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Brief paper

Distributed adaptive fault-tolerant control approach to cooperative output regulation for linear multi-agent systems[☆]

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ABSTRACT

In this paper, the cooperative output regulation problem for linear multi-agent systems with actuator faults is considered. It is assumed that the actuator faults are outage faults and loss-of-effectiveness faults. First, a distributed finite-time observer is designed to estimate the state of the exosystem. Based on the state of the finite-time observer, a distributed adaptive fault-tolerant controller is designed. Then, it is shown that the cooperative output regulation problem can be solved with the proposed fault-tolerant controller. Compared with the existing cooperative output regulation results, a novel lemma is introduced to guarantee the solvability of the regulator equations under actuator faults, and the developed controller is effective to compensate the actuator faults. Finally, a simulation example is presented to show the validity of the proposed method.

1. Introduction

The cooperative output regulation (COR) problem has been widely investigated in the last decade due to its potential application in various areas, such as mobile robot formation (Su & Huang, 2012b) and multiple spacecraft attitude synchronization (Cai & Huang, 2014). The primary task of the COR problem is to develop a distributed controller to achieve the trajectory tracking and disturbance rejection. In this regard, it can be viewed as an extension of the formation control problem (Fax & Murray, 2004; Ren & Sorensen, 2008) and the leader-following consensus problem (Hong, Hu, & Gao, 2006; Olfati-Saber & Murray, 2004).

It is known that the primary methods adopted to solve the COR problem can be divided into the distributed feedforward method (Cai, Lewis, Hu, & Huang, 2017; Ding, 2015; Li, Chen, & Ding, 2016; Su & Huang, 2012a) and the distributed internal model method (Kim, Shim, & Seo, 2011; Wang, Hong, Huang, & Jiang, 2010). For the distributed feedforward method, the controller consists of a feedback term and a feedforward term. The feedback term based on the local state/output of the system is employed to stabilize the system, while the feedforward term aims to track reference signals and to reject exogenous disturbances. The distributed feedforward method is proposed to solve the COR problem for linear multi-agent systems (MASs) based on local state and output information in Su and Huang (2012a) and Cai et al. (2017), respectively. In addition, this method is also used to solve the COR problem for nonlinear MASs in Ding (2015).

Although the feedforward method is effective to solve the COR problem, it can merely be adopted in the system with a determinate plant. However, plant uncertainty widely exists in practical application. To tolerate the plant uncertainty raised in MASs, the distributed internal model method is proposed in many valuable results (Kim et al., 2011; Su, Hong, & Huang, 2013; Wang et al., 2010; Xu, Wang, Hong, & Jiang, 2016; Yang, Huang, & Wang, 2016). In Kim et al. (2011), this method is used to solve the COR problem for single-input single-output linear MASs with uncertain parameters. In Wang et al. (2010), it is used to solve the robust output regulation problem for linear MASs under the assumption that the communication graph contains no cycle. In Su et al. (2013), the no cycle assumption is removed. Moreover, this method is also applied to solve the robust output regulation problem for nonlinear MASs, see Xu et al. (2016) and Yang et al. (2016). Although valuable results are presented, they can merely address the COR problem for MASs with plant uncertainty, which refer to minor changes of system parameters. In practical applications, actuator faults are inevitable and may jeopardize the performance and stability of the system (Yang, Staroswiecki, Jiang, & Liu, 2011; Yang & Ye, 2010). In particular, actuator faults may change actuator parameters

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seriously especially for actuator outage faults. Therefore, actuator faults cannot be considered as plant uncertainty. In this regard, the existing distributed internal model method in Kim et al. (2011), Su et al. (2013) and Wang et al. (2010) cannot be adopted to address the COR problem for MASs with actuator faults.

It is well known that the fault-tolerant control (FTC) method is effective to compensate for the actuator faults and to ensure reliability and safety of the dynamic system. For MASs, some effective FTC methods are proposed to compensate for the damaging effect caused by the actuator faults, as seen in Chen, Ho, Li, and Liu (2015), Shen, Jiang, Shi, and Zhao (2014), Yang et al. (2011), Yang and Ye (2010) and Zuo, Zhang, and Wang (2015). However, it is difficult to extend the existing FTC method to the COR problem. For the COR problem, the main challenges caused by the actuator faults can be summarized as follows: (i) whether the regulator equations are solvable when the MASs are affected by the actuator faults. (ii) how to develop a distributed fault-tolerant controller to compensate for the effect of the actuator faults? To solve the COR problem for linear MASs with actuator faults, the authors in Deng and Yang (2017) propose a lemma to guarantee the solvability of the regulator equations, and then a fault-tolerant controller is developed. However, this result is only applicable to the case of actuator loss-of-effectiveness faults. For the actuator loss-of-effectiveness faults, the fault matrix is invertible, which is important to guarantee the solvability of the regulator equations. It is universally known that actuator outage faults bring difficulties to theoretical analysis and often occur in practical applications (Yang & Ye, 2010). However, if the actuator outage faults occur, the fault matrix will be irreversible. Consequently, the existing method in Deng and Yang (2017) cannot guarantee the solvability of the regulator equations. Therefore, it is valuable to explore the sufficient condition for guaranteeing the solvability of the regulator equations under actuator outage faults and, further, to develop a distributed fault-tolerant controller to compensate for the effect of the actuator outage faults and loss-of-effectiveness faults.

In this context, the COR problem for linear MASs with actuator faults is studied. The actuator faults considered in this paper are outage faults and loss-of-effectiveness faults. Based on the matrix theory and the linear algebraic technique, a novel lemma is introduced to guarantee the solvability of the regulator equations. Moreover, a distributed finite-time observer is developed to estimate the state of the exosystem. Then, a distributed adaptive fault-tolerant controller is developed to solve the COR problem. The main contributions of this paper are summarized as follows: (i) An effective distributed adaptive fault-tolerant control approach is provided to solve the COR problem for linear multi-agent systems with actuator faults. (ii) Based on the matrix theory and the linear algebraic technique, a novel lemma is introduced to guarantee the solvability of the regulator equations under the effect of the actuator outage faults. However, the solvability of the regulator equations cannot be guaranteed by the existing methods in Cai et al. (2017), Deng and Yang (2017), Kim et al. (2011), Su et al. (2013), Su and Huang (2012a) and Wang et al. (2010). (iii) Different from the existing asymptotic observers and exponential observers in Cai et al. (2017), Li et al. (2016) and Su and Huang (2012a), a novel finite-time observer is developed to exactly estimate the state of the exosystem.

2. Preliminaries and problem statement

2.1. Notation and graph theory

Notation: I denotes the identity matrix with appropriate dimensions. $M > 0$ denotes M is a positive definite matrix. Given a vector $x = [x_1, \ldots, x_n]^T$, define $\|x\|_1 = \sum_{i=1}^{n} |x_i|$. Let $\|\cdot\|$ denote the Euclidean norm of a matrix or a vector. Let $diag\{A_1, \ldots, A_N\}$ be the block-diagonal matrix with matrices $A_1, \ldots, A_N$ on its principal diagonal. $A \otimes B$ denotes the Kronecker product of matrices A and B. In addition, $col\{\alpha_1, \ldots, \alpha_n\} = [\alpha_1^T, \ldots, \alpha_n^T]^T$ and $\mathbf{1} = col\{1, \ldots, 1\} \in R^n$.

Graph theory: The network considered in this paper consists of an exosystem labeled as 0 and N subsystems labeled as $1, \ldots, N$, respectively. The communication topology of the N subsystems can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ denotes the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of directed edges. A directed edge $(j, i) \in \mathcal{E}$ indicates that node j can send information to node i. Node j is called a neighbor of node i if $(j, i) \in \mathcal{E}$. The set consisting of all the neighbors of node i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, j \neq i\}$. A directed path between node j and node i is a sequence of adjacent edges in the form $\{(j, k), (k, l), \ldots, (m, i)\}$. The adjacency matrix $\mathcal{A} = [a_{ij}]$ of graph $\mathcal{G}$ satisfies $a_{ii} = 0$, $a_{ij} = 1 \Leftrightarrow (j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L}$ has the following elements $\mathcal{L}_{ij} = -a_{ij}$ if $i \neq j$ and $\mathcal{L}_{ii} = \sum_{j \neq i} a_{ij}$. The graph $\mathcal{G}$ is an undirected graph if $a_{ij} = a_{ji}$ for $\forall\, i, j \in \mathcal{V}$. The undirected graph is called connected if there exists a path between any different nodes i and j. Let $\mathcal{B} = diag\{a_{10}, \ldots, a_{N0}\}$, where $a_{i0} = 1$ means that agent i can receive information from the exosystem and $a_{i0} = 0$ otherwise. Define $\mathcal{L}_1 = \mathcal{L} + \mathcal{B}$.

Assumption 1. The undirected graph between the subsystems is connected and there exists a subsystem which can receive the information from the exosystem.

Lemma 1 (Hong et al., 2006)**.** *Suppose that Assumption 1 holds. Then all the eigenvalues of $\mathcal{L}_1$ are positive constants. In particular, there exists a unitary matrix M, such that $M^T \mathcal{L}_1 M = diag\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ with $0 < \lambda_1 \le \lambda_2 \le \cdots \le \lambda_N$.*

Similar to literature (Deng & Yang, 2017; Hong et al., 2006), Assumption 1 is used to ensure that each subsystem can receive information from the exosystem.

2.2. System description

The dynamics of the MASs are governed by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Ev(t), \quad i = 1, \ldots, N \tag{1}$$

$$e_i(t) = Cx_i(t) + Fv(t), \tag{2}$$

where $u_i(t) \in R^m$, $x_i(t) \in R^n$ and $e_i(t) \in R^q$ are the input, state and regulated output, respectively. $v(t) \in R^p$ represents the disturbance to be rejected or the reference signal to be tracked. The dynamic of $v(t)$ is generated by the exosystem:

$$\dot{v}(t) = Sv(t). \tag{3}$$

A, B, E, C, F and S are known matrices with appropriate dimensions.

2.3. Fault model

Similar to Chen et al. (2015) and Zuo et al. (2015), the following actuator faults are considered. For the actuator j of subsystem i under the fault model h, $u_{ij}(t)$ and $u_{ij}^{F_h}(t)$ represent the input and the output, respectively. It is presented as

$$u_{ij}^{F_h}(t) = \rho_{ij}^h u_{ij}(t) \tag{4}$$

where $i = 1, \ldots, N$, $j = 1, \ldots, m$ and $h = 1, \ldots, H$. H denotes the total of fault modes. ρ_{ij}^h is an unknown constant which satisfies $0 \le \underline{\rho}_{ij}^h \le \rho_{ij}^h \le \overline{\rho}_{ij}^h \le 1$.

The fault model (4) implies that: (i) $\underline{\rho}_{ij}^h = \overline{\rho}_{ij}^h = 1$ means the fault-free; (ii) $0 < \underline{\rho}_{ij}^h \le \overline{\rho}_{ij}^h < 1$ signifies the loss-of-effectiveness fault; (iii) $\rho_{ij}^h = 0$ represents the outage fault.

Denote

$$u_i^{F_h}(t) = \Lambda_i^h u_i(t),\ i = 1, \ldots, N,$$

where $h = 1, \ldots, H$, $u_i^{F_h}(t) = col\{u_{i1}^{F_h}(t), \ldots, u_{im}^{F_h}(t)\}$, $u_i(t) = col\{u_{i1}(t), \ldots, u_{im}(t)\}$ and $\Lambda_i^h = diag(\rho_{i1}^h, \ldots, \rho_{im}^h)$. For simplicity, the fault model is presented as

$$u_i^F(t) = \Lambda_i u_i(t) \tag{5}$$

where $\Lambda_i = diag\{\rho_{i1}, \ldots, \rho_{im}\}$ and $\Lambda_i \in \{\Lambda_i^1, \ldots, \Lambda_i^H\}$.

2.4. Control objective

The objective of this paper is to develop a distributed adaptive fault-tolerant controller for MASs (1)–(3) with actuator faults (5) such that

(i) The state of the resultant closed-loop system is asymptotically stable when $v = 0$;

(ii) For any initial states $v(0), x_1(0), \ldots, x_N(0)$, the regulated output converges asymptotically to zero, i.e. $\lim_{t\to\infty} e_i(t) = 0$.

Remark 1. Because of the communication constraint, the N subsystems in MASs (1) and (2) can be classified into two groups. The first group consists of those which can access the exogenous signal for feedback control, and the second consists of the rest of the N subsystems. As described in Cai et al. (2017), Ding (2015), Kim et al. (2011), Li et al. (2016), Su et al. (2013), Su and Huang (2012a), Wang et al. (2010), Xu et al. (2016) and Yang et al. (2016), the COR problem cannot be realized by the classical approach owing to the subsystems in the second group that cannot access the exogenous signal for feedback control. Therefore, the COR problem of MASs is considered in this paper.

Remark 2. The main purpose of control objective (ii) is $\lim_{t\to\infty} e_i(t) = 0$. In particular, by letting $E = 0$ and $F = -I$, this objective can reduce to the output tracking problem, i.e., $\lim_{t\to\infty}(Cx_i(t) - v(t)) = 0$ (where $Cx_i(t)$ can be seen the output of the ith follower). Furthermore, if the dimension of x_i and $v(t)$ are the same, this objective can reduce to the trajectory tracking problem ($\lim_{t\to\infty}(x_i(t) - v(t)) = 0$) by letting $E = 0$, $C = I$ and $F = -I$. Therefore, the considered COR problem can be used to solve the trajectory tracking and the disturbance rejection problem by choosing different matrix parameters S, E, C and F.

2.5. Assumptions and lemmas

To solve the COR problem for systems (1)–(3) with actuator faults (5), the following assumptions are made.

Assumption 2. rank $[B\Lambda_i]$ = rank $[B]$ for any $\Lambda_i \in \{\Lambda_i^1, \ldots, \Lambda_i^H\}$, $i = 1, \ldots, N$.

Assumption 3. The pair (A, B) is stabilizable.

Assumption 4. The matrix S has no eigenvalues with negative real parts.

Assumption 5. For all $\lambda \in \sigma(S)$, where $\sigma(S)$ denotes the spectrum of matrix S,

$$\text{rank}\left(\begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix}\right) = n + q.$$

Remark 3. Similar to literature (Deng & Yang, 2016; Li & Yang, 2012), the actuator outage faults satisfying redundant conditions in Assumption 2 can be compensated. Moreover, all the actuator loss-of-effectiveness faults can be compensated. In addition, Assumptions 3–5 are standard ones for guaranteeing the solvability of the linear output regulation problem in the classical output regulation results (Cai et al., 2017; Su et al., 2013; Su & Huang, 2012a; Wang et al., 2010).

To achieve the main results, the following lemmas are introduced.

Lemma 2. *Suppose that there exists a full column rank matrix $V \in R^{p\times r}$ such that (S, V) is controllable and there exists no matrix $W \in R^{p\times(r-1)}$ such that (S, W) is controllable. Then, there exists a coordinate transform $\eta_0(t) = Tv(t)$ such that the exosystem (3) is transformed into the following form*

$$\dot{\eta}_0(t) = \bar{S}\eta_0(t) \tag{6}$$

where

$$\bar{S} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \cdots & \bar{S}_{1r} \\ \bar{S}_{21} & \bar{S}_{22} & \cdots & \bar{S}_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{S}_{r1} & \bar{S}_{r2} & \cdots & \bar{S}_{rr} \end{bmatrix}, \bar{S}_{ii}^n = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & 0 & 1 & \cdots & 0 \\ & & \ddots & \ddots & \vdots \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix},$$

$\bar{S}_{ii} = \bar{S}_{ii}^n + T_i\alpha_{ii} \in R^{p_i\times p_i}$, $\bar{S}_{ij} = T_i\alpha_{ij} \in R^{p_i\times p_j}$ $(j \neq i)$, $\alpha_{ij} = [s_{ij}^1, s_{ij}^2, \ldots, s_{ij}^{p_j}]$, $T_i = col\{0, \ldots, 0, 1\} \in R^{p_i}$, $i = 1, \ldots, r$ and $j = 1, \ldots, r$.

Proof. See the Appendix.

Lemma 3 (*Li & Yang, 2012*)**.** *If Assumption 2 holds, there exist $\mu_1 > 0$, $\mu_2 > 0, \ldots, \mu_N > 0$ such that*

$$B\Lambda_i B^T \geq \mu_i BB^T,\ i = 1, 2, \ldots, N.$$

Lemma 4. *Suppose that Assumptions 2–5 hold. Then, there exist a matrix X and fault-dependent matrices $U_1^{\Lambda_1}, \ldots, U_N^{\Lambda_N}$ such that the following regulator equations are satisfied*

$$\begin{cases} XS - AX - E = B\Lambda_i U_i^{\Lambda_i}, \\ CX + F = 0, \quad i = 1, \ldots, N. \end{cases} \tag{7}$$

Proof. See the Appendix.

Remark 4. The existing cooperative fault-tolerant output regulation result (Deng & Yang, 2017) considers the actuator loss-of-effectiveness faults. Specifically, the diagonal element ρ_{ij} of the fault matrix Λ_i satisfies $0 < \rho_{ij} \leq 1$. Thus, the fault matrix Λ_i is invertible, which is important to guarantee the solvability of regulator equations (7). However, if the actuator outage faults occur, the fault matrix Λ_i will be irreversible. Consequently, the existing method in Deng and Yang (2017) cannot guarantee the solvability of the regulator equations. To address this issue, Lemma 4 is introduced.

Lemma 5 (*Bhat & Bernstein, 2000*)**.** *Suppose that there exists a continuous function $W(x)$ such that (i) $W(x)$ is positive definite; (ii) there exist real numbers $\varrho \in (0, 1)$ and $\iota > 0$ and an open neighborhood $\mathcal{D}$ of the origin such that*

$$\dot{W}(x) + \iota W^{\varrho}(x) \leq 0,\ x \in \mathcal{D}\backslash\{0\}.$$

Then, the origin of the system is finite-time stable, and the upper bound of the settling time T satisfies

$$T \leq \frac{V^{1-\varrho}(0)}{\iota(1-\varrho)}.$$

Lemma 6 (Bhat & Bernstein, 2005). *Consider the system: $\dot{x}_1 = x_2, \ldots, \dot{x}_{n-1} = x_n, \dot{x}_n = u$. Let $k_n, \ldots, k_1$ be positive constants such that the polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ is Hurwitz. There exists $\varepsilon \in (0, 1)$ such that, for any $\alpha \in (1-\varepsilon, 1)$, the origin is a globally finite-time stable equilibrium for the above system under the feedback*

$$u = -k_1 sign(x_1)|x_1|^{\alpha_1} - \cdots - k_n sign(x_n)|x_n|^{\alpha_n},$$

where α_i satisfies

$$\alpha_i = \frac{\alpha_{i+1}\alpha_{i+2}}{2\alpha_{i+2} - \alpha_{i+1}}, \quad i = n-1, \ldots, 1$$

with $\alpha_{n+1} = 1$ and $\alpha_n = \alpha$.

3. Main results

3.1. The finite-time observer design

Based on Lemma 2, the following distributed finite-time observers are developed

$$\dot{\eta}_i(t) = \bar{S}\eta_i(t) + \bar{T}\nu_i(t), \quad i = 1, \ldots, N \tag{8}$$

where $\bar{T} = diag\{T_1, \ldots, T_r\}$. $\nu_i(t) = col\{\nu_{i1}(t), \ldots, \nu_{ir}(t)\} \in R^r$ is the input vector which is given by

$$\nu_{ij}(t) = -\sum_{k=1}^{r} \alpha_{jk}\zeta_{ik}(t) - \sum_{k=1}^{p_j} \beta_{ij}^k sign(\zeta_{ij}^k(t))|\zeta_{ij}^k(t)|^{\theta_{ij}^k}, \tag{9}$$

where parameters β_{ij}^k and θ_{ij}^k will be designed in Lemma 7 and $\zeta_{ij}(t) = col\{\zeta_{ij}^1(t), \ldots, \zeta_{ij}^{p_j}(t)\} \in R^{p_j}$. The dynamic of $\zeta_i(t) = col\{\zeta_{i1}(t), \ldots, \zeta_{ir}(t)\}$ is described by

$$\dot{\zeta}_i(t) = \bar{S}\zeta_i(t) - c\bar{T}\bar{T}^T R\varphi_i(t) - \iota sign(R\varphi_i(t)) + \bar{T}\nu_i(t) \tag{10}$$

where $\varphi_i(t) = \sum_{j=0}^{N} a_{ij}[(\zeta_i(t) - \zeta_j(t)) - (\eta_i(t) - \eta_j(t))]$. ι is an arbitrary positive constant and $c \geq \frac{1}{\lambda_1}$. $\bar{T}$ is given in the proof of Lemma 2. $R > 0$ satisfies $R\bar{S} + \bar{S}^T R - 2R\bar{T}\bar{T}^T R = -Q_1$, where Q_1 is an arbitrary positive definite matrix.

To facilitate analysis, the following lemma is introduced:

Lemma 7. *Let ε be defined in Lemma 6. $\beta_{ij}^1, \ldots, \beta_{ij}^{p_j}$ are positive constants which are chosen such that the polynomial $s^{p_j} + \beta_{ij}^{p_j} s^{p_j - 1} + \cdots + \beta_{ij}^2 s + \beta_{ij}^1$ is Hurwitz. Suppose that the observer parameters in* (9) *and* (10) *are chosen as*

$$\theta_{ij}^k = \frac{\theta_{ij}^{k+1}\theta_{ij}^{k+2}}{2\theta_{ij}^{k+2} - \theta_{ij}^{k+1}}, \ j = 1, \ldots, r, \ k = p_j - 1, \ldots, 1$$

with $\theta_{ij}^{p_j+1} = 1$, $\theta_{ij}^{p_j} = \theta_{ij}$ and $\theta_{ij} \in (1-\varepsilon, 1)$. Then, the observer state $\eta_i(t)$ converges to $\eta_0(t)$ in finite-time, i.e., there exists $T_0 > 0$ such that $\eta_i(t) = \eta_0(t)$ for $t \geq T_0$.

Proof. See the Appendix.

3.2. The distributed adaptive controller design

Based on the observer state $\eta_i(t)$, the following distributed adaptive fault-tolerant controller is developed

$$u_i(t) = -\hat{\alpha}_i(t)B^T P\xi_i(t) - \frac{\hat{\beta}_i^2(t)B^T P\xi_i(t)\|\bar{\eta}_i(t)\|^2}{\hat{\beta}_i(t)\|\xi_i^T(t)PB\| \cdot \|\bar{\eta}_i(t)\| + \sigma(t)}, \tag{11}$$

where $\xi_i(t) = \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) + a_{i0}(x_i(t) - X\upsilon(t))$ and $\bar{\eta}_i = T^{-1}\eta_i$. $\sigma(t)$ is a uniform continuous function which satisfies $\lim_{T\to+\infty}\int_{t_0}^{T}\sigma(s)ds \leq \bar{\sigma} < +\infty$. $P > 0$ satisfies $PA + A^T P - 2PBB^T P = -Q_2$, where Q_2 is an arbitrary positive definite matrix. In addition, $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are the adaptive parameters which are updated by

$$\dot{\hat{\alpha}}_i(t) = \gamma_{\alpha_i}\|\xi_i^T(t)PB\|^2 - 2\gamma_{\alpha_i}\sigma(t)\hat{\alpha}_i(t), \tag{12}$$

$$\dot{\hat{\beta}}_i(t) = \gamma_{\beta_i}\|\xi_i^T(t)PB\| \cdot \|\bar{\eta}_i(t)\| - 2\gamma_{\beta_i}\sigma(t)\hat{\beta}_i(t), \tag{13}$$

where γ_{α_i} and γ_{β_i} are given positive constants and the initial states $\hat{\alpha}_i(t_0)$ and $\hat{\beta}_i(t_0)$ are positive constants.

Define $\bar{x}_i(t) = x_i(t) - X\upsilon(t)$. From (1), (3) and (7), one obtains

$$\dot{\bar{x}}_i(t) = A\bar{x}_i(t) + B\Lambda_i u_i(t) - BU_0\upsilon(t).$$

Let $\bar{x} = col\{\bar{x}_1(t), \ldots, \bar{x}_N(t)\}$. Accordingly,

$$\dot{\bar{x}} = (I_N \otimes A)\bar{x} + (I_N \otimes B)\Lambda u - (I_N \otimes BU_0)(\mathbf{1} \otimes \upsilon) \tag{14}$$

where $\Lambda = diag\{\Lambda_1, \ldots, \Lambda_N\}$ and $u = col\{u_1(t), \ldots, u_N(t)\}$. Define $\xi = col\{\xi_1(t), \ldots, \xi_N(t)\}$. Thus, $\xi = (\mathcal{L}_1 \otimes I)\bar{x}$. For brevity, the time variable t in all functions will be omitted in the remainder parts of this paper.

3.3. Stability analysis

Theorem 1. *If Assumptions 1–5 and the conditions in Lemma 7 are satisfied, the COR problem can be solved by the distributed fault-tolerant controller* (11) *with finite-time observers* (8)–(10) *and adaptive update laws* (12) and (13).

Proof. Consider the following Lyapunov function

$$V = \bar{x}^T(\mathcal{L}_1 \otimes P)\bar{x} + \sum_{i=1}^{N}\frac{\mu_i}{\gamma_{\alpha_i}}\tilde{\alpha}_i^2 + \sum_{i=1}^{N}\frac{\mu_i}{\gamma_{\beta_i}}\tilde{\beta}_i^2,$$

where $\tilde{\alpha}_i = \hat{\alpha}_i - \alpha_i$ and $\tilde{\beta}_i = \hat{\beta}_i - \beta_i$ with $\alpha_i = \frac{c}{\mu_i}$ and $\beta_i = \frac{\|U_0\|}{\mu_i}$. The time derivative of V along (14) is

$$\begin{aligned}\dot{V} =& 2\bar{x}^T(\mathcal{L}_1 \otimes PA)\bar{x} + 2\bar{x}^T(\mathcal{L}_1 \otimes PB)\Lambda u + 2\sum_{i=1}^{N}\frac{\mu_i}{\gamma_{\alpha_i}}\tilde{\alpha}_i\dot{\hat{\alpha}}_i \\ &- 2\bar{x}^T(\mathcal{L}_1 \otimes PBU_0)(\mathbf{1} \otimes \upsilon) + 2\sum_{i=1}^{N}\frac{\mu_i}{\gamma_{\beta_i}}\tilde{\beta}_i\dot{\hat{\beta}}_i \\ =& \Phi_1 + \Phi_2,\end{aligned} \tag{15}$$

where,

$$\Phi_1 = 2\bar{x}^T(\mathcal{L}_1 \otimes PA)\bar{x} - 2c\bar{x}^T(\mathcal{L}_1^2 \otimes PBB^T P)\bar{x},$$

$$\begin{aligned}\Phi_2 =& 2\sum_{i=1}^{N}(\xi_i^T PB\Lambda_i u_i + c\xi_i^T PBB^T P\xi_i - \xi_i^T PBU_0\upsilon \\ &+ \frac{\mu_i}{\gamma_{\alpha_i}}\tilde{\alpha}_i\dot{\hat{\alpha}}_i + \frac{\mu_i}{\gamma_{\beta_i}}\tilde{\beta}_i\dot{\hat{\beta}}_i).\end{aligned}$$

Define $\tilde{x} = (M^T \otimes I)\bar{x}$. According to Lemma 1, it yields

$$\begin{aligned}\Phi_1 =& \sum_{i=1}^{N}\tilde{x}_i^T(\lambda_i(PA + A^T P) - 2c\lambda_i^2 PBB^T P)\tilde{x}_i, \\ \leq& -\lambda_1\lambda_{min}(Q_2)\|\bar{x}\|^2.\end{aligned} \tag{16}$$

By using (11)–(13), one has

$$\begin{aligned}\Phi_2 \leq& 2\sum_{i=1}^{N}[-\mu_i\hat{\alpha}_i\|\xi_i^T PB\|^2 - \frac{\mu_i\hat{\beta}_i^2\|\xi_i^T PB\|^2 \cdot \|\bar{\eta}_i\|^2}{\hat{\beta}_i\|\xi_i^T PB\| \cdot \|\bar{\eta}_i\| + \sigma(t)} \\ &+ \|\xi_i^T PB\| \cdot \|U_0\| \cdot \|\bar{\eta}_i\| + \mu_i\tilde{\alpha}_i(\|\xi_i^T PB\|^2 - 2\sigma(t)\hat{\alpha}_i) \\ &+ c\|\xi_i^T PB\|^2 + \mu_i\tilde{\beta}_i(\|\xi_i^T PB\| \cdot \|\bar{\eta}_i\| - 2\sigma(t)\hat{\beta}_i) \\ &+ \|\xi_i^T PB\| \cdot \|U_0\|(\|\upsilon\| - \|\bar{\eta}_i\|)]\end{aligned}$$

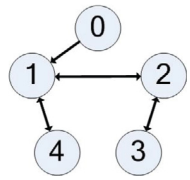

Fig. 1. The communication topology.

$$\leq 2\sum_{i=1}^{N}\mu_i\frac{\sigma(t)\hat{\beta}_i\|\xi_i^T PB\|\cdot\|\bar{\eta}_i\|}{\hat{\beta}_i\|\xi_i^T PB\|\cdot\|\bar{\eta}_i\|+\sigma(t)}-\sigma(t)\sum_{i=1}^{N}\mu_i\tilde{\alpha}_i^2$$
$$-\sigma(t)\sum_{i=1}^{N}\mu_i\tilde{\beta}_i^2+\sigma(t)\sum_{i=1}^{N}(\frac{c^2}{\mu_i}+\frac{\|U_0\|^2}{\mu_i})$$
$$+2\sum_{i=1}^{N}\|\xi_i^T PB\|\cdot\|U_0\|(\|v\|-\|\bar{\eta}_i\|). \tag{17}$$

According to Lemmas 2 and 7, $\bar{\eta}_i$ converges to v after $t\geq T_0$. Thus, for $t\geq T_0$, one obtains

$$\Phi_2\leq-\sigma(t)\sum_{i=1}^{N}\mu_i\tilde{\alpha}_i^2-\sigma(t)\sum_{i=1}^{N}\mu_i\tilde{\beta}_i^2+\kappa\sigma(t), \tag{18}$$

where $\kappa=\sum_{i=1}^{N}[2\mu_i+\frac{c^2}{\mu_i}+\frac{\|U_0\|^2}{\mu_i}]$. Substituting (16) and (18) into (15), it yields

$$\dot{V}\leq-\lambda_1\lambda_{min}(Q_2)\|\bar{x}\|^2-\sigma(t)\sum_{i=1}^{N}\mu_i(\tilde{\alpha}_i^2+\tilde{\beta}_i^2)+\kappa\sigma(t),$$

which implies that $V(t)\leq V(T_0)+\kappa\bar{\sigma}$. Therefore, $\bar{x}(t)$ is uniformly bounded, which implies that $\bar{x}(t)$ is uniformly continuous. Furthermore, one has

$$\lim_{t\to\infty}\int_{T_0}^{t}\lambda_1\lambda_{min}(Q)\|\bar{x}(\tau)\|^2 d\tau\leq V(T_0)+\kappa\bar{\sigma}.$$

By using the Barbalat's Lemma in Slotine and Li (1991), it is deduced that $\lim_{t\to\infty}\|\bar{x}(t)\|=0$, i.e., $\lim_{t\to\infty}(x_i-Xv)=0$. From the regulator equations (7), it yields

$$\lim_{t\to\infty}e_i=\lim_{t\to\infty}C(x_i-Xv)+\lim_{t\to\infty}(CX+F)v=0.$$

This completes the proof.

Remark 5. In this paper, the COR problem is considered for linear MASs with actuator outage and loss-of-effectiveness faults. The existing COR methods in Cai et al. (2017), Ding (2015), Kim et al. (2011), Su and Huang (2012a) and Wang et al. (2010), which design the time-invariant controllers, are unable to compensate for the effect of the actuator faults. To solve the problem, time-varying parameters $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are introduced in the distributed controller (11).

Remark 6. If asymptotic observers and exponential observers in Cai et al. (2017), Li et al. (2016) and Su and Huang (2012a) are adopted to compensate for the effect of the actuator faults, the last term in (17) cannot be counteracted. It means that the asymptotic convergence of $\bar{x}$ cannot be achieved. Consequently, the COR problem cannot be solved. To counteract the last term in (17), finite-time observers (8)–(10) are developed.

4. Simulation example

To show the effectiveness of the developed method, the MASs of the form (1)–(3) are considered, where $S=[0\ 1;\ -1\ 0]$, $A=$

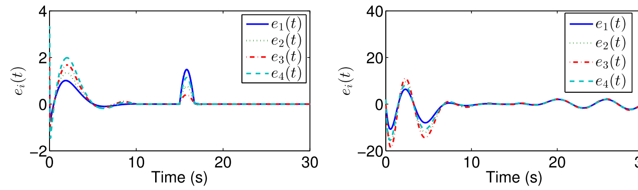

Fig. 2. Regulated output of the proposed method (left) and the method in Su et al. (2013) (right).

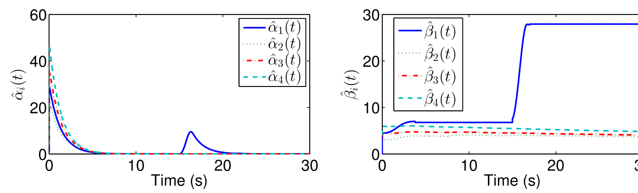

Fig. 3. The trajectories of parameters $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$.

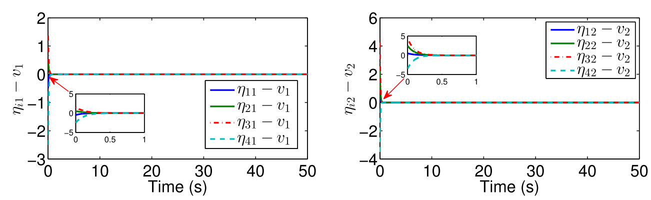

Fig. 4. The observer errors of $\eta_{i1}-v_1$ (left) and $\eta_{i2}-v_2$ (right).

$[0\ 1\ 0\ 0;\ 0\ 0\ 1\ 0;\ 0\ 0\ 0\ 1;\ 0\ 0\ 0\ 0]$, $B=[1\ 0\ 1;\ 0\ 0\ 0;\ 0\ 0\ 0;\ 0\ 1\ -1]$, $E=[1\ 0;\ 0\ 0;\ 0\ 0;\ 0\ 1]$, $C=[1\ 0\ 0\ 0]$ and $F=[-1\ 0]$. The information exchange of the communication network is depicted in Fig. 1. The regulator equations (7) are solved, which yields $X=[1\ 0;\ -1\ -1;\ 1\ -1;\ 1\ 1]$. The initial conditions are chosen as $v(0)=[1.5\ 1.5]^T$, $x_i(0)=i[1\ 1\ 1\ 1]^T$, $\eta_i(0)=i[1\ 2]^T$, $\hat{\alpha}_i(0)=\hat{\beta}_i(0)=i$ (for $i=1,\ldots,4$). Simulations were run with the following actuator faults: agents 2-4 are fault-free; before 13 s, agent 1 is fault-free; after 13 s, the actuator 1 of agent 1 loses 90% of its effectiveness, the actuator 2 loses 80% of its effectiveness and the actuator 3 suffers from the outage fault.

The regulated output $e_i(t)$ obtained using the fault-tolerant controller (11) and the robust output regulation controller in Su et al. (2013) can be seen in Fig. 2, which shows that the developed controller (11) is effective in compensating for the actuator outage faults and loss-of-effectiveness faults. In contrast, the robust output regulation controller in Su et al. (2013) is unable to compensate for the actuator faults. In addition, the trajectories of adaptive parameters $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are displayed in Fig. 3. The observer errors $\eta_{i1}(t)-v_1(t)$ and $\eta_{i2}(t)-v_2(t)$ are shown in Fig. 4.

5. Conclusions

The COR problem for linear MASs with actuator faults has been investigated in this paper. A novel lemma has been introduced to guarantee the solvability of the regulator equations under actuator faults. A distributed finite-time observer has been designed to estimate the state of the exosystem. Then, a distributed adaptive fault-tolerant controller has been developed. It has been shown that the cooperative output regulation problem can be solved with the developed method. In the future, the more generic directed or switching topology will be considered.

Appendix

A.1. Proof of Lemma 2

Proof. Since (S, V) is controllable, from the Luenberger canonical form (13) in Luenberger (1967), it is deduced that there exists a coordinate transformation $\zeta(t) = Tz(t)$ such that

$$\bar{S} = TST^{-1},\ \bar{T} = TV,$$

where $\bar{T} = diag\{T_1, \dots, T_r\}$. Thus, the exosystem (3) can be transformed into the system (6) by using the coordinate transformation $\eta_0(t) = Tv(t)$. This completes the proof.

A.2. Proof of Lemma 4

Proof. According to Assumptions 3–5, there exist matrices X and U_0 such that the following regulator equations are satisfied

$$\begin{cases} XS - AX - E = BU_0, \\ CX + F = 0, \quad i = 1, \dots, N. \end{cases} \tag{19}$$

The objective in the following is to show that there exist fault-dependent matrices $U_i^{\Lambda_i}$ $(i = 1, \dots, N)$ such that the matrix equations $B\Lambda_i U_i^{\Lambda_i} = BU_0$ are satisfied.

Based on matrix theory, it shows that the matrix equations $B\Lambda_i U_i^{\Lambda_i} = BU_0$ are solvable if and only if $\mathrm{rank}(B\Lambda_i) = \mathrm{rank}(B\Lambda_i, BU_0)$.

Let $B = [b_1, \dots, b_m]$. Since $\mathrm{rank}(B) = l \le m$, without loss of generality, we assume vectors $b_1, \dots, b_l$ are linear independent. Therefore, one has $b_i \in \mathcal{M}$ for $i = 1, \dots, m$, where

$$\mathcal{M} := \left\{x | x = \alpha_1 b_1 + \cdots + \alpha_l b_l, \forall \alpha_j \in R, (j = 1, \dots, l)\right\}.$$

Let $B\Lambda_i = [\bar{b}_1^i, \dots, \bar{b}_m^i]$. According to Assumption 2, it yields $\mathrm{rank}(B\Lambda_i) = l$. We assume, without loss of generality, that vectors $\bar{b}_1^i, \dots, \bar{b}_l^i$ are linear independent. Therefore, one has $\bar{b}_i \in \mathcal{N}_i$ for $i = 1, \dots, m$, where

$$\mathcal{N}_i := \left\{x | x = \alpha_1 \bar{b}_1^i + \cdots + \alpha_l \bar{b}_l^i, \forall \alpha_j \in R, (j = 1, \dots, l)\right\}.$$

For any $x \in \mathcal{N}_i$, there exist $\beta_1, \dots, \beta_l$ such that $x = \beta_1 b_1 + \cdots + \beta_l b_l$. Thus, the set $\mathcal{N}_i$ can be spanned by $b_1, \dots, b_l$. From the linear algebra technique, it shows that $\mathcal{M} = \mathcal{N}_i$.

Let $U_0 = [U_{01}, \dots, U_{0p}]$ and $U_{0j} = col\{U_{0j}^1, \dots, U_{0j}^m\}$ for $j = 1, \dots, p$. The vector BU_{0j} can be expressed as $BU_{0j} = U_{0j}^1 b_1 + \cdots + U_{0j}^m b_m$. Namely, $BU_{0j} \in \mathcal{N}_i$. Applying matrix theory, one obtains $\mathrm{rank}(B\Lambda_i, BU_0) = l$. Accordingly, the matrix equation $B\Lambda_i U_i^{\Lambda_i} = BU_0$ is solvable. This completes the proof.

A.3. Proof of Lemma 7

Proof. For simplicity, the proof is broken up into the following two steps.

(i) Define $\delta_i = \zeta_i - \tilde{\eta}_i$, where $\tilde{\eta}_i = col\{\tilde{\eta}_{i1}, \dots, \tilde{\eta}_{ir}\} = \eta_i - \eta_0$ is the observation error. From (10) and (20), it yields

$$\begin{aligned}\dot{\delta}_i =& \bar{S}\delta_i - c\bar{T}\bar{T}^T R\Big(\sum_{j=0}^N a_{ij}[(\zeta_i - \zeta_j) - (\eta_i - \eta_j)]\Big) \\ &- \iota sign\Big(\sum_{j=0}^N a_{ij} R[(\zeta_i - \zeta_j) - (\eta_i - \eta_j)]\Big).\end{aligned}$$

Let $\delta = [\delta_1, \dots, \delta_N]^T$. Accordingly,

$$\dot{\delta} = (I_N \otimes \bar{S})\delta - c(\mathcal{L}_1 \otimes \bar{T}\bar{T}^T R)\delta - \iota sign\big((\mathcal{L}_1 \otimes R)\delta\big).$$

We now construct a Lyapunov function $W = \delta^T(\mathcal{L}_1 \otimes R)\delta$. The time derivative of W is

$$\begin{aligned}\dot{W} =& 2\delta^T(\mathcal{L}_1 \otimes R\bar{S})\delta - 2c\delta^T(\mathcal{L}_1^2 \otimes R\bar{T}\bar{T}^T R)\delta \\ &- 2\iota\|(\mathcal{L}_1 \otimes R)\delta\|_1.\end{aligned}$$

Define $\varepsilon = (M^T \otimes I)\delta$. According to Lemma 1, it yields

$$\begin{aligned}\dot{W} =& 2\varepsilon^T(\Lambda \otimes R\bar{S})\varepsilon - 2c\varepsilon^T(\Lambda^2 \otimes R\bar{T}\bar{T}^T R)\varepsilon \\ &- 2\iota\|(\mathcal{L}_1 \otimes R)\delta\|_1 \\ =& \sum_{i=1}^N \lambda_i \varepsilon_i^T(R\bar{S} + \bar{S}^T R - 2c\lambda_i R\bar{T}\bar{T}^T R)\varepsilon_i \\ &- 2\iota\|(\mathcal{L}_1 \otimes R)\delta\|_1 \\ \le& -2\iota\sqrt{\lambda_1 \lambda_{min}(R)}W^{\frac{1}{2}}.\end{aligned}$$

Applying Lemma 5, it is deduced that δ converges to zero in finite-time.

(ii) From (6) and (8), the dynamics of $\tilde{\eta}_i$ are

$$\dot{\tilde{\eta}}_i = \bar{S}\tilde{\eta}_i + \bar{T}\nu_i. \tag{20}$$

From the definitions of $\tilde{\eta}_i$, $\bar{S}$, $\bar{T}$ and ν_i, one obtains

$$\dot{\tilde{\eta}}_{ij} = \bar{S}_{jj}^n \tilde{\eta}_{ij} + T_i\big(\nu_{ij} + \sum_{k=1}^r \alpha_{jk}\tilde{\eta}_{ik}\big).$$

According to the definition of ν_{ij} in (9), it yields

$$\begin{aligned}\dot{\tilde{\eta}}_{ij} =& \bar{S}_{jj}^n \tilde{\eta}_{ij} + T_i\Big(-\sum_{k=1}^{p_j} \beta_{ij}^k sign(\zeta_{ij}^k)|\zeta_{ij}^k|^{\theta_{ij}^k} \\ &- \sum_{k=1}^r \alpha_{jk}(\zeta_{ik} - \tilde{\eta}_{ik})\Big).\end{aligned} \tag{21}$$

Therefore, after finite-time, (21) is reduced to

$$\dot{\tilde{\eta}}_{ij} = \bar{S}_{ii}^n \tilde{\eta}_{ij} - T_i \sum_{k=1}^{p_j} \beta_{ij}^k sign(\zeta_{ij}^k)|\zeta_{ij}^k|^{\theta_{ij}^k}. \tag{22}$$

Applying the conclusions of (i) and Lemma 6, it shows that $\tilde{\eta}_{ij}$ converges to zero in finite time. Thus, the observer state η_i converges to η_0 in finite-time. This completes the proof.

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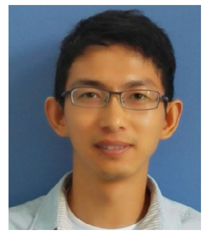

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