Linear Control
Laboratory report 1

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Question 1 Explain the chosen values for $q_{\bar{P}3}$ and $\bar{h_3}$. Explain the calculation of the surface S_{S30}.

We define $q_{\bar{P}3}$ to be the (constant) flow rate chosen for the open loop experiment (i.e. 30mL/s) and $\bar{h_3}$ to be the value of h_3 when the water has stabilized in the third tank. This corresponds to an equilibrium for h_3.

In a state of equilibrium, h_3 is constant and thus:

$$\frac{\mathrm{d}\bar{h_3}}{\mathrm{d}t} = 0.$$

From that last equation and the model of the system, we can derive the value of S_{S30}:

$$\begin{aligned}\frac{\mathrm{d}\bar{h_3}}{\mathrm{d}t} = 0 = \frac{1}{S_R}q_{\bar{P}3} - \frac{1}{S_R}q_{\bar{S}30} &\Leftrightarrow \frac{1}{S_R}q_{\bar{P}3} = \frac{1}{S_R}q_{\bar{S}30} \\ &\Leftrightarrow q_{\bar{P}3} = q_{\bar{S}30} \\ &\Leftrightarrow q_{\bar{P}3} = S_{S30}\sqrt{2g\bar{h_3}} \\ &\Leftrightarrow S_{S30} = \frac{q_{\bar{P}3}}{\sqrt{2g\bar{h_3}}}\end{aligned}$$

This gives us the following experimental values:

$q_{\bar{P}3}$	$\bar{h_3}$	S_{S30}

Question 2 Detail the linearized model of the system and the transfer functions of $G(s)$ and $H(s)$.

Let us first identify the input, disturbance, inner state and output of the system:

	Input	**Disturbance**	**Inner state**	**Output**
Variable	q_{P3}	S_{F30}	h_3	h_3
Alias	u	v	x	y

For clarity, we shall use the "Alias" notation for these variables. The (nonlinear) system for this experiment is thus:

$$\begin{aligned}\dot{x} &= \frac{S_{S30}}{S_R}\sqrt{2gx} + \frac{1}{S_R}u - \frac{1}{S_R}v\sqrt{2gx} \\ y &= x.\end{aligned}$$

We can obtain a linear system by doing an approximation using the first order Taylor series centered around the equilibrium on the right-hand side of the first equation. Let us begin by computing the gradiant of $\dot{x}$ with respect to (x, u, v):

$$\nabla_{(x,u,v)}\dot{x} = \left(\frac{S_{S30} - v}{S_R}\sqrt{\frac{g}{2x}}, \frac{1}{S_R}, -\frac{\sqrt{2gx}}{S_R} \right).$$

The linearization of the system is then given by the Taylor series:

$$\begin{aligned}
\dot{x} &\approx \nabla_{(x,u,v)}\dot{x}\big|_{(x,u,v)=(\bar{x},\bar{u},\bar{v})} \bullet (x - \bar{x}, u - \bar{u}, v - \bar{v}) \\
&= \frac{S_{S30} - \bar{v}}{S_R}\sqrt{\frac{g}{2\bar{x}}}(x - \bar{x}) + \frac{1}{S_R}(u - \bar{u}) - \frac{\sqrt{2g\bar{x}}}{S_R}(v - \bar{v}) \\
&= \frac{S_{S30} - \bar{v}}{S_R}\sqrt{\frac{g}{2\bar{x}}}x + \frac{1}{S_R}u - \frac{\sqrt{2g\bar{x}}}{S_R}v + \frac{1}{S_R}\left(S_{S30}\sqrt{\frac{g\bar{x}}{2}} + \bar{u} - \sqrt{\frac{5g\bar{x}}{2}}\bar{v} \right).
\end{aligned}$$