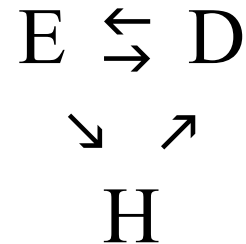


Stationary distribution:  $\sum_x \pi(x) q_{xy} = 0, \quad y \in S.$

1. Suppose the time spent in Anytown Emergency Room is exponentially distributed with mean 4 hours (rate  $q_E = 6$ ). Suppose also that  $1/3$  of the ER patients are admitted to the Anytown Hospital, and  $2/3$  are discharged. The time spent in the Hospital is exponentially distributed with mean 2 days (rate  $q_H = 0.5$ ). People in Anytown are extremely accident-prone; the time until a person goes to the Emergency Room is exponentially distributed with mean 20 days (rate  $q_D = 0.05$ ). Consider a Markov pure jump process  $X(t)$  with three states  $\{E(\text{mergency Room}), H(\text{ospital}), D(\text{ischarged})\}$ . Find the long-term distribution of  $X(t)$ .



2. Sue's sewing machine is very old, and it malfunctions often. When a machine fails, it needs either a small repair (which happens with probability 0.75) or a large repair (probability 0.25). If the machine needs a small repair, the time of the repair is exponentially distributed with mean 3 minutes (rate = 20). If the machine needs a large repair, the time of the repair is exponential with mean 6 minutes (rate = 10). After a repair, the machine works for an exponentially distributed time with mean 15 minutes (rate = 4). Assume that all times are independent. Consider a Markov pure jump process  $X(t)$  with three states  $\{W(\text{orking}), S(\text{mall repair}), L(\text{arge repair})\}$ . Find the long-term distribution of  $X(t)$ .

$$q_W = 4$$

$$q_S = 20$$

$$q_L = 10$$

$$Q_{WS} = 0.75, \quad Q_{WL} = 0.25$$

$$Q_{SW} = 1$$

$$Q_{LW} = 1$$

$$q_{WW} = -4$$

$$q_{WS} = 3$$

$$q_{WL} = 1$$

$$q_{SW} = 20$$

$$q_{SS} = -20$$

$$q_{SL} = 0$$

$$q_{LW} = 10$$

$$q_{LS} = 0$$

$$q_{LL} = -10$$

3. The Department of Statistics has two photocopy machines. The time to breakdown for each machine has an exponential distribution with parameter  $\lambda$ . The time to repair for each machine has an exponential distribution with parameter  $\mu$ . (The two machines could be undergoing repairs at the same time.) Assume that all times to breakdown and all times to repair are independent. For each machine, let 1 denote the *working* condition, and 0 denote the *broken* condition. Then the status of both machines can be described using 4 states, i.e.,

$$0 = (0, 0) \quad 1 = (1, 0) \quad 2 = (0, 1) \quad 3 = (1, 1).$$

Let  $X(t)$  denote the conditions of both machines at time  $t$ .

- a) Identify all infinitesimal parameters of  $X(t)$ .
- b) Find the stationary distribution for  $X(t)$ .

4. The Department of Statistics has three printers. Each printer breaks down independently at rate  $\mu$ , then it is sent to the repair shop. The repair shop can only repair one printer at a time and each printer takes an exponential amount of time with parameter  $\lambda$  to repair. Let  $X(t)$  denote the number of working printers.
- a) Identify all infinitesimal parameters of  $X(t)$ .
- b) Find the stationary distribution for  $X(t)$ .

Birth and death process:

$$\pi_0 = 1, \quad \pi_x = \frac{\lambda_0 \dots \lambda_{x-1}}{\mu_1 \dots \mu_x}, \quad x \geq 1.$$

$$\pi(x) = \frac{\pi_x}{\sum_y \pi_y}, \quad x \geq 0.$$

Birth and death process:

$$\gamma_0 = 1, \quad \gamma_x = \frac{\mu_1 \cdots \mu_x}{\lambda_1 \cdots \lambda_x}, \quad x \geq 1.$$

$$\sum_{x=0}^{\infty} \gamma_x = \infty, \quad \sum_{x=0}^{\infty} \pi_x < \infty, \quad \Rightarrow \quad \text{Positive recurrent.}$$

$$\sum_{x=0}^{\infty} \gamma_x = \infty, \quad \sum_{x=0}^{\infty} \pi_x = \infty, \quad \Rightarrow \quad \text{Null recurrent.}$$

$$\sum_{x=0}^{\infty} \gamma_x < \infty, \quad \sum_{x=0}^{\infty} \pi_x = \infty, \quad \Rightarrow \quad \text{Transient.}$$

**5.** HPS 3.16

**16** Consider a birth and death process on the nonnegative integers whose death rates are given by  $\mu_x = x$ ,  $x \geq 0$ . Determine whether the process is transient, null recurrent, or positive recurrent if the birth rates are

(a)  $\lambda_x = x + 1$ ,  $x \geq 0$ ;

(b)  $\lambda_x = x + 2$ ,  $x \geq 0$ .