STAT 433 | **Examples for 03/29/2022 (1)** | Spring 2022, A. Stepanov

Stationary distribution:

Markov pure jump process:

$$\sum_x \pi(x) q_{xy} = 0, \quad y \in S.$$

$$-\pi(y)\, q_y + \sum_{x \neq y} \pi(x)\, q_x\, \mathrm{Q}_{xy} = 0, \quad y \in S.$$

Embedded chain:

$$\sum_x \pi_{\text{emb}}(x)\, \mathrm{Q}_{xy} = \pi_{\text{emb}}(y), \quad y \in S.$$

$$-\pi_{\text{emb}}(y) + \sum_{x \neq y} \pi_{\text{emb}}(x)\, \mathrm{Q}_{xy} = 0, \quad y \in S.$$

$\pi(x)\, q_x$ is proportional to $\pi_{\text{emb}}(x)$; $\qquad \pi(x)$ is proportional to $\pi_{\text{emb}}(x)/q_x$.

1. Consider a Markov pure jump process on $\{1, 2, 3\}$ with $q_x = x^2$, $x = 1, 2, 3$,
$\mathrm{Q}_{13} = 1$, $\mathrm{Q}_{21} = 1/4$, $\mathrm{Q}_{23} = 3/4$, $\mathrm{Q}_{31} = 1/9$, $\mathrm{Q}_{32} = 8/9$.

a) Identify all infinitesimal parameters of $\mathrm{X}(t)$.
Find the stationary distribution π of this process using $\sum_x \pi(x) q_{xy} = 0$, $y \in S$.

b) Find the stationary distribution π_{emb} of the embedded chain of this process. That is, find the stationary distribution of the Markov chain with the transition probability matrix Q.

$$Q = \begin{bmatrix} 0 & 0 & 1 \\ 1/4 & 0 & 3/4 \\ 1/9 & 8/9 & 0 \end{bmatrix}$$

Now find the probability vector $\pi(x)$ proportional to $\pi_{\text{emb}}(x)/q_x$.

$\pi_{\text{emb}}\,\mathbf{Q} = \pi_{\text{emb}}$.

2. At Anytown State University, students arrive to never-ending advising office hours for Sociomechanics major according to a Poisson process with rate λ students per hour. Students then talk to the one and only advisor one at a time, and the time of the conversation has an exponential distribution with mean θ hours. If a new student arrives, and the advisor is busy talking to another student, the new student waits in line until the advisor becomes available. Find the long-term distribution of $X(t)$, the number of students at the advising office hours, if it exists and the condition(s) on λ and θ when it does exist.

3. At Anytown State University, students arrive to never-ending advising office hours for for Philosophical Engineering major according to a Poisson process with rate λ students per hour. There are three advisors answering students' questions. A student talks to an advisor one-on-one, and the time of the conversation has an exponential distribution with mean θ hours. If all three advisors are busy when a student arrives, the student would wait until an advisor becomes available. Find the long-term distribution of $X(t)$, the number of students at the advising office hours, if it exists and the condition(s) on λ and θ when it does exist.