STAT 433

Examples for 01/27/2022

Spring 2022
A. Stepanov

Example 1: Consider p and q such that $p > 0$, $q > 0$, $p + q < 1$.

Consider a Markov Chain with the states being all integer numbers (i.e., …, – 2, – 1, 0, 1, 2, …), having transition function $P(k, k+1) = p > 0$, $P(k, 0) = q > 0$, and $P(k, k) = 1 - p - q > 0$. For each state, determine whether it is recurrent or transient.

Let C be an irreducible closed set of recurrent states.

$\rho_C(x) = P_x(T_C < \infty)$, $\qquad x \in S$.

(probability that the Markov chain that starts in x would be absorbed into C)

$\rho_C(x) = 1$, $\qquad x \in C$.

$\rho_C(x) = 0$, $\qquad x$ is recurrent, $\qquad x \notin C$,

$$\rho_C(x) = \sum_{y \in C} P(x, y) + \sum_{y \in S_T} P(x, y) \cdot \rho_C(y), \qquad x \in S_T.$$

Example 2: Consider a Markov chain with the following transition probability matrix:

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 2 & 0 & \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \\ 3 & 0 & 0 & 0 & 1 \end{array}$$

a) Determine which states are recurrent and which are transient. Identify all irreducible closed sets of recurrent states.

b) Find ρ_{xy},

$x = 0, 1, 2, 3,$

$y = 0, 1, 2, 3.$

Example 3: Consider a Markov chain with the following transition probability matrix:

$$
\begin{array}{c c}
 & \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} &
\begin{bmatrix}
\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 \\
\frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} \\
0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

a) Determine which states are recurrent and which are transient. Identify all irreducible closed sets of recurrent states.

b) Find $\rho_C(x)$ for each transient state x and each irreducible closed set C of recurrent states.

$$\rho_C(x) = \sum_{y \in C} P(x,y) + \sum_{y \in S_T} P(x,y) \cdot \rho_C(y), \qquad x \in S_T.$$

c) Find ρ_{xy} for each transient state x and $y = 0, 1, 2, 3, 4, 5, 6.$

$$\rho_{xy} = P(x,y) + \sum_{\substack{z \in S_T \\ z \neq y}} P(x,z) \cdot \rho_{zy}, \qquad x, y \in S_T.$$