Symbolic Computation via Program Transformation

Henrich Lauko, Petr Ročkai and Jiří Barnat

ParaDiSe
Parallel & Distributed
Systems Laboratory

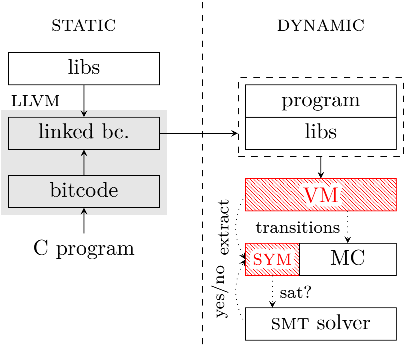
Masaryk University

Brno, Czech Republic

Symbolic Computation Motivation

- verify programs with inputs from the environment
- symbolic execution, concolic testing, test generation etc.

Interpretation-based



Symbolic Computation

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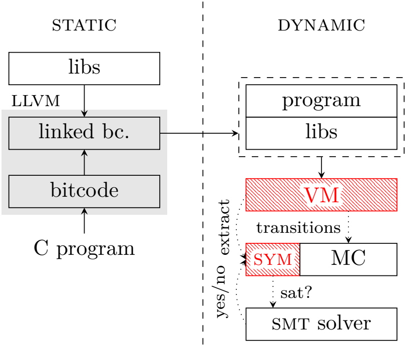


```
1 a ← input()
2 if (a > 0)
3   b ← a + 1
4 else
5   b ← a - 1
```

Symbolic Computation

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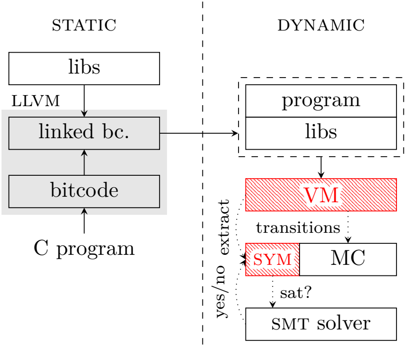


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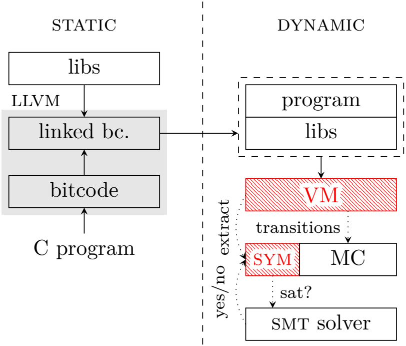


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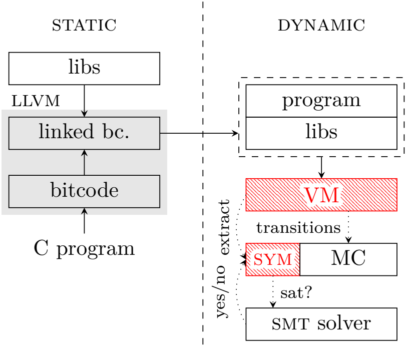


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a ← input()      [true]
if (a > 0)       [a > 0]
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     [a > 0 ∧ b = a + 1]
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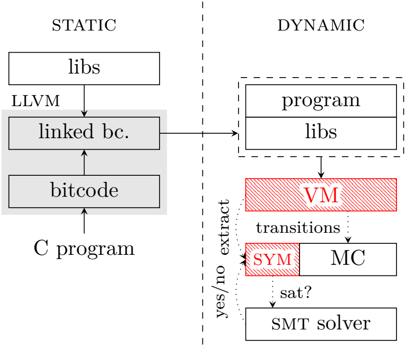


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    [a ≤ 0 ∧ b = a − 1]
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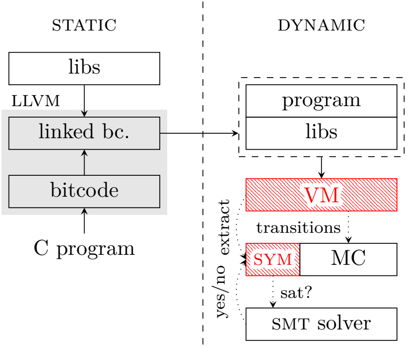


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                     [a > 0 ∧ b = a + 1]
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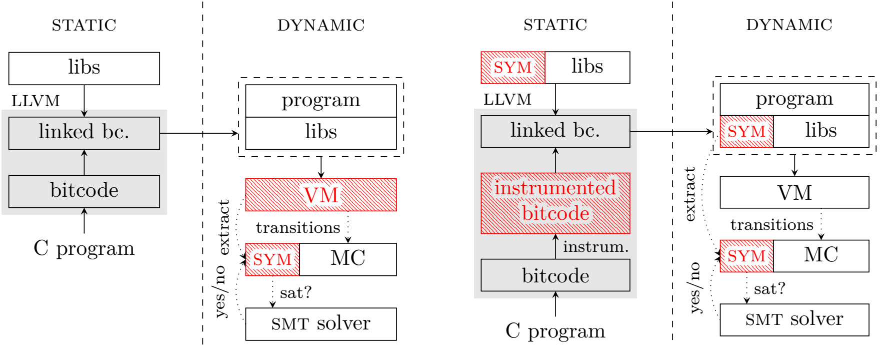
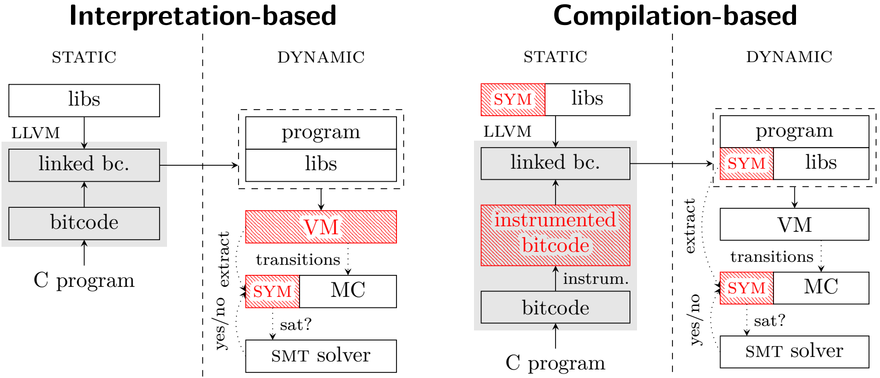
```

- multiple possible paths
- maintained in interpreter
- program does not know about symbolic values

Proposed Symbolic Computation

Interpretation-based **Compilation-based**





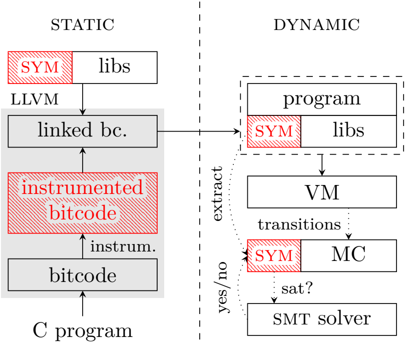
Motivation: minimize complexity of the verification tool

Proposed Symbolic Computation

```
1 a ← input()
2 if (a > 0)
3   b ← a + 1
4 else
5   b ← a - 1
```

```
1 a ← sym_input()
2 if (sym_gt(a, 0))
3   b ← sym_add(a, 1)
4 else
5   b ← sym_sub(a, 1)
```

Compilation-based



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Goals

1. mixing of explicit and symbolic computation

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3. impose minimal run-time overhead

Transformation

Transformation of Program

1. **syntactically abstract the input program**

```
x:int  ← input()
y:int  ← factorial(7)
z:int  ← x + y
b:bool ← z < 0
```

```
x: a_int  ← lift(*)
y: int    ← factorial(7)
z: a_int  ← a_add(x, lift(y))
b: a_bool ← a_lt(z, lift(0))
```

Transformation of Program

1. **syntactically abstract the input program**

```
x:int  ← input()             x: a_int  ← lift(*)
y:int  ← factorial(7)        y: int    ← factorial(7)
z:int  ← x + y               z: a_int  ← a_add(x, lift(y))
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```

- transform instructions, types, functions
- preserve concrete computation
- lift concrete values

Transformation of Program

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x:int  ← input()              x: a_int  ← lift(*)
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b:bool ← z < 0                b: a_bool ← a_lt(z, lift(0))
```

- transform instructions, types, functions
- preserve concrete computation
- lift concrete values

2 concretely realize abstraction

```
x: sym_int  ← lift(*)
y: int      ← factorial(7)
z: sym_int  ← sym_add(x, lift(y))
b: sym_bool ← sym_lt(z, lift(0))
```

- replace abstract calls with provided implementation

Control Flow of Symbolic Program

Problem: constrained values by control flow

```
x: int ← input()
cond: bool ← x < 0
if (cond)
  y: int ← x + 1
else
  ...
```

- both paths can happen
- x is not constrained

Control Flow of Symbolic Program

Problem: constrained values by control flow

```
x: int ← input()
cond: bool ← x < 0
if (cond)
  y: int ← x + 1
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```

- both paths can happen
- x is not constrained

Solution: instrument constraint propagation

```
x: sym_int ← lift(*)
cond: sym_bool ← sym_lt(x, 0)
if (*) // nondeterministic
  x': sym_int ← assume(cond)
  y : sym_int ← sym_add(x', 1)
else
  x': sym_int ← assume(!cond)
  ...
```

- assumes extend a path condition

Types in the Symbolic Program

Problem: how to deal with aggregate types?

```
arr: int[]  ← [1, 2, 3]
arr[1]: int ← input()
```

- we want to minimize the number of symbolic values

Types in the Symbolic Program

Problem: how to deal with aggregate types?

```
arr: int[]  ← [1, 2, 3]
arr[1]: int ← input()
```

- we want to minimize the number of symbolic values

Solution: use discriminated union type

- realize abstract value as union of *concrete* and *symbolic* value

```
arr: union[]  ← [1, 2, 3] // either int or sym_int
arr[1]: union ← lift(*)
```

- similarly deal with recursive structures

Function Calls

Problem: how to transform functions with symbolic arguments?

```
int foo(a: int, b: int, c: int)
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int foo(a: int, b: int, c: int)
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- may produce exponentially many duplicates:

```
int foo(a: sym_int, b: int, c: int)
int foo(a: int, b: sym_int, c: int)
int foo(a: int, b: int, c: sym_int)
int foo(a: sym_int, b: sym_int, c: int)
...
```

- resolve return type

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int foo(a: int, b: int, c: sym_int)
int foo(a: sym_int, b: sym_int, c: int)
...
```

- resolve return type

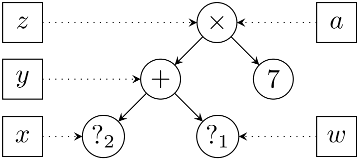
Solution: static analysis + use discriminated union

```
union foo(a: union, b: union, c: int)
```

Symbolic Runtime

Symbolic execution:

```
a: pointer ← malloc()
w: sym_int ← lift(*)
x: sym_int ← lift(*)
y: sym_int ← sym_add(w, x)
z: sym_int ← sym_mul(y, 7)
store z → a
```



Symbolic execution:

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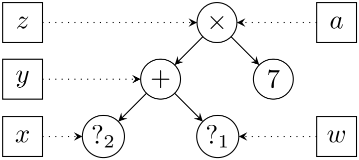
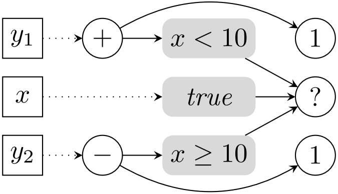


Branching example:

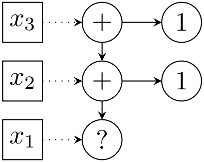
```
x : sym_int ← lift(*)
if (*) // nondeterministic
  x': sym_int ← assume(x < 10)
  y : sym_int ← sym_add(x', 1)
else
  x': sym_int ← assume(x >= 10)
  y : sym_int ← sym_sub(x', 1)
```



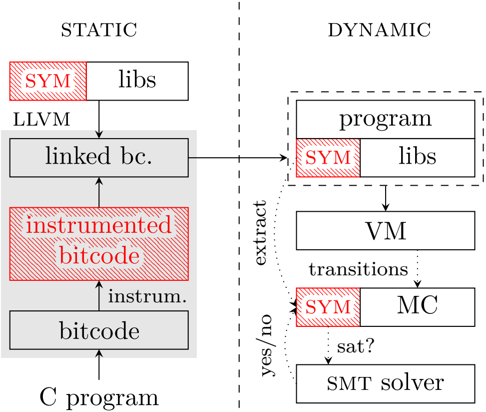
Data Representation II.

Cycle example:

```
x : sym_int ← lift(*)
for i: int ← 1 .. 2
  x: sym_int ← sym_add(x, 1)
```



Symbolic Verification Algorithm



Required support in a tool:

- nondeterminism
- feasibility check
- equality check
- values metadata

Simpler domains do not even need *SMT* support (sign domain).

Results

Integrated with DIVINE model checker:

- LLVM-to-LLVM transformation
- STP SMT solver

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- LLVM-to-LLVM transformation
- STP SMT solver

Component sizes: (lines of code)

	DIVINE*	KLEE	SymDIVINE	CBMC
symbolic support	5.4	24.2	7	39.8
shared code	136.5	125	423	27.5

- reduced complexity of verification tool

SV-COMP Benchmarks:

tag	total	DIVINE*	SymDIVINE	CBMC
array	190	**96**	68	93
bitvector	32	**17**	9	2
loops	178	**72**	67	9
product-lines	575	336	**411**	234
pthread	45	**9**	0	1
recursion	81	**47**	43	22
systemc	59	14	**27**	0
total	1160	591	**625**	361

Conclusion

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Summary

- introduced compilation-based symbolic verification
- generalized approach to the abstraction of programs