Feedback Linearization Regulator with Coupled Attitude and Translation Dynamics Based on Unit Dual Quaternion

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Abstract— **The main contribution of this paper is the design of a unit dual quaternion-based attitude and position regulator. The dynamic model of rigid body represented by unit dual quaternion is derived firstly from the rotational and the translational dynamics. And then an output feedback regulator, which ensures asymptotical stability, is proposed using unit dual quaternion. To the best of our knowledge, this study is the first to relate dual quaternion to dynamic control of rigid body without requiring decoupling attitude and position. The simulation results are provided to verify the performance.**

I. INTRODUCTION

Attitude and position control is a critical issue of rigid body in 6 DOF, which has been studied in many applications, such as autonomous underwater vehicle (AUV) [1], [2], and spacecraft [3], [4]. In most existing literature, the dynamic equations of motion are separated into the rotational and the translational motions, and consequently, the control problem is divided into the attitude control problem and the position control problem respectively. Position is specified by a three-dimensional vector, whereas various representations of attitude have been discussed, such as Euler angles, Rodrigues parameters, unit equivalent axis/angle, and unit quaternion. Unit quaternion is the most popular tool in attitude control, as the unit quaternion uses the least possible number of parameters (four) to represent orientation globally [5], [6]. However, when using unit quaternion, it just focuses on attitude control, and the translational motion has not been addressed. The second method, but uncommon, to control attitude and position is utilizing the geometric structure of SE(3) [7]. All the homogenous transformation matrices construct SE(3). However, the homogenous transformation matrix is a 4×4 matrix, which leads to the controller being more complex.

In this paper, we use a new mathematical tool, i.e. unit dual quaternion to deal with attitude and position control problem simultaneously. Unit dual quaternion is the natural extension of unit quaternion. Unit quaternion provides an efficient global representation for rotations without singularities, and unit dual quaternion inherits this property. In addition, unit dual quaternion can represent rotation and translation simultaneously. It holds more advantages than the popular tool - homogenous transformation matrix. The homogenous transformation matrix is with 16 entries, while dual quaternion is with only 8 numbers, so dual quaternion is much more compact than homogenous transformation matrix. Further, it has also been revealed by existing work that the calculation efficiency of unit dual quaternion is much higher than homogenous transformation matrix [8]. Dual quaternion has been applied into various domains, such as computer-aided geometric design [9], image-based localization [10], hand-eye calibration [11] and navigation [12]. As mentioned by Funda in [8], dual quaternions offer a potentially significant advantage in robotic system, however it is a pity that dual quaternions seldom play a certain role in the control of rigid bodies, as quaternions do in attitude control. The primary use of dual quaternion in control is reported in [13], in which, parallel (but not equivalent) to [7], the generalized proportional controller on kinematics is derived based on unit dual quaternion Lie-group and its logarithm mapping. To the best of our knowledge, our study is the first to relate dual quaternion to dynamic control of rigid body.

This paper is organized as follows. The mathematical preliminaries of dual quaternion are introduced in Section II, and then the dynamic model of rigid body represented by unit dual quaternion is derived from the rotational and the translational dynamics in Section III. By utilizing unit dual quaternion, a stable output feedback regulator for rigid-body without decoupling attitude and position is proposed in Section IV. Section V demonstrates the simulation results and the last section states conclusion.

II. MATHEMATICAL PRELIMINARIES

In this section, a brief introduction to dual quaternion is given. As two foundations of dual quaternion, quaternion and dual number are defined firstly. More details on these subjects can be found in literature [10], [12].

Quaternion is an extension of the complex number to $\mathbb{R}^4$. Formally, quaternion q can be defined as

$$q = a + bi + cj + dk \qquad (1)$$

where $i^2 = j^2 = k^2 = -1, ij = k, jk = i, ki = j$. A convenient shorthand notation is $q = [s, \vec{v}]$, where s is a scalar (called *scalar part*), and $\vec{v}$ is a three-dimensional vector (called *vector part*). Obviously, a vector can be treated equivalently as a quaternion with vanishing real part, which is called *vector quaternion*.

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Quaternions satisfy the following operations:

$$\begin{array}{rcl} q_1 + q_2 & = & [s_1 + s_2,\, \vec{v}_1 + \vec{v}_2] \\ \lambda q & = & [\lambda s,\, \lambda \vec{v}] \\ q_1 \circ q_2 & = & [s_1 s_2 - \vec{v}_1 \cdot \vec{v}_2,\, s_1 \vec{v}_2 + s_2 \vec{v}_1 + \vec{v}_1 \times \vec{v}_2] \end{array} \tag{2}$$

where λ is a scalar.

The conjugation of a quaternion is defined as

$$q^* = [s,\, -\vec{v}] \tag{3}$$

The norm of a quaternion is defined as

$$\|q\|^2 = q \circ q^* = a^2 + b^2 + c^2 + d^2 \tag{4}$$

The multiplicative inverse element of quaternion q is

$$q^{-1} = (1/\|q\|^2) \circ q^* \tag{5}$$

If $\|q\|^2 = 1$, the quaternion is called *unit quaternion*. For unit quaternion, $q^{-1} = q^*$.

Dual number is defined as

$$\hat{a} = a + \epsilon b \quad \text{with} \quad \epsilon^2 = 0, \text{ but} \quad \epsilon \neq 0 \tag{6}$$

where a and b are real numbers, called *real part* and *dual part*, respectively, and ϵ is nilpotent such as $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

Dual vectors are a generalization of dual numbers whose real and dual parts are both three-dimensional vectors. Here, we define a new operation, i.e. dot-product, between dual vectors.

Definition 1: Let $\hat{\vec{v}} = \vec{v}_r + \epsilon \vec{v}_d$ and $\hat{\vec{k}} = \vec{k}_r + \epsilon \vec{k}_d = (k_{r1}, k_{r2}, k_{r3})^T + \epsilon (k_{d1}, k_{d2}, k_{d3})^T$ be dual vectors, then

$$\hat{\vec{k}} \cdot \hat{\vec{v}} = K_r \vec{v}_r + \epsilon K_d \vec{v}_d \tag{7}$$

where K_r and K_d are both 3×3 diagonal matrices with diagonal entries k_{r1}, k_{r2}, k_{r3} or k_{d1}, k_{d2}, k_{d3}, namely $K_r = diag(k_{r1}, k_{r2}, k_{r3})$ and $K_d = diag(k_{d1}, k_{d2}, k_{d3})$. For convenience and simplicity, the sign '$\cdot$' is sometimes omitted in the following.

Dual quaternion is the quaternion with dual number components, i.e. $\hat{q} = [\hat{s},\, \hat{\vec{v}}]$, where $\hat{s}$ is a dual number and $\hat{\vec{v}}$ is a dual vector. Obviously, a dual vector can be treated equivalently as a dual quaternion with vanishing real part, which is called *dual vector quaternion*. A dual quaternion can be rewritten into $\hat{q} = q_r + \epsilon q_d$, where q_r and q_d are both quaternions. Dual quaternions satisfy the following operations:

$$\begin{array}{rcl} \hat{q}_1 + \hat{q}_2 & = & [\hat{s}_1 + \hat{s}_2,\, \hat{\vec{v}}_1 + \hat{\vec{v}}_2] \\ & = & (q_{r1} + q_{r2}) + \epsilon (q_{d1} + q_{d2}) \\ \lambda \hat{q} & = & [\lambda \hat{s},\, \lambda \hat{\vec{v}}] = \lambda q_r + \epsilon \lambda q_d \end{array} \tag{8}$$

The multiplication between dual quaternions is defined as

$$\begin{aligned} \hat{q}_1 \circ \hat{q}_2 =& [\hat{s}_1 \hat{s}_2 - \hat{\vec{v}}_1 \hat{\vec{v}}_2, \hat{s}_1 \hat{\vec{v}}_2 + \hat{s}_2 \hat{\vec{v}}_1 + \hat{\vec{v}}_1 \times \hat{\vec{v}}_2] \\ =& q_{r1} \circ q_{r2} + \epsilon (q_{r1} \circ q_{d2} + q_{d1} \circ q_{r2}) \end{aligned} \tag{9}$$

The dual quaternion multiplication is associative and distributive but not commutative as well as quaternion multiplication.

The *conjugation* of dual quaternion $\hat{q}$ is defined as

$$\hat{q}^* = [\hat{s},\, -\hat{\vec{v}}] \tag{10}$$

The *norm* of dual quaternion $\hat{q}$ is defined as

$$\|\hat{q}\|^2 = \hat{q} \circ \hat{q}^* \tag{11}$$

If $\|\hat{q}\|^2 = 1 + \epsilon 0$, then dual quaternion is called *unit dual quaternion*. The multiplicative inverse element of dual quaternion $\hat{q}$ is

$$\hat{q}^{-1} = (1/\|\hat{q}\|^2) \circ \hat{q}^* \tag{12}$$

Thus, for unit dual quaternion, $\hat{q}^{-1} = \hat{q}^*$.

Unit quaternions can be used to describe rotation. For the frame rotation about a unit axis $\vec{n}$ with an angle $|\theta| < 2\pi$, there is a unit quaternion

$$q = [\cos(\frac{|\theta|}{2}),\, \sin(\frac{|\theta|}{2})\vec{n}] \tag{13}$$

relating a fixed vector expressed in the original frame r^o with the same vector expressed in the new frame r^n by

$$r^n = q^* \circ r^o \circ q \tag{14}$$

r^o and r^n are two vector quaternions. Similarly, unit dual quaternion can be used to represent transformation (rotation and translation simultaneously). Suppose that there is a rotation q succeeded by a translation p^b, then the whole transformation can be represented using unit dual quaternion as follows [12].

$$\hat{q} = q + \frac{\epsilon}{2} \cdot q \circ p^b \tag{15}$$

Unit quaternion q serves as a rotation, taking coordinates of a point from one frame to another. On the other hand, every attitude of a rigid body that is free to rotate relative to a fixed frame can be identified with a unique unit quaternion q. Analogous to the rotational case, an unit dual quaternion $\hat{q}$ serves as both a specification of the configuration (consisting of attitude and position) of a rigid body and a transformation taking the coordinates of a point from one frame to another.

Unit quaternions form a Lie-group over multiplication with the conjugate being the inverse, denoted by Q_u. The same goes for unit dual quaternions, which form a Lie-group over dual quaternion multiplication, denoted by DQ_u. The logarithm of unit quaternion, whose space denoted by *v*, is defined as

$$\ln q = \frac{\theta}{2} \tag{16}$$

Here, $\theta = |\theta|\vec{n}$ is a vector quaternion. Similarly, the logarithm mapping of the unit dual quaternion is defined as

$$\ln \hat{q} = \frac{1}{2}(\theta + \epsilon p^b) \tag{17}$$

which is a dual vector quaternion. The space consisting of all unit dual quaternion logarithm is denoted by $\hat{v}$. $\hat{v}$ is the Lie-algebra of DQ_u as well as *v* is the Lie-algebra of Q_u.

III. DYNAMIC MODEL AND PROBLEM STATEMENT

Little work has been done on dynamics problems with dual quaternions. [14] gave the formulation of a general dynamics problem using dual quaternion components. But the equations of motion are quite complicated and the physical significance of the variables is not intuitively apparent. In this section, we deduced dual-quaternion-based single rigid body dynamics from traditional rotational dynamics and translational dynamics, which can act as the basis for the solution of any rigid body dynamic problem. Control objective is also stated in this section.

Following [15], [16], the rotational dynamics of rigid body can be written in the body frame in the following form.

$$J\dot{\omega}^b + \omega^b \times (J\omega^b) = \tau \tag{18}$$

where $\omega^b \in \mathbb{R}^3$ is the angular velocity vector in body frame; $J \in \mathbb{R}^{3\times 3}$ is the rigid body inertia matrix in body frame; and $\tau \in \mathbb{R}^3$ is the control torque vector associated with rigid body.

The translational dynamics of rigid body relative to body frame is

$$m\ddot{p}^b = f \tag{19}$$

where $p^b \in \mathbb{R}^3$ is the position of rigid body, $m \in \mathbb{R}$ and $f \in \mathbb{R}^3$ are the mass and the control force in body frame, respectively [17].

Thus, from (18) and (19), we obtain

$$\begin{aligned} \dot{\omega}^b &= -J^{-1}\omega^b \times J\omega^b + J^{-1}\tau && (20)\\ \ddot{p}^b &= m^{-1}f && (21) \end{aligned}$$

Kinematic equation of a rigid body expressed with unit dual quaternion is

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} \circ \xi^b \tag{22}$$

where

$$\xi^b = \omega^b + \epsilon \cdot (\dot{p}^b + \omega^b \times p^b) \tag{23}$$

ξ^b is twist in body frame [12].

Differentiating the twist in body frame in (23), we obtain

$$\dot{\xi}^b = \dot{\omega}^b + \epsilon(\ddot{p}^b + \dot{\omega}^b \times p^b + \omega^b \times \dot{p}^b) \tag{24}$$

Substituting (20) and (21) into (24), then

$$\begin{aligned} \dot{\xi}^b &= a + J^{-1}\tau \\ &\quad + \epsilon(m^{-1}f + (a + J^{-1}\tau) \times p^b + \omega^b \times \dot{p}^b) \\ &= J^{-1}\tau + \epsilon(m^{-1}f + J^{-1}\tau \times p^b) \\ &\quad + (a + \epsilon(a \times p^b + \omega^b \times \dot{p}^b)) \\ &= \hat{F} + \hat{U} \end{aligned} \tag{25}$$

where $a = -J^{-1}\omega^b \times J\omega^b$ and

$$\begin{cases} \hat{F} = a + \epsilon(a \times p^b + \omega^b \times \dot{p}^b) \\ \hat{U} = J^{-1}\tau + \epsilon(m^{-1}f + J^{-1}\tau \times p^b) \end{cases} \tag{26}$$

Thus, (22), (23), (25) and (26) constitute the dynamic model of rigid body in body-frame. They provide a basis for any rigid body dynamic problem. In the following, we propose a regulator on the basic of this dynamic model.

In regulation problem, the control objective is to design a stable output feedback regulator, without requiring decoupling the rotational and translational dynamics, to stabilize the attitude and position simultaneously. Mathematically, when $t \to \infty$, $\hat{q}(t) \to (1,0,0,0) + \epsilon(0,0,0,0)$, i.e. $q(t) \to (1,0,0,0)$ and $p^b(t) \to (0,0,0)$, with initial conditions $\xi^b(0) = (0,0,0)+\epsilon(0,0,0)$, $\xi'^b(0) = (0,0,0)+\epsilon(0,0,0)$ and $\hat{q}(0) \neq (1,0,0,0) + \epsilon(0,0,0,0)$. Perfect and instantaneous measurements of $\hat{q}$ and ξ^b are assumed.

IV. REGULATOR AND STABILITY

Considering model (25) and (26), a regulator is proposed based on the principle of feedback linearization as follows.

$$\hat{U} = 2\hat{k}_p \ln \hat{q} + \hat{k}_v \xi^b - \hat{F} \tag{27}$$

where $\hat{k}_p = \vec{k}_{pr} + \epsilon\vec{k}_{pd} = (k_{pr1}, k_{pr2}, k_{pr3})^T + \epsilon(k_{pd1}, k_{pd2}, k_{pd3})^T$ and $\hat{k}_v = \vec{k}_{vr} + \epsilon\vec{k}_{vd} = (k_{vr1}, k_{vr2}, k_{vr3})^T + \epsilon(k_{vd1}, k_{vd2}, k_{vd3})^T$. k_{pri}, k_{pdi}, k_{vri} and $k_{vdi} (i = 1,2,3)$ will be designed later.

Theorem 1: Regulator (27) with appropriate $\hat{k}_p$ and $\hat{k}_v$ ensures that $\hat{q}(t)$ in model (25) and (26) converges to the identity element $DQ_I = (1,0,0,0) + \epsilon(0,0,0,0)$ of DQ_u asymptotically .

Proof: Substituting (27) into (25), then

$$\dot{\xi}^b = 2\hat{k}_p \ln \hat{q} + \hat{k}_v \xi^b \tag{28}$$

Substituting (17) into (28), then

$$\dot{\xi}^b = \hat{k}_p(\theta + \epsilon p^b) + \hat{k}_v(\omega^b + \epsilon(\dot{p}^b + \omega^b \times p^b)) \tag{29}$$

Comparing (29) with (24), we obtain (30) and (31)

$$\dot{\omega}^b = K_{pr}\theta + K_{vr}\omega^b \tag{30}$$

$$\ddot{p}^b = K_{pd}p^b + K_{vd}(\dot{p}^b + \omega^b \times p^b) - \dot{\omega}^b \times p^b - \omega^b \times \dot{p}^b \tag{31}$$

where $K_a = diag(k_{a1}, k_{a2}, k_{a3})$ (as K_{pr}, K_{vr}, K_{pd} and K_{vd}). We will discuss the stabilities of (30) and (31) respectively. First for (30) and then for (31).

It is easy to verify $\dot{\theta} = \omega^b$, thus (30) can be rewritten as

$$\ddot{\theta} - K_{vr}\dot{\theta} - K_{pr}\theta = 0 \tag{32}$$

If the values of $\vec{k}_{vr}$ and $\vec{k}_{pr}$ are chosen carefully so that the characteristic roots of (32) have negative real parts, then θ approaches zeros asymptotically as well as ω^b and $\dot{\omega}^b$.

As the same, (31) also can be rewritten as

$$\ddot{p}^b - K_{vd}\dot{p}^b - K_{pd}p^b + F(\omega^b, \dot{\omega}^b) = 0 \tag{33}$$

where $F(\omega^b, \dot{\omega}^b) = -K_{vd}(\omega^b \times p^b) + \dot{\omega}^b \times p^b + \omega^b \times \dot{p}^b$.

When $t \to \infty$, ω^b and $\dot{\omega}^b \to (0,0,0)^T$, so $F(\omega^b, \dot{\omega}^b) \to (0,0,0)^T$, then (33) can be simplified as

$$\ddot{p}^b - K_{vd}\dot{p}^b - K_{pd}p^b = 0 \tag{34}$$

Similarly, if the values of $\vec{k}_{vd}$ and $\vec{k}_{pd}$ are so chosen that the characteristic roots of (34) have negative real parts, then p^b also approaches zeros asymptotically.

Synthesizing the discussions about stabilities of (30) and (31), when $t \to \infty$, $\hat{q}(t) = q(t) + \epsilon q(t) \circ p^b(t)$ will converge to $DQ_I = (1,0,0,0) + \epsilon(0,0,0,0)$ asymptotically . ■

Comparing with conventional decoupling controls, the proposed regulator uniquely deal with the rotational and translational dynamic control simultaneously without decoupling and hold the interconnection between rotation and translation. By extracting f and τ explicitly from (27), it shows coupling items between attitude and position exists in f, which does not exist in conventional attitude control problem and position control problem. Therefore, the proposed regulator refers to natural interconnection between rotation and translation to some extent. Comparing with the SE(3) method in [7], dual quaternions are used instead of matrices and matrix operations are replaced by dual quaternion operations in proposed regulator. As revealing by existing work, these changes provide higher computational efficient. Furthermore, control law in [7] demands k_p to have a lower bound to avoid singularities, which is unnecessary in this study, as dual quaternion can represent arbitrary transformation without singularity and its logarithmic mapping is properly defined everywhere on DQ_u.

Remark 2: In some sense, $2\ln\hat{q}$ can be treated as $\int \xi^b$. If replacing $2\ln\hat{q}$ by $\int \xi^b$ in (28), then

$$\dot{\xi}^b - \hat{k}_v \xi^b - \hat{k}_p \int \xi^b = 0 \tag{35}$$

Therefore, by choosing appropriate $\hat{k}_v$ and $\hat{k}_p$, (35) ensures asymptotical stability.

Remark 3: The dynamic model and the regulator are both discussed in body frame in this study. In fact, the similar dynamic model and the regulator can be set up in spatial frame.

Remark 4: The above result uniquely deals with the rotational and translational dynamic control simultaneously without decoupling. It may be very useful in some fields where it is hard to decouple the dynamic model into the rotational and the translational dynamics or the coupling terms strongly influence the control performances.

V. SIMULATION RESULT

To illustrate the result presented in Section IV, a simple example extended from [5] is considered here.

We assume the moment J of the inertia matrix in the body frame is given by

$$J = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.63 & 0.0 \\ 0.0 & 0.0 & 0.85 \end{bmatrix} \tag{36}$$

and the mass of the rigid body is $m = 1$.[1]

For direct illustration, we assume the initial attitude and the initial position are both in spatial frame. The same goes for Fig. 1 and Fig. 2. The regulator (27) acts in body frame, so we translate the initial attitude and position into body frame in simulations.

[1] J, m and p^b, p^s in the following are all with the International System of Units (SI Units), such as $kg * m^2$, kg, m. The units are omitted therein.

The initial attitude is described by equivalent axis and angle, where the equivalent axis is $\vec{k} = (0.4896, 0.2032, 0.8480)^T$ and the equivalent angle is $\phi = 2.4648rad(141.22^o)$. Thus, the initial attitude $q(0) = (\cos(\phi/2), \sin(\phi/2)\vec{k}) = (0.3320, 0.4618, 0.1917, 0.7999)$. The initial position $p^s(0) = (2,2,1)^T$. The control objective is that $\hat{q}(t)$ converges asymptotically to the identity element DQ_I, i.e. $q(t) \to (1,0,0,0)$ and $p^b(t) \to (0,0,0)^T$, when $t \to \infty$. The simulation is performed with Simulink for a time span of 20s. The parameters in (27) are $\hat{k}_p = (-0.75, -0.75, -0.75)^T + \epsilon(-1,-1,-1)^T$ and $\hat{k}_v = (-1,-1,-1)^T + \epsilon(-1,-1,-1)^T$.

Fig. 1 shows the evolution of the four components of the attitude q with respect to time t, and Fig. 2 shows the evolution of the three components of the position p in spatial space versus time. q converges into $Q_I = (1,0,0,0)$ and p^s converges to $(0,0,0)^T$ asymptotically, so $\hat{q}$ converges into DQ_I asymptotically.

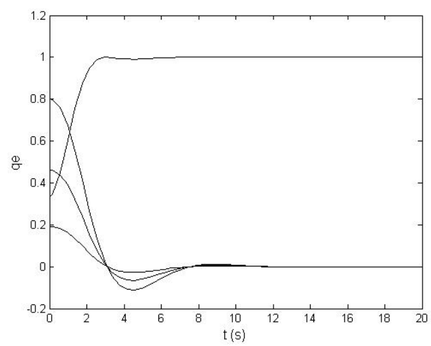

Fig. 1. Evolution of the four components of q with respect to time

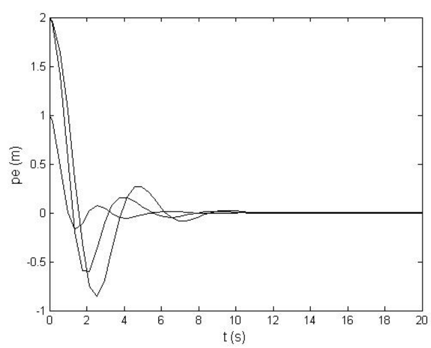

Fig. 2. Evolution of the three components of p in spatial frame with respect to time

VI. CONCLUDING REMARKS

In this paper, we propose a regulator on the basis of feedback linearization principle for rigid body with coupled rotational and translational dynamics by using a new mathematical tool, viz. unit dual quaternion. As a preliminary study about attitude and position control problem simultaneously without decoupling, our feedback linearization regulator is just a first attempt. Many other mature control laws can be transcribed as well. In addition, the dynamic control in this paper is with full state measurements (i.e. unit dual quaternion and twist), whereas, control law without twist, namely removing the requirement of angular velocity and velocity measurements, is also a worth considering issue. Some of these work has been done in [16] and [6] with only attitude control and in [17] with attitude and position control. Finally, if the inertia matrix J in (18) and the mass m in (19) are unknown or poorly known, the adoptive control law can be adopt as the work in [5].

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