

Symbolic Computation via Program Transformation

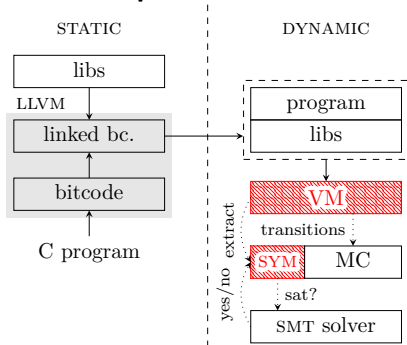
Henrich Lauko, Petr Ročkal and Jiří Barnat



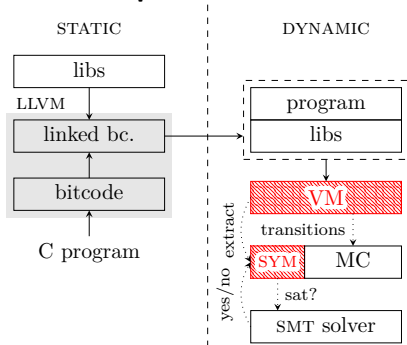
Masaryk University
Brno, Czech Republic

18th October 2018

Interpretation-based

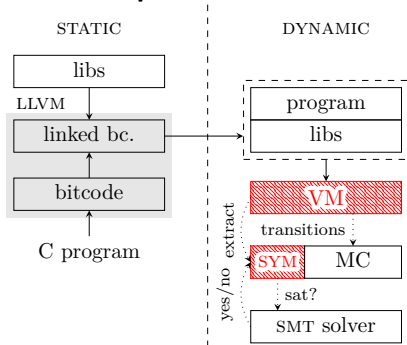


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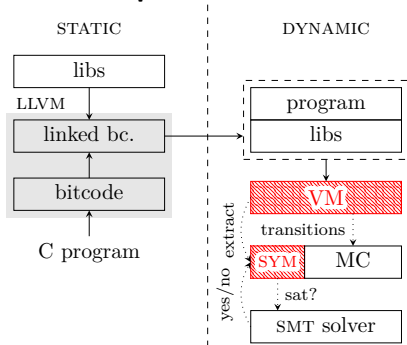
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1 a ← input()
2 if (a > 0)
3   b ← a + 1
4 else
5   b ← a - 1
```

Interpretation-based



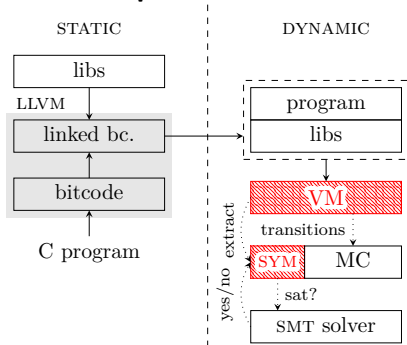
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Interpretation-based



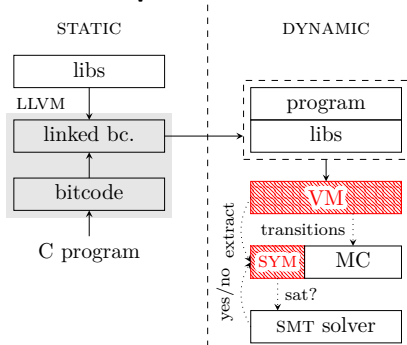
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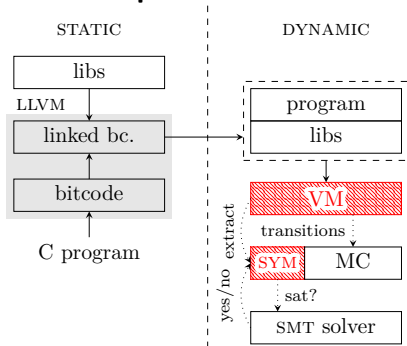
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4   [a > 0 ∧ b = a + 1]
5 else
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Interpretation-based



```
1 a ← input()    [true]
2 if (a > 0)      [a > 0]
3   b ← a + 1
4   [a > 0 ∧ b = a + 1]
5 else
6   b ← a - 1
7   [a ≤ 0 ∧ b = a - 1]
```

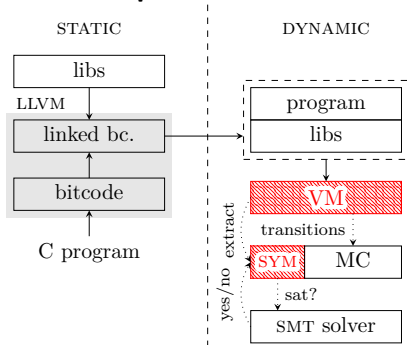
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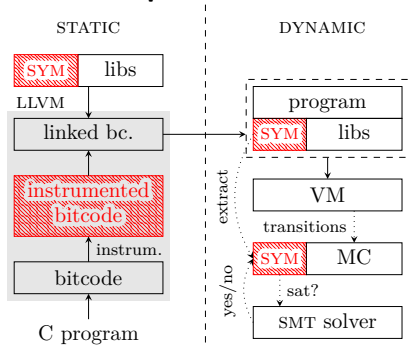
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1 a ← input()    [true]
2 if (a > 0)      [a > 0]
3   b ← a + 1
4 else
5   b ← a - 1
   [a ≤ 0 ∧ b = a - 1]
```

- multiple possible paths
- maintained in interpreter
- program does not know about symbolic values

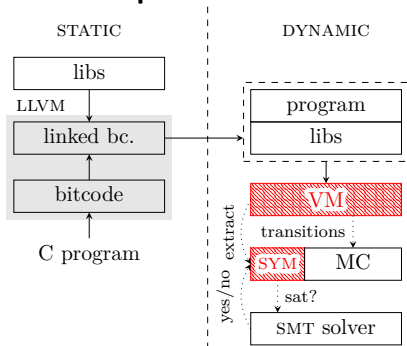
Interpretation-based



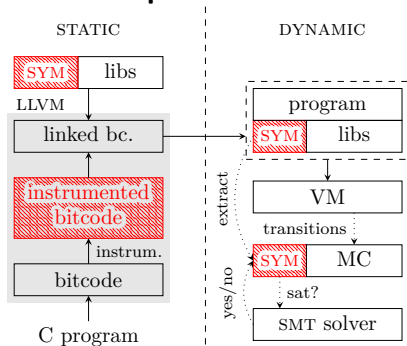
Compilation-based



Interpretation-based



Compilation-based



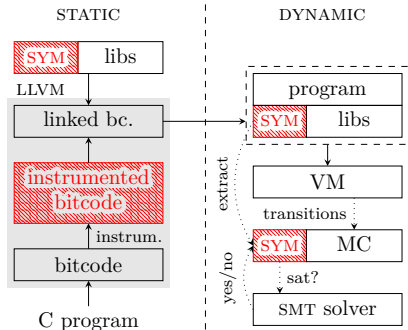
Motivation: minimize complexity of the verification tool

```
1 a ← input()
2 if (a > 0)
3   b ← a + 1
4 else
5   b ← a - 1
```



```
1 a ← sym_input()
2 if (sym_gt(a > 0))
3   b ← sym_add(a, 1)
4 else
5   b ← sym_sub(a, 1)
```

Compilation-based



Motivation: minimize complexity of the verification tool



- 1 mixing of explicit and symbolic computation



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- 3 impose minimal run-time overhead



1 syntactically abstract the input program

<code>x:int</code>	<code>← input()</code>	<code>x: a_int</code>	<code>← lift(*)</code>
<code>y:int</code>	<code>← factorial(7)</code>	<code>y: int</code>	<code>← factorial(7)</code>
<code>z:int</code>	<code>← x + y</code>	<code>z: a_int</code>	<code>← a_add(x, lift(y))</code>
<code>b:bool</code>	<code>← y < z</code>	<code>b: a_bool</code>	<code>← a_lt(x, z)</code>

1 syntactically abstract the input program

<code>x:int ← input()</code>	<code>x: a_int ← lift(*)</code>
<code>y:int ← factorial(7)</code>	<code>y: int ← factorial(7)</code>
<code>z:int ← x + y</code>	<code>z: a_int ← a_add(x, lift(y))</code>
<code>b:bool ← y < z</code>	<code>b: a_bool ← a_lt(x, z)</code>

- transform instructions, types, functions
- preserve concrete computation
- lift concrete values

1 syntactically abstract the input program

<code>x:int ← input()</code>	<code>x: a_int ← lift(*)</code>
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<code>b:bool ← y < z</code>	<code>b: a_bool ← a_lt(x, z)</code>

- transform instructions, types, functions
- preserve concrete computation
- lift concrete values

2 concretely realize abstraction

```
x: sym_int ← lift(*)  
y: int      ← factorial(7)  
z: sym_int  ← sym_add(x, lift(y))  
b: sym_bool ← sym_lt(x, z)
```

- replace abstract calls with provided implementation



Problem: constrained values by control flow

```
x: int ← input()
cond: bool ← x < 0
if (cond)
  y: int ← x + 1
else
  ...
```

- both paths can happen
- x is not constrained

Problem: constrained values by control flow

```
x: int ← input()
cond: bool ← x < 0
if (cond)
  y: int ← x + 1
else
  ...
```

- both paths can happen
- x is not constrained

Solution: instrument constraint propagation

```
x: sym_int ← lift(*)
cond: sym_bool ← sym_lt(x, 0)
if (*) // nondeterministic
  x': sym_int ← assume(cond)
  y : sym_int ← sym_add(x', 1)
else
  x': sym_int ← assume(!cond)
  ...
```

- assumes extend a path condition



Problem: how to deal with aggregate types?

```
arr: int[] ← [1, 2, 3]  
arr[1]: int ← input()
```

- we want to minimize the number of symbolic values



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```
arr: int[] ← [1, 2, 3]
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- we want to minimize the number of symbolic values

Solution: use discriminated union type

- realize abstract value as union of *concrete* and *symbolic* value

```
arr: union[] ← [1, 2, 3] // either int or sym_int
arr[1]: union ← lift(*)
```

- similarly deal with recursive structures

TODO image of metadata



Problem: how to transform functions with symbolic arguments?

```
int foo(a: int, b: int, c: int)
```

- may produce exponentially many duplicates:

```
int foo(a: sym_int, b: int, c: int)
```

```
int foo(a: int, b: sym_int, c: int)
```

```
int foo(a: int, b: int, c: sym_int)
```

```
int foo(a: sym_int, b: sym_int, c: int)
```

```
...
```

- resolve return type

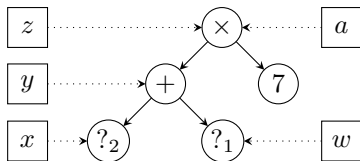
Solution: static analysis + use discriminated union

```
union foo(a: union, b: union, c: int)
```

Symbolic Runtime

Symbolic execution:

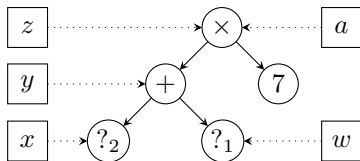
```
a: pointer ← malloc()  
w: sym_int ← lift(*)  
x: sym_int ← lift(*)  
y: sym_int ← sym_add(w, x)  
z: sym_int ← sym_mul(y, 7)  
store z → a
```



Symbolic execution:

```

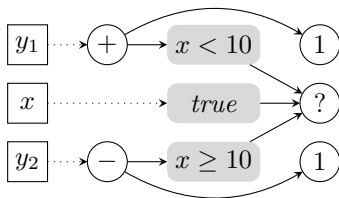
a: pointer ← malloc()
w: sym_int ← lift(*)
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store z → a
    
```



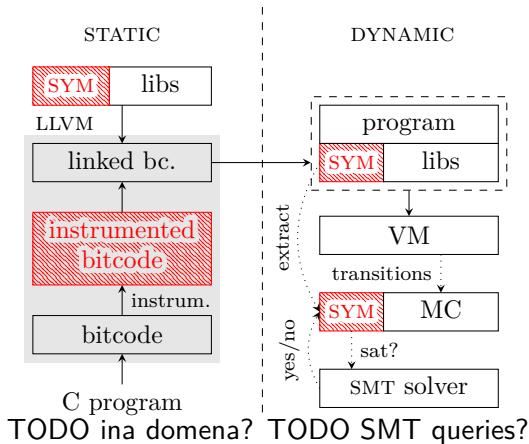
Branching example:

```

x : sym_int ← lift(*)
if (*) // nondeterministic
  x': sym_int ← assume(x < 10)
  y : sym_int ← sym_add(x, 1)
else
  x': sym_int ← assume(x < 10)
  y : sym_int ← sym_sub(x, 1)
    
```



TODO cycle example?



Required support in a tool:

- nondeterminism
- feasibility check
- equality check
- values metadata

Simpler domains do not even need *SMT* support (sign domain).



Integrated with DIVINE model checker:

- LLVM-to-LLVM transformation
- STP SMT solver.

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Component sizes: (lines of code)

	DIVINE*	KLEE	SymDIVINE	CBMC
symbolic support	5.4	24.2	7	39.8
shared code	136.5	125	423	27.5

- reduced complexity of verification tool

SV-COMP Benchmarks:

tag	total	DIVINE*	SymDIVINE	CBMC
array	190	96	68	93
bitvector	32	17	9	2
loops	178	72	67	9
product-lines	575	336	411	234
pthread	45	9	0	1
recursion	81	47	43	22
systemc	59	14	27	0
total	1160	591	625	361

TODO grafy



Goals

- 1 mixing of explicit and symbolic computation ✓
- 2 expose a small interface to the rest of the system ✓
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Summary

- introduced compilation-based symbolic verification
- generalized approach to the abstraction of programs