bar{\Lambda} \otimes PK^T B^T \\ &+ I_{N-1} \otimes B_2 B_2^T + \epsilon\delta^2(I_{N-1} \otimes DD^T) \\ &+ \epsilon^{-1}\bar{\Lambda}^2 \otimes PK^T E^T EKP < 0.\end{aligned} \tag{14}$$

and

$$\mathrm{tr}(I_{N-1} \otimes C_1 P C_1^T) < \gamma^2. \tag{15}$$

Let $Q = KP$. Then it follows that (14) and (15) hold, if

$$\begin{aligned}&AP + PA^T + \lambda_i BQ + \lambda_i Q^T B^T + B_2 B_2^T \\ &+ \epsilon\delta^2 DD^T + \epsilon^{-1}\lambda_i^2 Q^T E^T E Q < 0, \ i = 2, \cdots, N,\end{aligned} \tag{16}$$

and

$$\mathrm{tr}(C_1 P C_1^T) < \frac{\gamma^2}{N}. \tag{17}$$

Using the Schur Complement Lemma [33], it is not hard to get the condition in Theorem 1. ■

Remark 1: Contrary to the previous related works [23], [25], [26], [27], [28], [29], [34] where the H_2 consensus performance of nominal and homogeneous multi-agent systems is considered, in this paper we investigate the robust H_2 consensus problem under norm-bounded parametric uncertainties. Note that in the presence of parametric uncertainties, the multi-agent network becomes heterogeneous, which makes the robust H_2 consensus problem much more challenging.

Remark 2: The lower bound for the H_2 performance of the consensus network (5) can be obtained by solving the following optimization problem:

$$\text{Minimize } \gamma,$$

$$\text{subject to LMIs (6), (7) with } P > 0, \epsilon > 0.$$

III. Robust Consensus as a Special Case

In this section, we consider the case where there are no noises, i.e., $\omega_i = 0$. Let $x = [x_1, x_2, \cdots, x_N]^T$. The closed-loop network dynamics can be derived from (1) and (4) as follows:

$$\dot{x} = (I_N \otimes A + cL \otimes BK + (I_N \otimes D)\Delta(L \otimes EK))x. \tag{18}$$

The objective of this section is to design a distributed controller (4) such that the close-loop system (18) reaches robust consensus for all admissible uncertainties ΔB_i satisfying (2).

Denote $\xi = (M \otimes I_n)x$, then it follows that x reaches robust consensus if and only if ξ is quadratically stable. From (18), it is not difficult to get that

$$\dot{\xi} = (I_N \otimes A + cL \otimes BK + (M \otimes D)\Delta(L \otimes EK))\xi. \tag{19}$$

Before moving on, we first introduce some useful lemmas.

Lemma 3: [30] If $\Delta \in \mathbf{R}^{N\times N}$ with $\Delta^T\Delta \leq \delta^2 I_N$ and $\tilde{\Delta} = \Delta_{2:N;2:N}$, where $\Delta_{2:N;2:N}$ denotes the submatrix of Δ by deleting the first column and the first row, then it follows that $\tilde{\Delta}^T\tilde{\Delta} \leq \delta^2 I_{N-1}$.

Lemma 4: [30] For the uncertain system

$$\dot{x} = (A + \Delta A)x$$

where $\Delta A = F\Delta G$ with Δ being Lebesgue measurable and $\Delta^T\Delta \leq \delta^2 I$, the state x is quadratically stable if and only if A is Hurwitz and $\|G(sI - A)^{-1}F\|_\infty \leq \frac{1}{\delta}$.

As a special case of Theorem 1, we can obtain the following result.

Theorem 2: Under Assumption 1, the agents in (1) achieve robust consensus under the state feedback controller (4) if

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and only if $A+\lambda_i BK$ are Hurwitz and $\|\lambda_i EK(sI_n - A - \lambda_i BK)^{-1}D\|_\infty < \frac{1}{\delta}$ for $i = 2, \cdots, N$.

This theorem actually converts the robust consensus problem of the uncertain network (18) to the quadratic stability problem of the following $N-1$ subsystems:

$$\dot{\tilde{x}}_i = (A + \lambda_i BK + \lambda_i D\mathbf{\Delta}_i EK)\tilde{x}_i, \quad i = 2, \cdots, N,$$

where $\mathbf{\Delta}_i^T \mathbf{\Delta}_i \le \delta^2 I$.

Theorem 3: Let $X = X^T > 0$ and Y be the solution to the following LMI:

$$\begin{bmatrix} A_{cl} & \lambda_i Y^T E^T \\ \lambda_i EY & -I \end{bmatrix} < 0, \quad i = 2, N, \tag{20}$$

where $A_{cl} = AX + XA^T + \lambda_i BY + \lambda_i Y^T B^T + \delta^2 DD^T$ and $K = YX^{-1}$. Then the agents in (1) reach robust consensus under the state feedback controller (4).

Proof 2: In light of the Bounded Real Lemma [32] and Theorem 2, it can be shown by following similar steps in proving Theorem 1 that the closed-loop system (19) reaches robust consensus, if

$$AX + XA^T + \lambda_i BY + \lambda_i Y^T B^T + \lambda_i^2 Y^T E^T EY + \delta^2 DD^T < 0$$

for $i = 2, \cdots, N$. Using Schur Complement Lemma [33], it then follows that robust consensus is achieved, if the LMIs in (20) hold for $i = 2, N$. ■

Remark 3: Different from [22] where only the robust stabilization problem is considered, Theorem 1 solves the robust consensus problem, which is more general and useful in, e.g., distributed optimization and formation control.

Remark 4: In contrast to dynamic uncertainties considered in [11] and coprime factor perturbations studied in [12], this work focuses on linear multi-agent systems whose input matrices are subject to norm-bounded parametric uncertainties. This paper considers time-domain uncertainties, which complements the results on frequency-domain uncertainties in [11] and [12].

IV. Simulation Example

In this section, a simulation example will be presented to verify the theoretical results.

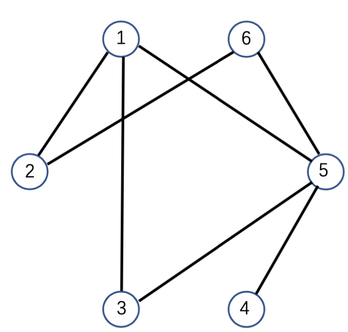

Fig. 1: The communication topology.

Consider a multi-agent system with 6 agents whose communication graph is shown in Fig. 1. The nominal dynamics of each agent is described by (1), with

$$A = \begin{bmatrix} 1.5000 & 0.5 & 2 \\ 2.5 & -2 & 1.3 \\ 1.7 & -2.5 & 3.9 \end{bmatrix}, \quad B = \begin{bmatrix} 1.0 \\ 0.5 \\ 0.6 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0.3 \\ 0.7 \\ 1.0 \end{bmatrix}.$$

The uncertainty associated with agent i is $\Delta B_i = D\Delta_i E$, with

$$D = \begin{bmatrix} 0.3124 & 0.3787 \\ 0.2556 & 0.5680 \\ 0.4071 & 0.4733 \end{bmatrix}, \quad E = \begin{bmatrix} 0.3190 \\ 0.9478 \end{bmatrix}.$$

Furthermore, we set

$$\Delta_1 = \begin{bmatrix} 0.10\sin(t) & 0 \\ 0 & 0.10 \end{bmatrix}, \ \Delta_2 = \begin{bmatrix} 0.11\sin(3t) & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\Delta_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.13\cos(4t) \end{bmatrix}, \ \Delta_4 = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.11\cos(10t) \end{bmatrix},$$
$$\Delta_5 = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11\cos(1.3t) \end{bmatrix}, \ \Delta_6 = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.09\cos(2.3t) \end{bmatrix}.$$

It is easy to see that the upper bound δ of the norms of the uncertainties is 0.15. Suppose that we want the H_2 norm of the network to be less than 7, i.e., $\gamma = 7$. Solving the LMI in Theorem 1 to get P and Q, we then select $K = QP^{-1} = \begin{bmatrix} 606.9 & 1747.5 & -2975 \end{bmatrix}$. Fig. 1 depicts the evolution of the consensus error $\xi = (M \otimes I_3)x$ with respect to time t when ω is set to be 0. Evidently, the uncertain multi-agent system reaches consensus asymptotically.

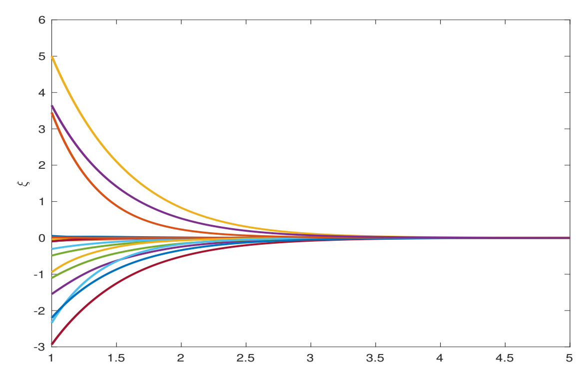

Fig. 2: The evolution of the consensus error ξ.

Next, we verify that when there exists noise signal ω, the network (5) has an H_2 norm less than 7. According to the definition of the H_2 norm, if we set ω to be the white gaussian noise with unit variance and zero mean, it then follows that $\lim_{t\to\infty} \mathbb{E}(z(t)^T z(t)) = \|T_{\omega z}\|_2^2$ [32]. As depicted in Fig. 3, the performance variable z fluctuates within a small bound, when it comes to the steady state. It is not difficult to calculate that the steady state value of $\sqrt{\mathbb{E}(z(t)^T z(t))}$ is 2.1707, which is smaller than 7.

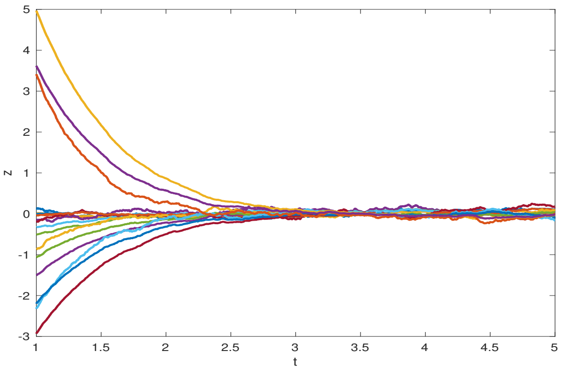

Fig. 3: The evolution of the performance variable z.

V. Conclusion

In this paper, we have studied the distributed robust H_2 consensus control problem for linear multi-agent systems with uncertain input matrices and measurement noises. We have developed LMI conditions to construct distributed state feedback controllers to achieve a prescribed H_2 performance level. There are many potential extensions to this work. One is to consider the directed graph case. Another extension is to consider other kinds of parametric uncertainties, such as the state matrix uncertainty. Though very promising these extensions are, each of them seems quite intricate and needs further study.

References

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[2] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks," *IEEE Transactions on Circuits and System I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2010.

[3] Y. F. Gong, G. G. Wen, Z. X. Peng, T. W. Huang, and Y. W. Chen, "Observer-based time-varying formation control of fractional-order multi- agent systems with general linear dynamics," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 1, pp. 82–86, 2020.

[4] B. Cheng, Z. Li, and X. Wang, "Cooperative output regulation of heterogeneous multi-agent systems with adaptive edge-event-triggered strategies," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 10, pp. 2199–2203, 2020.

[5] Y. Gong, G. Wen, Z. Peng, T. Huang, and Y. Chen, "Observer-based time-varying formation control of fractional-order multi-agent systems with general linear dynamics," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 1, pp. 82–86, 2020.

[6] D. Wang, D. Wang, and W. Wang, "Necessary and sufficient condition for containment control of multi-agent systems with time delay," *Automatica*, vol. 103, pp. 418–423, 2019.

[7] J. Lian, and F. Wu, "Stabilization of switched linear systems subject to actuator saturation via invariant semi-ellipsoids", *IEEE Transactions on Automatic Control*, vol. 65, no. 10, pp. 4332–4339, 2020.

[8] X. Wang, L. Shen, Z. Liu, S. Zhao, Y. Cong, Z. Li, S. Jia, H. Chen, Y. Yu, Y. Chang, and Y. Wang, "Coordinated flight control of miniature fixed-wing UAV swarms: Methods and experiments," *SCIENCE CHINA Information Sciences*, vol. 62, no. 11, p. 212204, 2019.

[9] B. Ning, Q. L. Han, and Z. Zuo, "Practical fixed-time consensus for integrator-type multi-agent systems: A time base generator approach", *Automatica*, vol. 105, pp. 406–414, 2019.

[10] R. Carli, A. Chiuso, L. Schenato, and S. Zampieri, "Optimal synchronization for networks of noisy double integrators," *IEEE Transactions on Automatic Control*, vol. 56, no. 5, pp. 1146–1152, 2011.

[11] H. L. Trentelman, K. Takaba, and N. Monshizadeh, "Robust synchronization of uncertain linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 6, pp. 1511–1523, 2013.

[12] H. J. Jongsma, H. L. Trentelman, and M. K. Camlibel, "Robust synchronization of coprime factor perturbed networks," *Systems and Control Letters*, vol. 95, pp. 62–69, 2016.

[13] X. Li, Y. C. Soh, and L. Xie, "Robust consensus of uncertain linear multi-agent systems via dynamic output feedback," *Automatica*, vol. 98, pp. 114–123, 2018.

[14] Z. Li, and J. Chen, "Robust consensus of linear feedback protocols over uncertain network graphs," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, 2017.

[15] D. Zelazo, and M. Burger, "On the robustness of uncertain consensus networks," *IEEE Transactions on Control of Network Systems*, vol. 4, no. 2, pp. 170–178, 2017.

[16] Z. Li, and J. Chen, "Robust consensus for multi-agent systems communicating over stochastic uncertain networks," *SIAM Journal on Control and Optimization*, vol. 57, no. 5, pp. 3553–3570, 2019.

[17] S. Z. Khong, E. Lovisari, and A. Rantzer, "A unifying framework for robust synchronization of heterogeneous networks via integral quadratic constraints," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1297–1309, 2016.

[18] H. Kim, H. Shim, and J. H. Seo, "Output consensus of heterogeneous uncertain linear multi-agent systems," *IEEE Transactions on Automatic Control* , vol. 56, no. 1, pp. 200–206, 2011.

[19] I. Lestas, and G. Vinnicombe, "Heterogeneity and scalability in group agreement protocols: Beyond small gain and passivity approaches," *Automatica*, vol. 46, no. 7, pp. 1141–1151, 2010.

[20] Y. Tang, H. Gao, W. Zhang, and J. Kurths, "Leader-following consensus of a class of stochastic delayed multi-agent systems with partial mixed impulses," *Automatica*, vol. 53, pp. 346–354, 2015.

[21] K. Z. Liu and Y. Yao, *Robust Control: Theory and Applications*. Singapore, Wiley, 2016.

[22] Z. Li, Z. Duan, L. Xie, and X. Liu, "Distributed robust control of linear multi-agent systems with parameter uncertainties," *International Journal of Control*, vol. 85, no. 8, pp. 1039–1050, 2012.

[23] M. Siami, S. Bolouki, B. Bamieh, and N. Motee, "Centrality measures in linear consensus networks with structured network uncertainties," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 3, pp. 924–934, 2018.

[24] Z. Li and Z. Duan, *Cooperative Control of Multi-agent Systems: A Consensus Region Approach*. CRC Press, 2015.

[25] B. Bamieh, M. R. Jovanovic, P. Mitra, and S. Patterson, "Coherence in large-scale networks: Dimension-dependent limitations of local feedback," *IEEE Transactions on Automatic Control*, vol. 57, no. 9 pp 2235–2249, 2012.

[26] M. Siami and N. Motee, "New spectral bounds on H_2 norm of linear dynamical networks," *Automatica*, vol. 80, pp 305–312, 2017.

[27] S. Patterson and B. Bamieh, "Consensus and coherence in fractal networks," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 4, pp 338–348, 2014.

[28] J. Jiao, H. L. Trentelman, and M. K. Camlibel, "A suboptimality approach to distributed H_2 control by dynamic output feedback," *Automatica*, vol. 121, p. 109164, 2020.

[29] Z. Li, Z. Duan, and G. Chen, "On H_∞ and H_2 performance regions of multi-agent systems," *Automatica*, vol. 47, pp 797–803, 2011.

[30] L. Xie, M. Fu, and Carlos E. de Souza, "H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback," *IEEE Transactions on Automatic Control*, vol. 37, no. 8, pp. 1253–1256, 1992.

[31] L. Xie, "Output feedback H_∞ control of systems with parameter uncertainty," *International Journal of control*, vol. 63, no. 4, pp. 741–750, 1996.

[32] K. Zhou and J. C. Doyle, *Essentials of Robust Control*. Prentice Hall, Upper Saddle River, NJ, 1998.

[33] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia, PA, 1994.

[34] H. K. Mousavi, and N. Motee, "Explicit characterization of performance of a class of networked linear control systems," *IEEE Transactions on Control of Network Systems*, in press, 2020.