Fairness Modulo Theory: A New Approach to LTL Software Model Checking

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Presented by Henrich Lauko

December 5, 2016

Talk outline

1. Ultimate Automizer algorithm

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2. Reachability solving

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3. Interpolants and infeasibility proofs

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3. Interpolants and infeasibility proofs
4. LTL model checking

Ultimate Automizer

Programs as languages

A program defines a language over program statements.

- alphabet = the set of program statements
- finite automaton = control flow graph
- accepting states = error locations

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```
int z:=0;
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while(x<20){
     if(i!=0){
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     x++;
}
assert(z<=5)
```
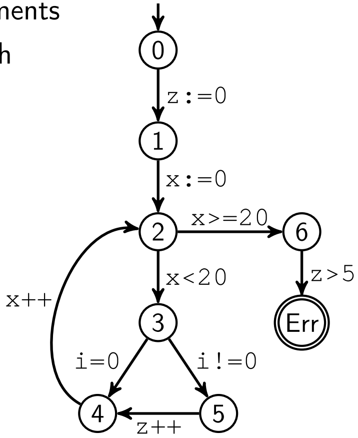
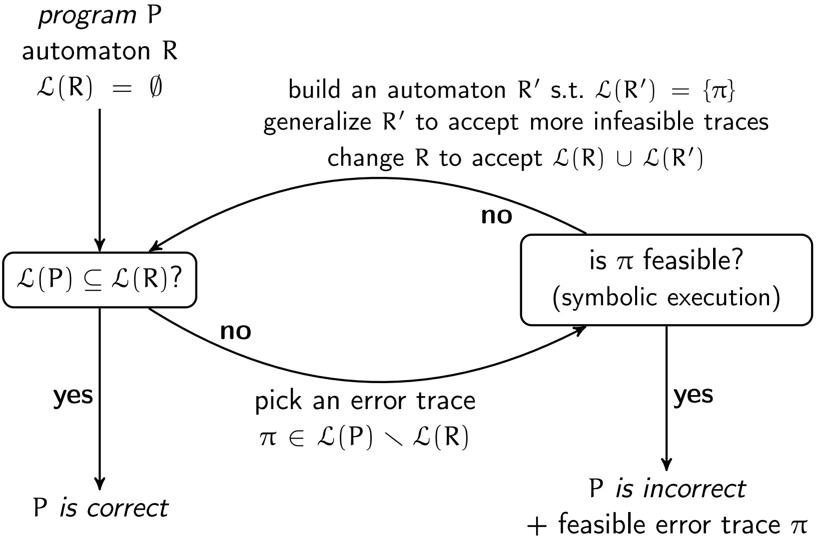



Automizer algorithm



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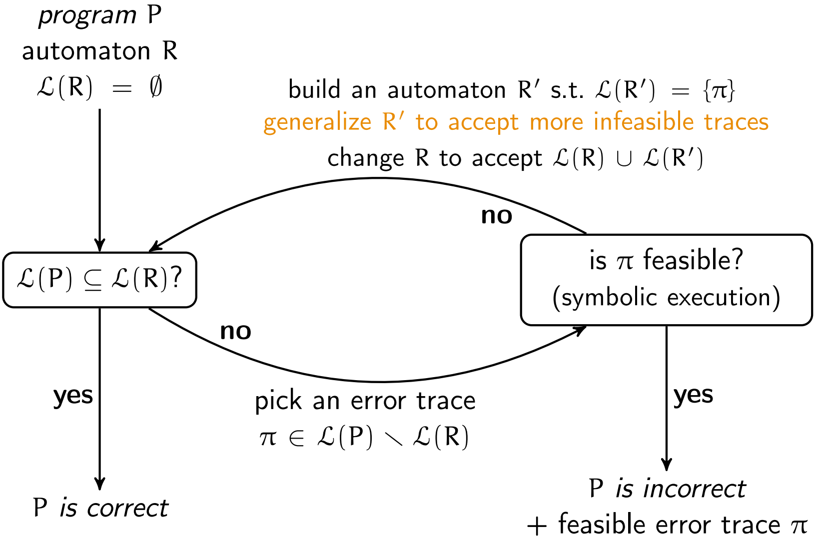

Generalizition of infeasible paths

Craig's interpolants

Let φ, ψ be two first-order formulae such that $\varphi \implies \psi$. Then there exists a first order-formula θ called *interpolant* such that

- all non-logical symbols in θ occur in both φ and ψ,
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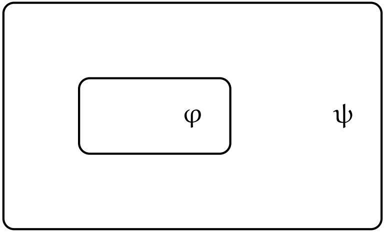

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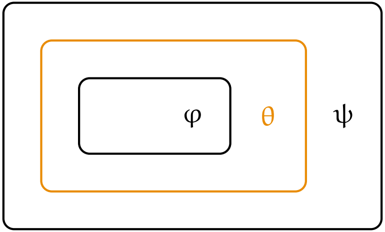

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$\boxed{\varphi}$ $\qquad$ $\boxed{\neg\psi}$

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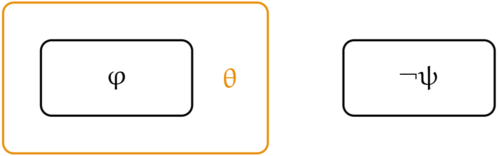

Example: Error path

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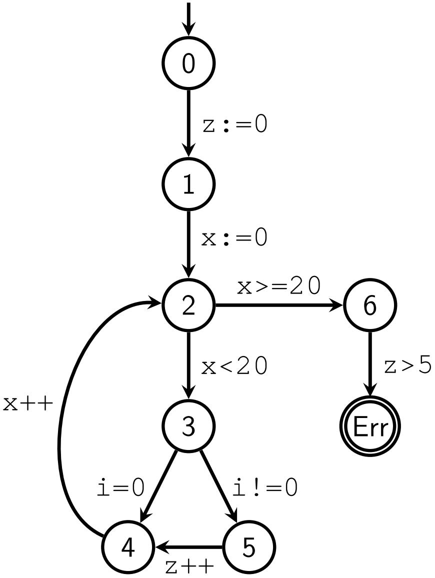

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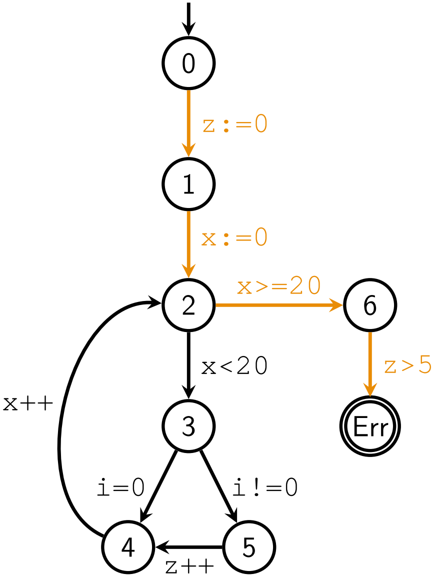

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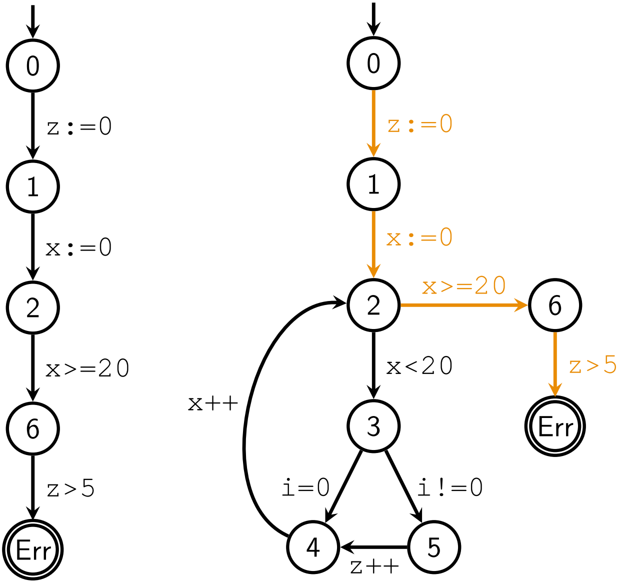
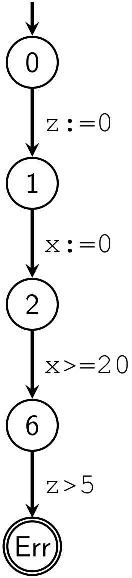


Error feasibility analysis

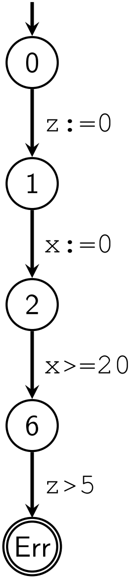


Symbolic execution produces:

$$z = 0 \wedge x = 0 \wedge x \geqslant 20 \wedge z > 5 \equiv \text{false}$$

So the error trace is *infeasible*.

Generalization of infeasible trace



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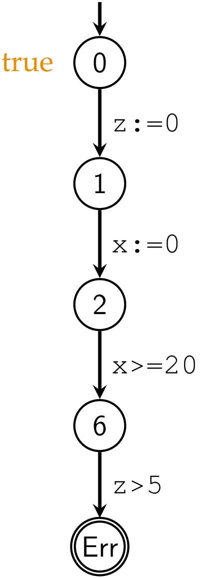

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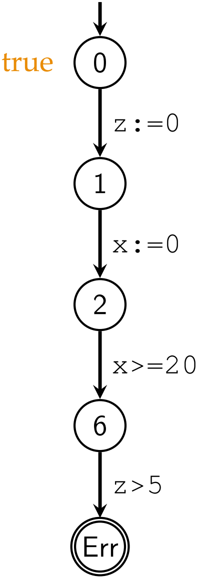

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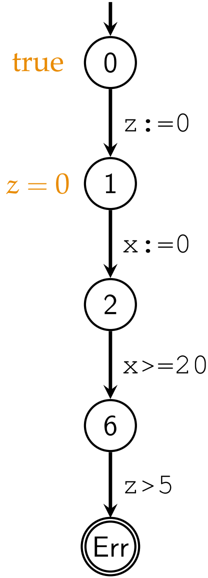

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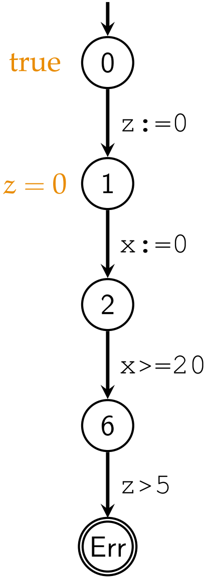

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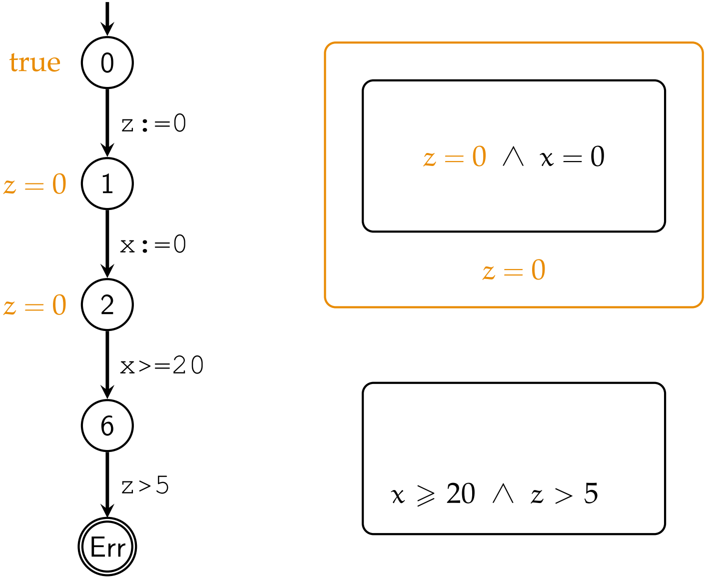

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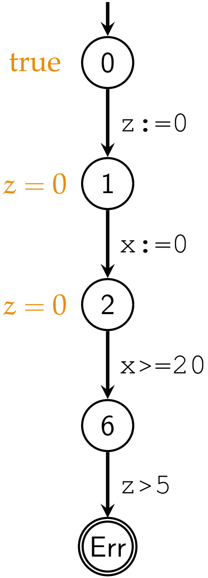

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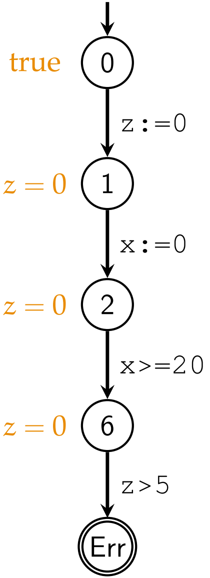

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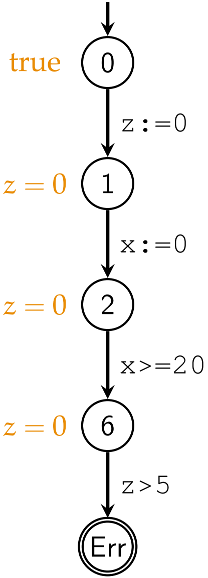

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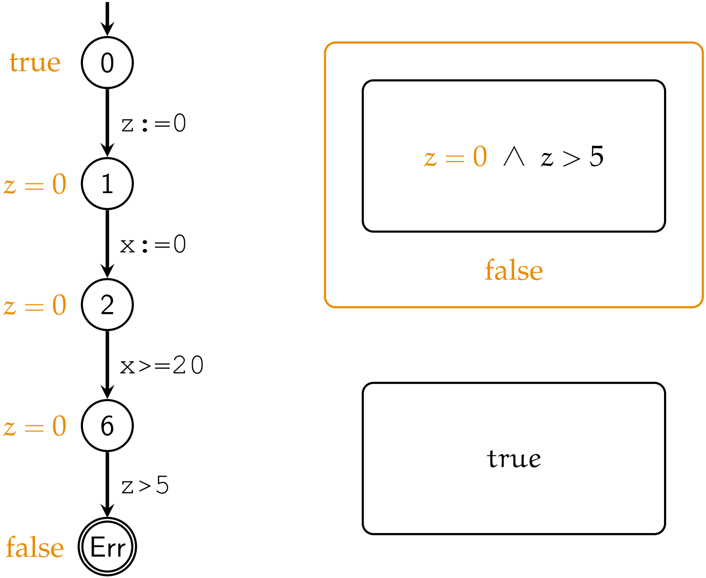

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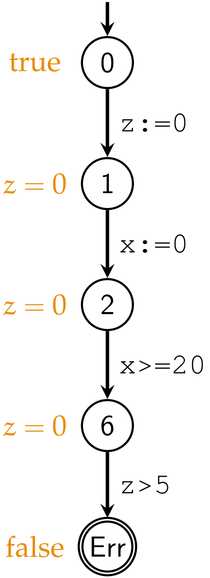

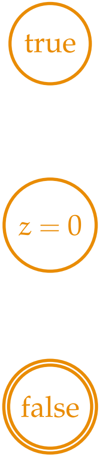

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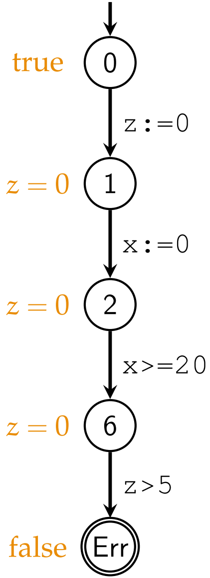

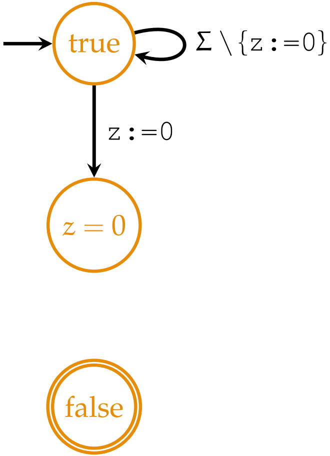

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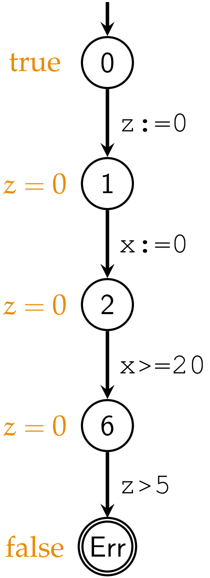

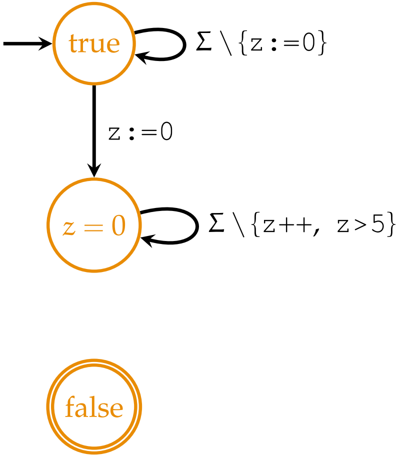

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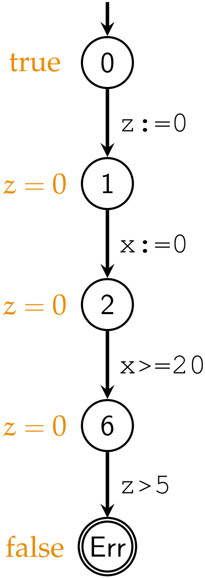

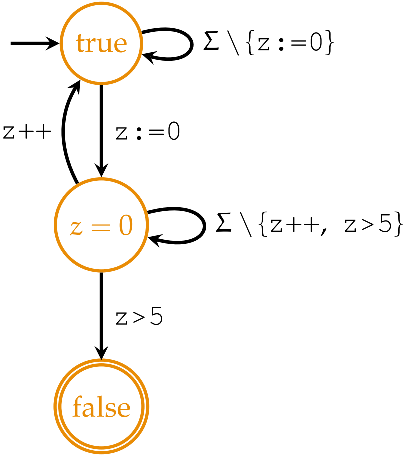

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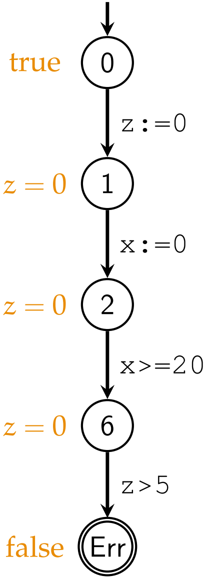

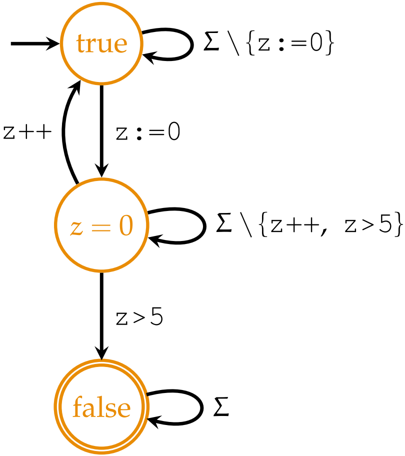

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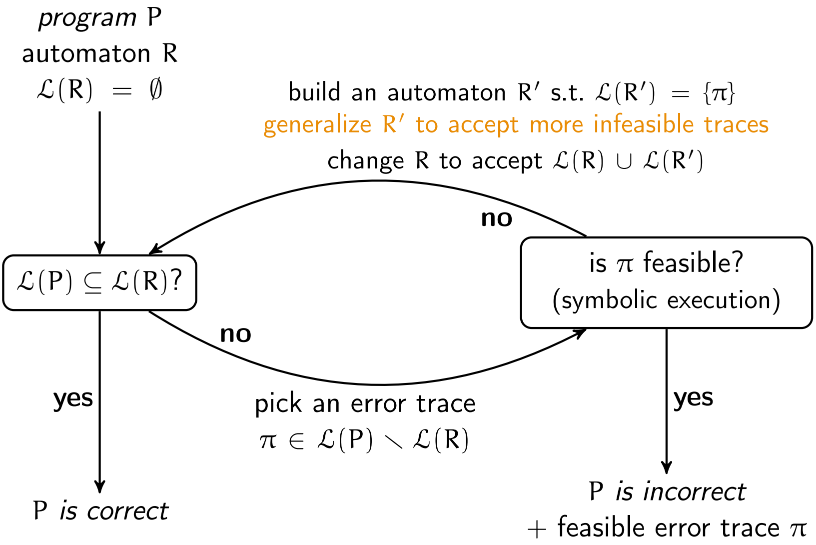
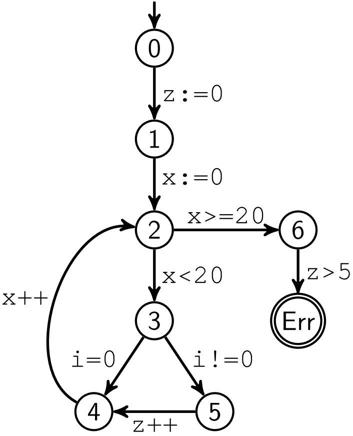
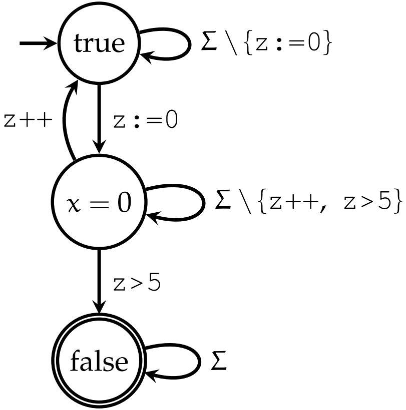


$\mathcal{L}(P) \subseteq \mathcal{L}(R)$?





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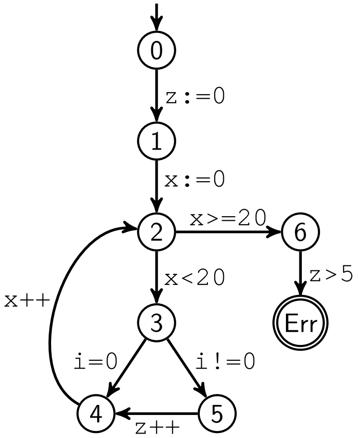

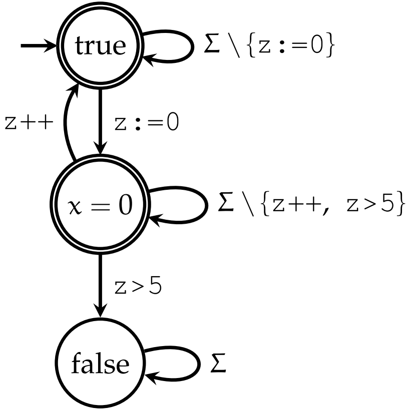

$\mathcal{L}(P) \subseteq \mathcal{L}(R)$ iff the language of the synchronous product of P and complemented R is empty.

LTL model checking

LTL model checking problem

Does the program $\mathcal{P}$ *satisfy* the property φ?

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Fair path

Path is fair if it visits set of accepting states infinitely often.

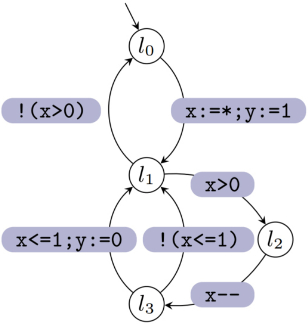
Example: Büchi program

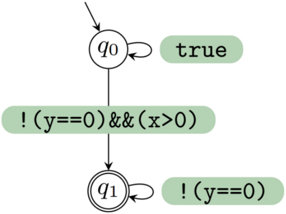
Program:

```
int x, y;
while( true ){
    x := *;
    y := 1;
    while( x > 0 ){
        x--;
        if( x <= 1 )
            y := 0;
    }
}
```

LTL property:

$$\varphi \equiv G(x > 0 \implies F(y = 0))$$





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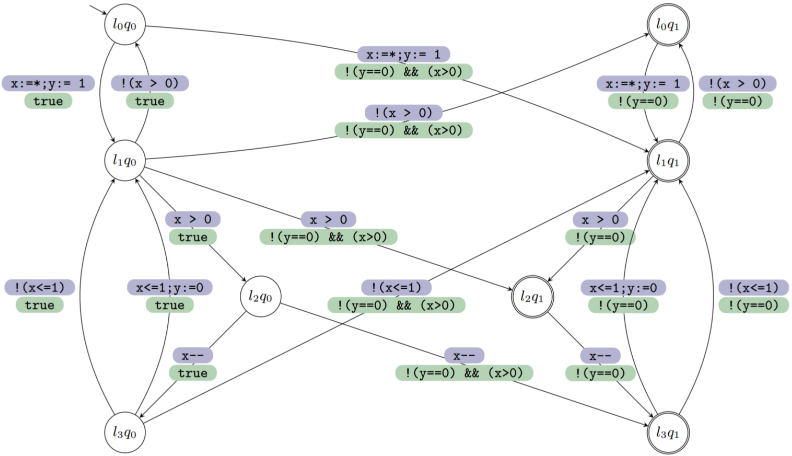

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   Termination proof: $r(x, y) = x$

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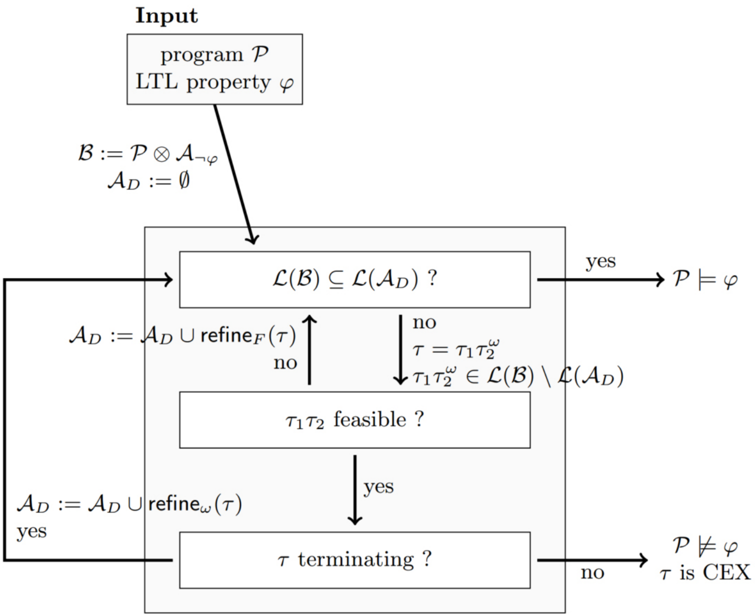
Ultimate LTL Automizer



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- own source-to-source transformations
- own trace abstraction, ranking function synthesis, automata manipulations

Some benchmarks

Program	Lines	φ	Term. [21] Time (s)	Term. [21] Result	DP [23] Time (s)	DP [23] Result	ULTIMATE LTLAUTOMIZER Time (s)	ULTIMATE LTLAUTOMIZER Result	$\lvert\mathbf{r}_F\rvert$	$\lvert\mathbf{r}_\omega\rvert$	Inc. (%)
Ex. Sec. 2 of [23]	5	$\Diamond\Box p$	2.32	✔	1.98	✔	0.51	✔	1	0	122
Ex. Fig. 8 of [21]	34	$\Box(p \Rightarrow \Diamond q)$	209.64	✔	27.94	✔	0.72	✔	2	0	186
Toy acquire/release	14	$\Box(p \Rightarrow \Diamond q)$	103.48	✔	14.18	✔	0.44	✔	1	1	129
Toy linear arith. 1	13	$p \Rightarrow \Diamond q$	126.86	(✔)	34.51	(✔)	1.10	✗	5	1	0.28
Toy linear arith. 2	13	$p \Rightarrow \Diamond q$	**T.O.**	**T.O.**	6.74	✔	0.82	✔	4	2	0.24
PostgreSQL strmsrv	259	$\Box(p \Rightarrow \Diamond\Box q)$	**T.O.**	**T.O.**	9.56	✔	1.04	✔	2	0	216
PostgreSQL strmsrv + bug	259	$\Box(p \Rightarrow \Diamond\Box q)$	87.31	(✗)	47.16	(✗)	0.66	✔	2	0	216
PostgreSQL pgarch	61	$\Diamond\Box p$	31.50	(✔)	15.20	(✔)	0.33	✗	2	0	209
PostgreSQL dropbuf	152	$\Box p$	**T.O.**	**T.O.**	1.14	(✔)	3.57	✗	1	1	148
PostgreSQL dropbuf	152	$\Box(p \Rightarrow \Diamond q)$	53.99	✔	27.54	✔	1.37	✔	2	1	168
Apache `accept()`	314	$\Box p \Rightarrow \Box\Diamond q$	**T.O.**	**T.O.**	197.41	✔	502.15	**OOM**	-	-	209
Apache progress	314	$\Box(p \Rightarrow (\Diamond q_1 \vee \Diamond q_2))$	685.34	✔	684.24	✔	2.01	✔	4	0	209
Windows OS 1	180	$\Box(p \Rightarrow \Diamond q)$	901.81	✔	539.00	✔	43.59	✔	1	1	178
Windows OS 2	158	$\Diamond\Box p$	16.47	✔	52.10	✔	0.11	✔	1	0	176
Windows OS 2 + bug	158	$\Diamond\Box p$	26.15	✗	30.37	✗	0.22	✗	1	0	174
Windows OS 3	14	$\Diamond\Box p$	4.21	✔	15.75	✔	0.08	✔	2	0	220
Windows OS 4	327	$\Box(p \Rightarrow \Diamond q)$	**T.O.**	**T.O.**	1,114.18	✔	1.86	✔	1	3	207
Windows OS 4	327	$(\Diamond p) \vee (\Diamond q)$	1,223.96	✔	100.68	✔	-	**N.R.**	-	-	-
Windows OS 5	648	$\Box(p \Rightarrow \Diamond q)$	**T.O.**	**T.O.**	**T.O.**	**T.O.**	20.76	✔	1	16	190
Windows OS 6	13	$\Diamond\Box p$	149.41	✔	59.56	✔	**T.O.**	**T.O.**	6	8	158
Windows OS 6 + bug	13	$\Diamond\Box p$	6.06	✗	22.12	✗	0.05	✗	0	0	61
Windows OS 7	13	$\Box\Diamond p$	**T.O.**	**T.O.**	55.77	✔	0.91	✔	2	11	161
Windows OS 8	181	$\Diamond\Box p$	**T.O.**	**T.O.**	5.24	✔	53.55	✔	4	55	168

Some benchmarks

Program set	Avg. Lines	|Set|	✔	✗	**T.O.**	**OOM**	★ (**N.R.**)	Statistics for ✔ and ✗: Avg. Time (s)	Avg. $\lvert \mathbf{r}_F \rvert$	Avg. $\lvert \mathbf{r}_\omega \rvert$	Inc. (%)
RERS P14	514	50	19	21	2	0	8	107.21	21	< 1	329
RERS P15	1353	50	24	0	11	12	3	103.46	17	< 1	369
RERS P16	1304	50	15	1	16	14	4	297.34	32	< 1	362
RERS P17	2100	50	26	0	9	9	6	56.38	12	< 1	324
RERS P18	3306	50	21	0	17	10	2	262.03	24	< 1	297
RERS P19	8079	50	0	0	28	17	5	-	-	-	-
coolant	65	18	6	10	2	0	0	1.75	2	1	258
Benchmarks from Tab. 1	157	23	15	5	1	1	0 (1)	16.78	2	5	184