3F1 Flight Control Report

Investigation on control methods of a simulated aircraft model

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1 Overview

In order to analyse and optimize a real time control system, a mixture of discrete-time and continous-time concepts are used. In this experiment, the plant is an aeroplane with predefined models of dynamics, the plant is discrete-time while the manual controller acts in continous time. Block diagram shown in figure 1 shows the set up of our simple aeroplane control system with nomenclators defined as follows:

1. $r(t)$: Reference signal, set to be 0 (horizon) for our experiment
2. $e(t)$: Error signal
3. $u(t)$: Controller output
4. $d(t)$: Input disturbances
5. $x(t)$: Plant (aeroplane) input
6. $y(t)$: Plant (aeroplane) output

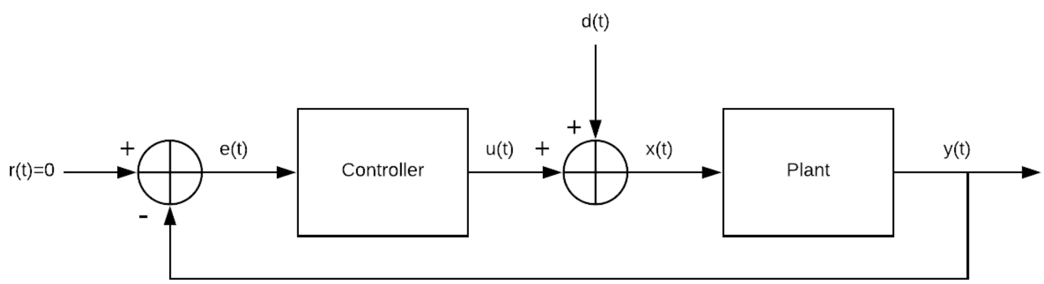


Figure 1: Aeroplane control block diagram

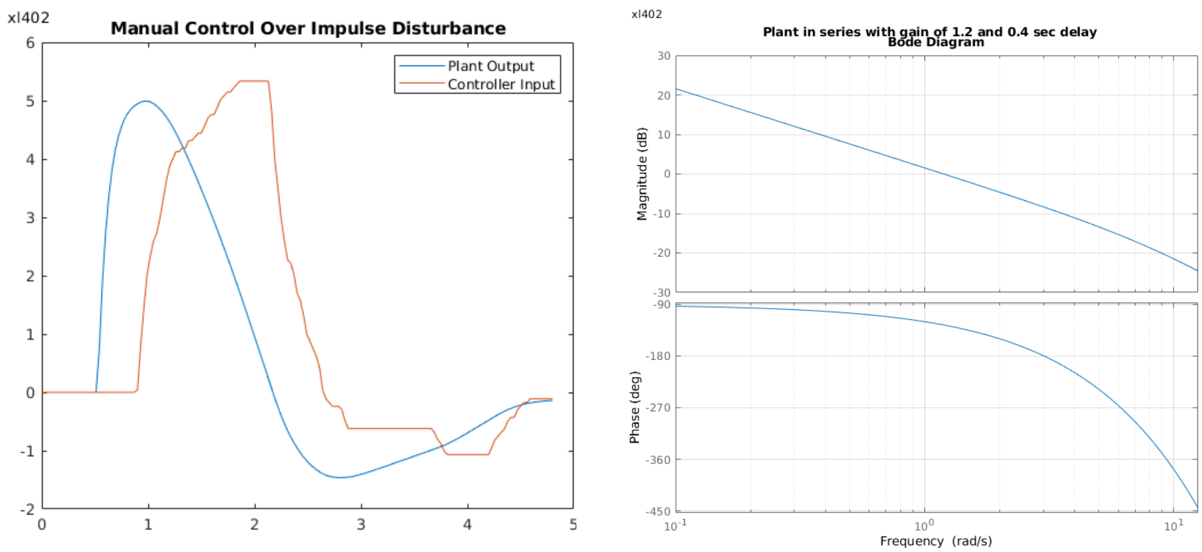
In order to investigate different methods of control (manual, PID) over different models of the aircraft (naturally stable, unstable), we devise a series of investigations below:

1. Manual control of the aircraft
2. Pilot induced oscillation
3. Sinusoidal disturbances
4. Unstable aircraft model
5. Proportional feedback controller
6. PID controller

2 Manual Controller Behaviour Analysis

Starting with a simplified aircraft mode; defined by:

$$\ddot{y}(t) + M\dot{y}(t) = Nx(t) \tag{1}$$



(a) Typical response graph

(b) Open loop Bode plot

Figure 2: Manual Control

Where M is a coefficient of aerodynamic damping and N is a coefficient of aerodynamic effectiveness of the elevators. The laplace transfer function of this plane is therefore:

$$G = \frac{N}{s^2 + Ms} \tag{2}$$

Now we need to model and estimate the parameters of the manual controller, figure 2a depicts a typical response graph of a user to a impulse disturbance of magnitude 5 (units) so that $d(t) = 5\delta(0)$, note from the previous block diagram in figure 1, as the reference siganl is always set to zero, the plant output is $y(t)$ is identical to the negative of the error signal $e(t)$, in our case they are interchangable.

A simple model for the manual controller is a proportional gain k in series with a pure time delay D. The laplace transfer function of this controller is:

$$K(s) = ke^{-sD} \tag{3}$$

From the response graph in figure 2a, we can estimate parameter k and D where k is estimated as the step impulse the pilot applies after experiencing the impulse disturbance divided by the magnitude of the disturbance, in our case it is:

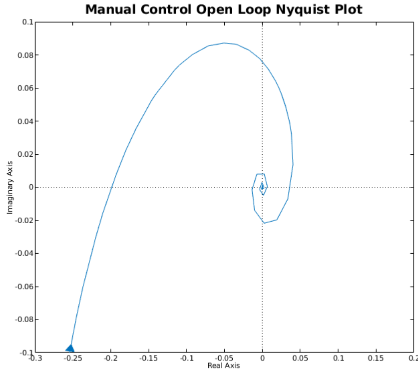
$$k = 5.5/5 = 1.1 \tag{4}$$

the time delay is read off the x axis to be

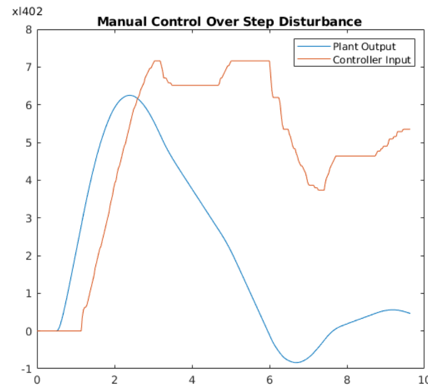
$$D = 0.25 \text{ seconds} \tag{5}$$

The Bode diagram for the open loop including the controller is plotted in figure 2b, from which we can estimate the phase margin by finding the phase corresponding the $0dB$ gain and measure its distance to 180 degrees. The phase margin estimate is $\phi = 50$degrees $= 0.87$rad. Hence the amount of extra time delay in this control loop would tolerate before going unstable is:

$$t_{\text{tolerance}} = \frac{\phi}{\omega} = 0.87/1.1 = 0.79 \text{ seconds} \tag{6}$$

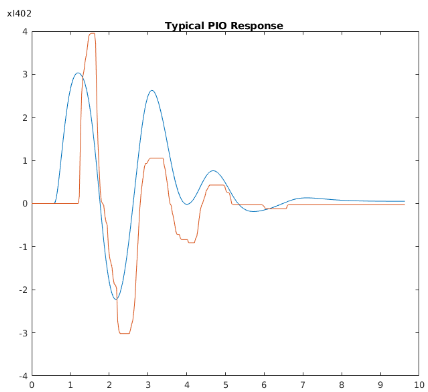


(a) Nyquist diagram of open loop control

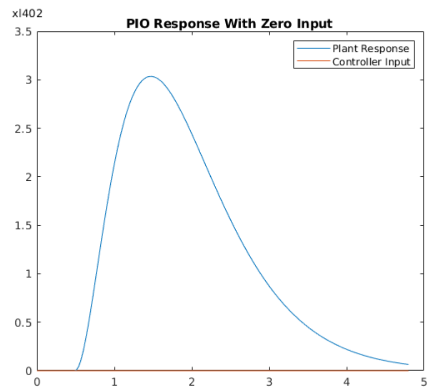


(b) Typical manual step response

Figure 3: Manual control responses



(a) Typical pilot plant PIO responses



(b) Plant response with zero input

Figure 4: PIO behaviour analysis

Where ω is the frequency at which the gain is $0dB$. Figure 3a shows the Nyquist diagram sketched from the Bode plot. Some key points are: we know as $\omega \to 0$, $\phi \to 90$ so the plot starts from the bottom right quadrant. The gain margin is approximataly $4dB$ hence the Nyquist x axis intercept in the left quadrant is $-1/4 \approx -0.25$ (the magnitude of this Nyquist plot is also in units of dB). Lastly, the magnitude and phase are always decreasing.

The basic aircraft model is given a step disturbance of magnitude 5, a typical step response of manual control is depicted in figure 3b, we can see that integral is used, since there is a near constant gradient in the gain, this is to compensate for the fact that the error signal does not decrease as quickly enough (since it is a step instead of an impulse disturbance).

3 Pilot Induced Oscillation Analysis

A badly designed airraft has the following plant transfer function:

$$G_1(s) = \frac{c}{(Ts+1)^3} \tag{7}$$

With parameters $T = 4D/pi$ and $c = \sqrt{8}/k$, k and D are gain and time delay of the controller (pilot) modeled previously. A system designed like this, exhibits a phenominom called *pilot induced oscillation*, figure 4a depicts a typical pilot-plant behaviour when the aircraft undergoes a impulse disturbance. Figure 4b depicts the plant behaviour under the impulse disturbance without pilot input. We can see that the aircraft is naturally stable, but the pilot's intention to stabalize the aircraft actually causes oscillatary behaviour. To under stand this, the open loop bode plot is shown in figure 5 is generated with a suitable frequency range, we can see that the phase margin is merely 2 degrees. This explains why the aircraft is very oscillatary.

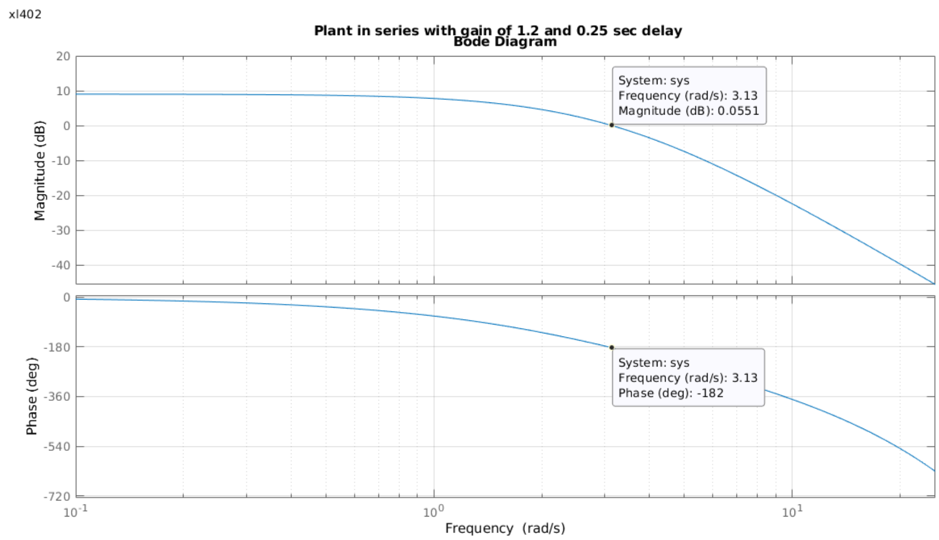
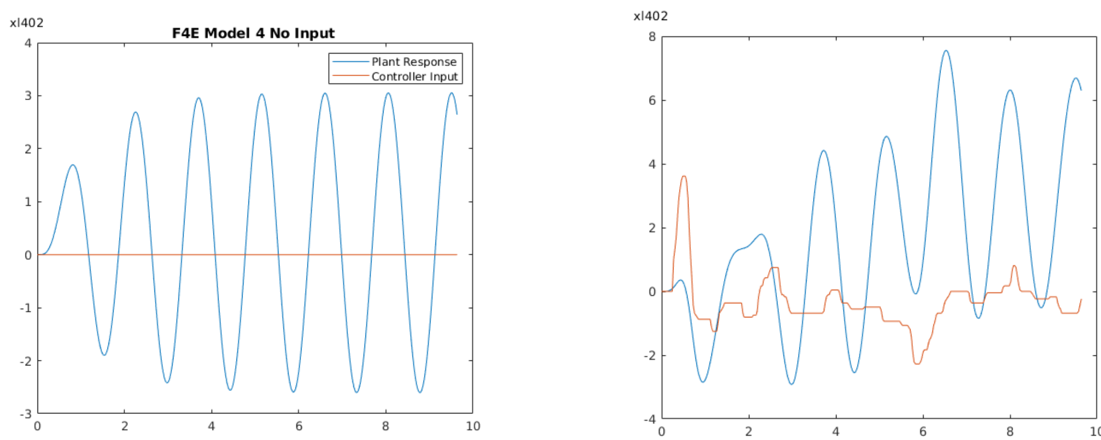


Figure 5: PIO system open loop Bode plot



(a) Typical pilot plant PIO responses

(b) Typical manual pilot-plant responses

Figure 6: F4E Model 4 fighter aircraft Responses

4 Sinusoidal Disturbances Over F4E Fighter Aircraft

Some more realistic models of the F4E fighter aircraft is analysed, there are four models representing the open loop dynamics, linearised about 4 different operating points (different altitudes and Mach numbers). As three models out of four are naturally unstable, only the fourth operating point is examined here (*altitude=35000ft. Mach 1.5*).
Given a sinusoidal disturbance of $0.66Hz$ of magnitude 1, figure 6a shows the aircraft's responses without any pilot input. figure 6b shows its behaviour when the pilot attempts to stabalize it manually, with a gain of 1, and a time delay D calculate previously. Clearly, the pilot is unable to reduce the oscillation of the aircraft. The open loop Bode plot is shown in figure 7. From this we can calculate the maximum proportional gfain for which the closed system is stable, reading from the -180 line, we find that for a gain of 1, the aircraft is in fact unstable (above $0dB$), the gain margin is actually 0.11. This is confirmed from figure 6b as the pilot is more careful at using the joystick. Also the gain and phase and gain at $0.66Hz = 4.14\text{rad/s}$ is $9.03dB$ and -164 degrees respectively. For a plant transfer function of G and a controller transfer function of K, the open loop transfer function from d to y is simply G as the reference signal r is zero. For the closed loop transfer function L:

$$L = \frac{G}{1 + KG} \tag{8}$$

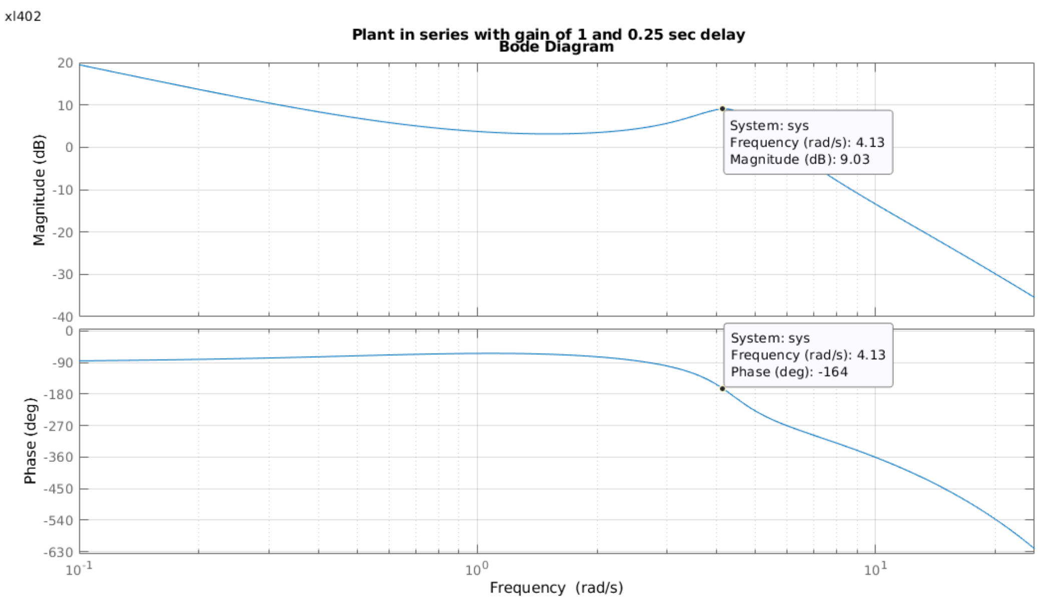


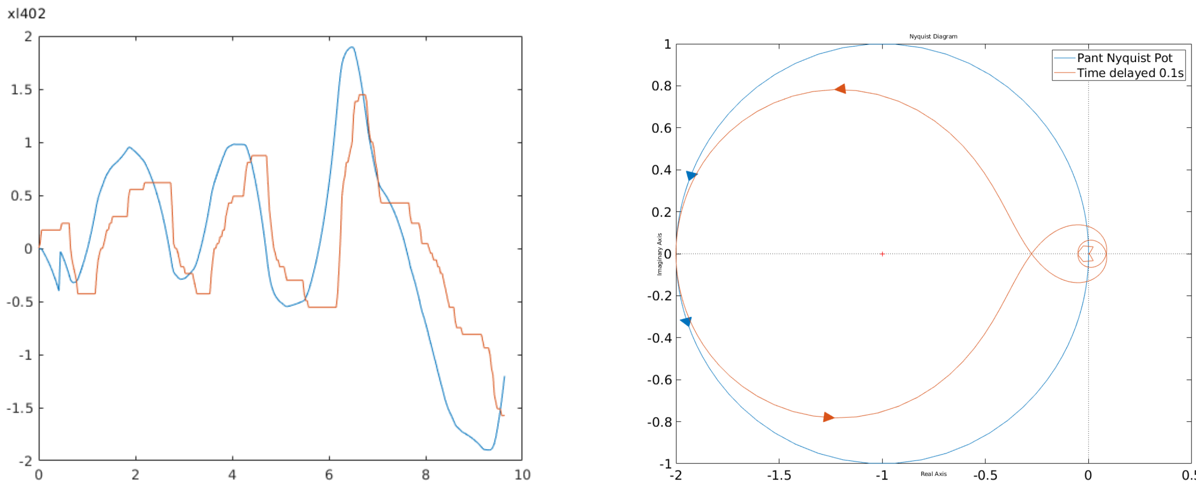
Figure 7: F4E Model 4 open loop Bode plot

5 Unstable Aircraft Behaviour Analysis

The behaviour of a unstable aircraft is investigated, taking the simplified model:

$$G(s) = \frac{2}{sT - 1} \tag{9}$$

Which has an unstable at $1/T$, manual control with some trial and errors can achieve the stabalisation of this model with $T = 0.5$ for full 5 seconds. The response of the pilot-plant response is shown in 8a. The nyquist diagram is shown in figure 8b, we can see that there is a circle with anti-clockwise encirclement of the -1 point, therefore the plant itself is unstable, the nyquist plot reaches the x-axis at -2 point, therefore a proportional gain greater than $1/2$ will be able to stabalise this aircraft. The orange line indicated on the same graph is the time delayed (0.1 seconds) nyquist plot of the same plant, we can see that the gain required to stabalise it does not change.



(a) Typical manual stabalisation response

(b) Open loop nyquist plot

Figure 8: Unstable aircraft behaviour

6 Autopilot

Consider the following model for a transport aircraft on approach to landing:

$$G_3(s = \frac{6.3s^2 + 4.3s + 0.28}{s^5 + 11.2s^4 + 19.6s^3 + 16.2s^2 + 0.91s + 0.27} \tag{10}$$

The response of this aircraft model to an impulse disturbance of weight 2 is shown in figure 9a. We devise a simple proportional negative feedback controller with gain of 5. This controller is tested with a disturbance impulse of weight 2 and a runtime of 15 seconds. It is observed that the oscillatary response disappears. The gain is slowly interesed until $K_c = 17$ where the closed loop system begins to oscillate at a period of 1.81 seconds.Therefore the gain margin of the loop when a proportional controller with a gain of 5 is used is $17/5 = 3.4 = 10.62dB$.

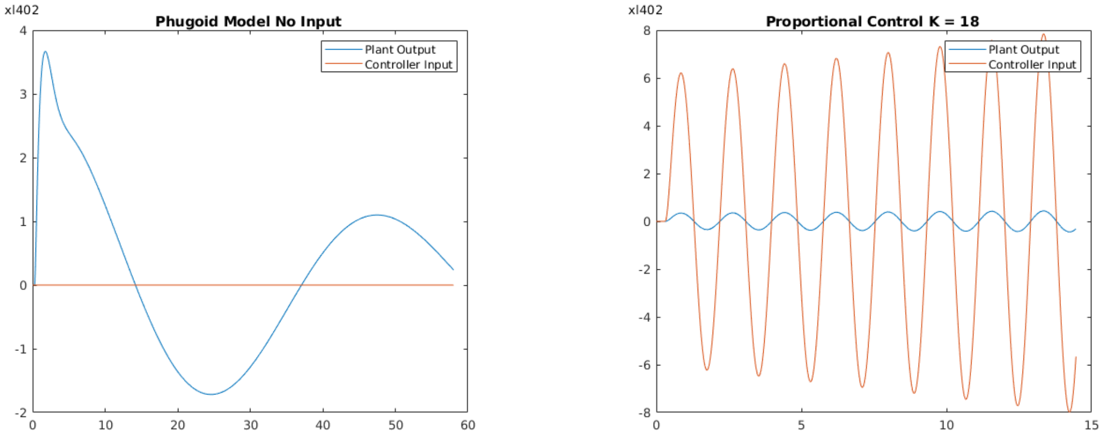
7 PID Controller

The autopilot is to be implemented with a proportional-integral-derivative (PID) controller of the form:

$$u(t) = K_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de}{dt}\right) \tag{11}$$

The transfer function of this controller is therefore:

$$K = \frac{K_p(1 + sT_i + s^2T_iT_d)}{sT_i} \tag{12}$$



(a) No input impulse response

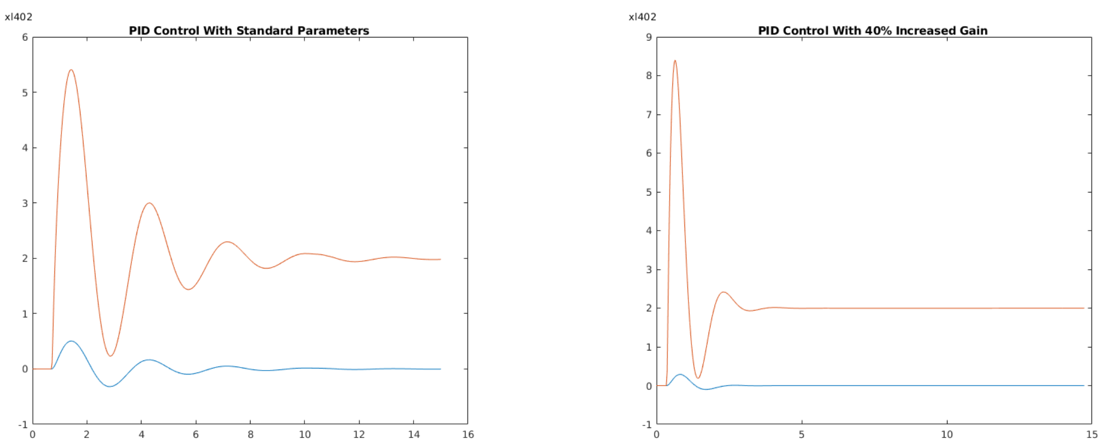
(b) Oscillatary response at high gain

Figure 9: Proportional control of transport aircraft

According to the *Zeigler-Nichols* rules in process control, the parameters should be set as:

$$K_p = 0.6K_c, \quad T_i = 0.5T_c, \quad T_d = 0.125T_c \tag{13}$$

In our case following T_c (time period of proportional feedback induced oscillation) and K_c (maximum feedback gain before oscillation) fom the section above, $K_p = 0.6 \times 17 = 10.2$, $T_i = 0.5 \times 1.81 = 0.905$ and $T_d = 0.125 \times 1.81 = 0.226$. Figure 10a shows the response after implementing such feedback system, figure **??** shows the response after increasing the gain by 40%, note the slight increase in damping (reduced oscillation).



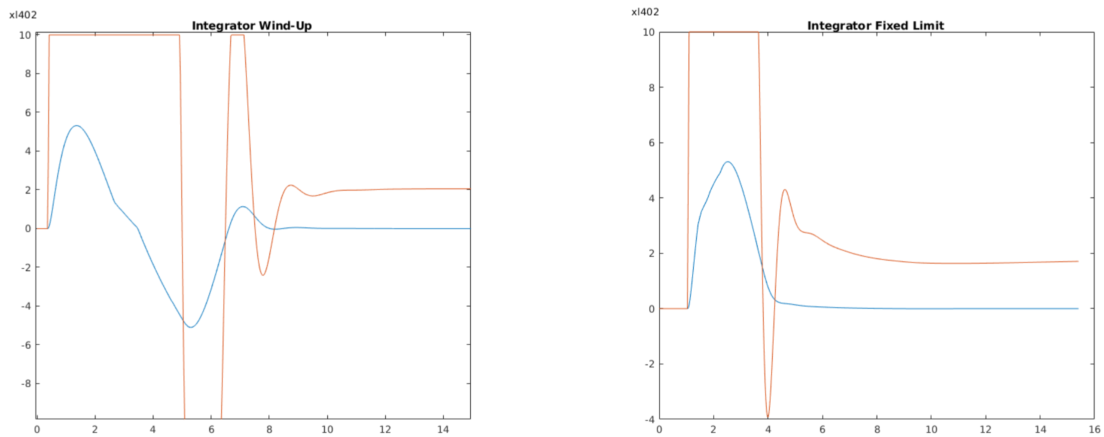
(a) Standard Ziegler-Nichols parameters

(b) Increased gain (40%)

Figure 10: PID Controlled aircraft response

8 Integrator Wind-Up

When the disturbance is set to a high impulse, a phenomena called the integrator wind-up is observed, this can cause unwanted overshoot/undershoot after a period of input saturation due to the integrator being 'wound up' and having to unload again. One simple corrective method is to set a limit on the integrator. Figure 11a shows this issue, note how it takes the controller several large swings to stabalise the aircraft. Figure 11b shows the outcome of setting a threshold of the integrator. Following the code given, the smallest number which guarantees a zero steady-state error in response to a step disturbance of weight 2 is 0.1.



(a) Integrator wind-up issue

(b) Integrator limit set

Figure 11: Aircraft response over large disturbance

9 Appendix: MATLAB Code

Code below is from the *flysim1.m* file:

```
num=[6.3 4.3 0.28];
den=[1 11.2 19.6 16.2 0.91 0.27];

runtime=15;
wght=[0,1,5,0.66];

samper=30;

srate=(samper+1.3)/1000;

grphc1

integ=0;deriv=0;yprev=0;
Kp=10.2*1.7; Ti=0.98; Td=0.4;
for i=1:count
        set(hh,'Xdata',hx,'Ydata',hy+y*hz);
    integ = integ+0.5*(y+yprev)*srate;
    integ=sign(integ)*min(abs(integ),0.10);
    deriv=(y-yprev)/srate;
    pp=-Kp*(y+integ/Ti+deriv*Td);
    yprev=y;
        pp=sign(pp)*min(max(0,abs(pp)-0.0),10);
```

```
        set(jh,'Xdata',jx,'Ydata',jy+pp*jz);
        drawnow;

        ylist(i)=y;
        ulist(i)=pp;

        x=adis*x + bdis*(pp+disturb(i));
        y=cdis*x + ddis*(pp+disturb(i));

        while (time2-time1<samper)
                time2=clock;time2=1000*(60*time2(5)+time2(6));
        end
        thetimes(i)=time2;
        time1=time2;

        if (y<-10 | y>10 )
                flg=1;crashind=i+1;
                thetimes(i+1)=thetimes(i)+samper;
                ylist(i+1)=y;
                ulist(i+1)=sign(p(1,2))*min(abs(p(1,2)),10);
                break;
        end

end

grphc2
```