STAT 433

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Examples for 03/10/2022

1. On a highway, cars pass according to a Poisson process with rate 5 per minute. Trucks pass according to a Poisson process with rate 3 per minute. The two processes are independent. Let $N_C(t)$ and $N_T(t)$ denote the number of cars and trucks that pass in t minutes, respectively. Then $N(t) = N_C(t) + N_T(t)$ is the number of vehicles that pass in t minutes.

a) Find $P(N_C(3) = 20)$.

b) Find $P(N(3) = 20)$.

c) Find $P(N(3) = 20 \mid N(1) = 8)$.

d) Find $P(N(1) = 8 \mid N(3) = 20)$.

e) Find $P(N_T(3) = 7 \mid N(3) = 20)$.

f) Find $E(N(4) \mid N_T(3) = 7)$.

Compound Poisson Process:

2. Let $X(t)$ be a Poisson process with rate λ.

Let $S(t) = \sum_{i=1}^{X(t)} Y_i$, where $Y_1, Y_2, \ldots$ are independent, identically distributed random variables (independent of $X(t)$) with mean μ and variance σ^2.

a) Find the mean and the variance of $S(t)$.

Hint 1: $$E[(S(t))^k] = \sum_{x=1}^{\infty} P(X(t)=x) \cdot E\left[\left(\sum_{i=1}^{x} Y_i\right)^k\right], \quad k = 1, 2.$$

Hint 2: $$\sigma^2 = Var(Y) = E(Y^2) - [E(Y)]^2 = E(Y^2) - \mu^2.$$

b) A person makes shopping trips according to a Poisson process with rate λ. The number of purchases he makes during each shopping trip is distributed according to a Geometric distribution with probability of "success" p. What are the mean and variance of the number of purchases made by time t?

c) Suppose that cars arrive to a fair according to a Poisson process with rate λ. The number of passengers (in addition to the driver) in a car has a Binomial$(n = 3, p)$ distribution. What are the mean and the variance of the number of people who have arrived by time t?

Hint: $Y = 1 + \text{Binomial}(n = 3, p) = \text{driver} + \text{passengers}$.