CS5335 HW1

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HW Q1:

Based on the description, B' is obtained by rotate $\frac{\pi}{2}$ about x axis from B. Thus:

$$^{B}R_{B'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

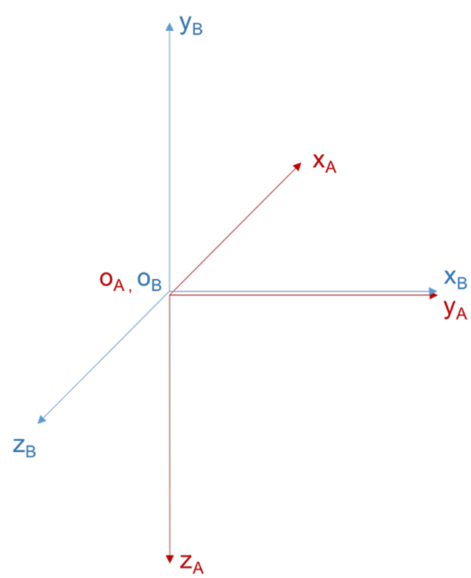
Then, A is obtained by rotate $\frac{\pi}{2}$ about original y axis, which is current negative z axis from B'. Thus:

$$^{B'}R_{A} = \begin{bmatrix} \cos-\frac{\pi}{2} & -\sin-\frac{\pi}{2} & 0 \\ \sin-\frac{\pi}{2} & \cos-\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, rotation can be represented as:

$$^{B}R_{A} = {}^{B}R_{B'}{}^{B'}R_{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

The original frame B and final frame A can be plotted as following:



HW Q2:

As there is difference between the description and the figure regarding the frame 2 location (In description, o_2 is at the center of the cube, while it is at the surface of the table in the figure.), all of the following calculations are based on the assumption that o_2 is located at center of the cube, which is 10 celimeter above the table surface.

For Frame1($o_1x_1y_1z_1$), it can be obtained from Frame0($o_0x_0y_0z_0$) by translating 1 meter along both positive y_0 and positive z_0 directions. Thus:

$$^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Frame2($o_2x_2y_2z_2$), it can be obtained from Frame1($o_1x_1y_1z_1$) by translating 50 celimeter along both negative x_1 and positive y_1 directions, and 10 celimeter along positive z_1 direction. Thus:

$$^1T_2 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_2 = {}^0T_1\, {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Frame3($o_3x_3y_3z_3$), it can be obtained from Frame2($o_2x_2y_2z_2$) by translating 1.9 meter along the positive z_2 direction, followed by rotating $\frac{\pi}{2}$ about the z_2 axis, and then followed by rotating π about current x axis, which is along the original positive y_2 direction. Thus:

$$^2R_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} ^2R_3 & ^2d_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = {}^0T_2\, {}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$