

Symbolic Computation via Program Transformation

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- 1 Context of symbolic computation
- 2 Transformation-based approach
- 3 Integration to tool with explicit computation



Explicit computation

- variables represent concrete values
- compiled or interpreted programs

```
x ← input() // x = 7
if (x > 0)
  ...
else
  ...
```

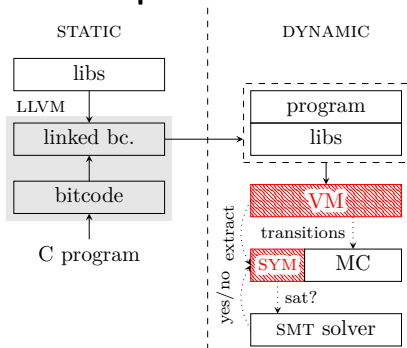
Symbolic computation

- variables represent sets of possible values
- mostly interpreted

```
x ← input() // x = {...}
if (x > 0)
  ... // x = {v | v > 0}
else
  ... // x = {v | v ≤ 0}
```

- verification, test generation, concolic testing

Interpretation-based

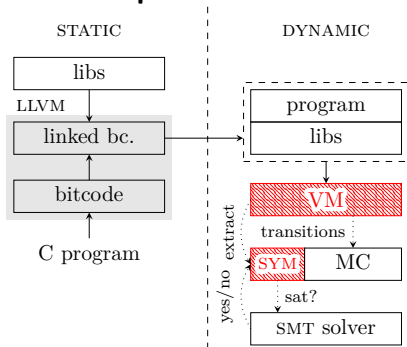


```
a ← input()
pc ≡ [true]
b ← input()
pc ≡ [true]
if (a > 0)
  pc ≡ [a > 0]
  if (b < 0)
    pc ≡ [a > 0 ∧ b < 0]
    b ← a + 1
  pc ≡ [a > 0 ∧ b < 0]
  data ≡ [b = a + 1]
else
  b ← a - 1
pc ≡ [a > 0 ∧ b ≥ 0]
data ≡ [b = a - 1]
```

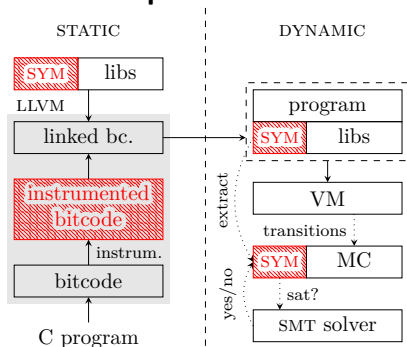
- program does not know anything about symbolic values

- let the program build *data representation* and *path condition*

Interpretation-based



Compilation-based



- minimizes complexity of the verification algorithm



- 1 mixing of explicit and symbolic computation
- 2 expose a small interface to the rest of the system
- 3 impose minimal run-time overhead

<code>x:int</code>	<code>← input()</code>	<code>x: sym_int</code>	<code>← lift(*)</code>
<code>y:int</code>	<code>← factorial(7)</code>	<code>y: int</code>	<code>← factorial(7)</code>
<code>z:int</code>	<code>← x + y</code>	<code>z: sym_int</code>	<code>← sym_add(x, lift(y))</code>
<code>b:bool</code>	<code>← y < z</code>	<code>b: sym_bool</code>	<code>← sym_lt(x, z)</code>

- transform instructions, types, functions
- preserve concrete computation
- lift concrete values
- provide library with implementation of symbolic operations:
 - `lift`, `lower`, `sym_add`, ...

1 syntactically abstract the input program:

- values: `int` \rightarrow `abstract_int`
- instructions: `add` \rightarrow `abstract_add`

2 concretely realize abstraction:

- values: `abstract_int` \rightarrow `sym_int`
- instructions: `abstract_add` \rightarrow `sym_add`

- realization inserts an arbitrary domain that is provided

1 Branching

```
cond: bool ← x < 0
if (cond)
  ...
else
  ...
```

```
cond: sym_bool ← sym_lt(x, 0)
if (*)
  x': sym_int ← assume(cond)
  ...
else
  x': sym_int ← assume(!cond)
  ...
```

- assume constrains values of x
- extend *path condition*



2 Aggregate types

```
arr: int[] ← [1, 2, 3]
arr[1]: int ← input()
```

- we want to minimize the number of symbolic values

Solution: use discriminated union type

- realize abstract value as union of *concrete* and *symbolic* value

```
arr: union[] ← [1, 2, 3] // either int or sym_int
arr[1]: int ← lift(*)
```

- similarly deal with recursive structures

3 Function Calls

- how to transform functions with symbolic arguments?

```
int foo(a: int, b: int, c: int)
```

- may produce exponentially many duplicates:

```
int foo(a: sym_int, b: int, c: int)
int foo(a: int, b: sym_int, c: int)
int foo(a: int, b: int, c: sym_int)
int foo(a: sym_int, b: sym_int, c: int)
...
```

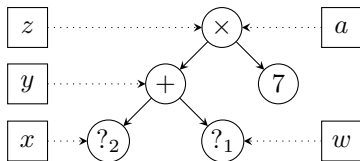
- resolve return type

Solution: static analysis + use discriminated union

```
union foo(a: union, b: union, c: int)
```

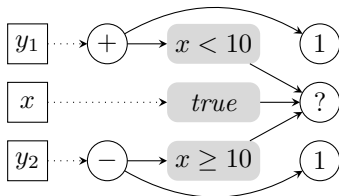
Symbolic execution:

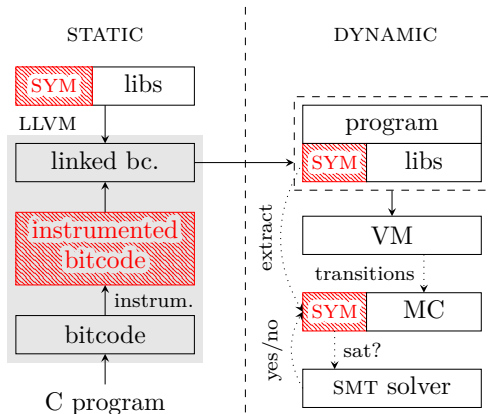
```
a: pointer ← malloc()
w: sym_int ← lift(*)
x: sym_int ← lift(*)
y: sym_int ← sym_add(w, x)
z: sym_int ← sym_mul(y, 7)
store z → a
```



Branching example:

```
x : sym_int ← lift(*)
if (*) // nondeterministic
  x': sym_int ← assume(x < 10)
  y : sym_int ← sym_add(x, 1)
else
  x': sym_int ← assume(x < 10)
  y : sym_int ← sym_sub(x, 1)
```





Required support in a tool:

- nondeterminism
- feasibility check
- equality check
- values metadata

Simpler domains do not even need *SMT* support (sign domain).

- integrated with DIVINE model checker and STP SMT solver (denoted DIVINE*)

Component sizes: (lines of code)

	DIVINE*	KLEE	SymDIVINE	CBMC
symbolic support	5.4	24.2	7	39.8
shared code	136.5	125	423	27.5

- reduced complexity of verification tool

SV-COMP Benchmarks:

tag	total	DIVINE*	SymDIVINE	CBMC
array	190	96	68	93
bitvector	32	17	9	2
loops	178	72	67	9
product-lines	575	336	411	234
pthread	45	9	0	1
recursion	81	47	43	22
systemc	59	14	27	0
total	1160	591	625	361



Goals

- 1 mixing of explicit and symbolic computation ✓
- 2 expose a small interface to the rest of the system ✓
- 3 impose minimal run-time overhead ✓

Summary

- introduced compilation-based symbolic verification
- generalized approach to the abstraction of programs