Balancing Trees

Tricks to amaze your friends

Background

- BSTs where introduced because in theory they give nice fast search time.
- We have seen that depending on how the data arrives the tree can degrade into a linked list
- So what is a good programmer to do.
- Of course, they are to balance the tree

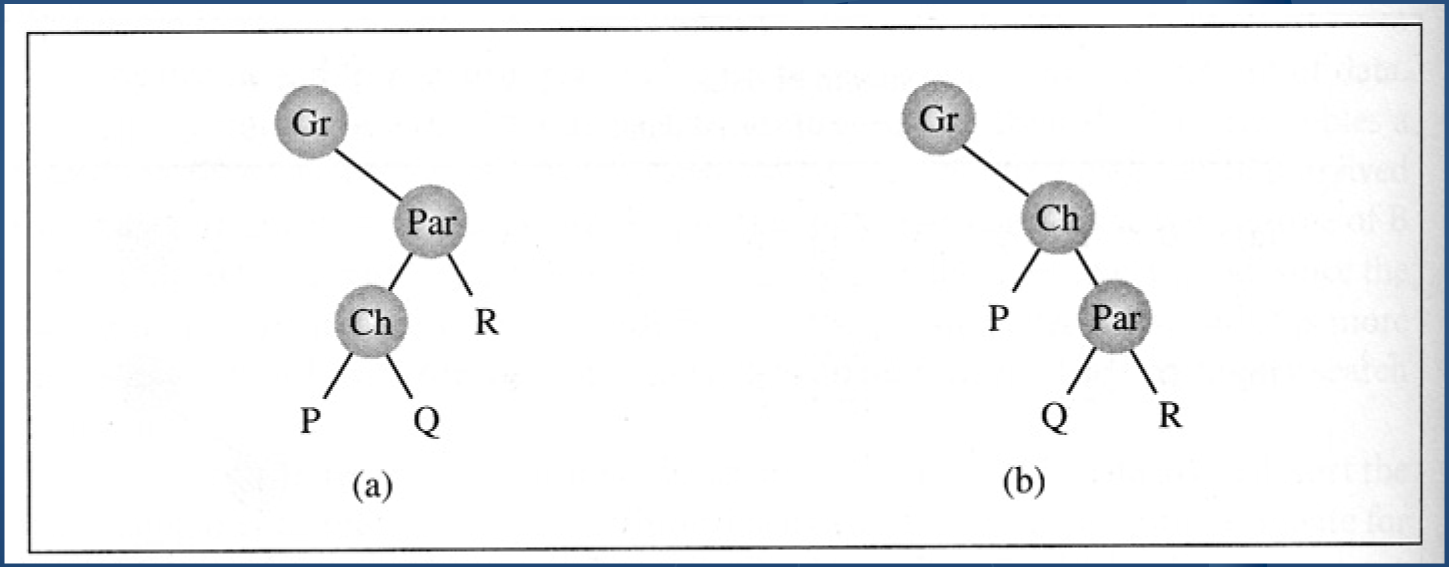
Ideas

- One idea would be to get all of the data first, and store it in an array
- Then sort the array and then insert it in a tree
- Of course this does have some drawbacks
- Ok, we need another idea

DSW Trees

- Named for Colin Day and then for Quentin F. Stout and Bette L. Warren, hence DSW.
- The main idea is a rotation
- rotateRight(Gr, Par, Ch)
  - If Par is not the root of the tree
    - Grandparent Gr of child Ch, becomes Ch's parent by replacing Par;
  - Right subtree of Ch becomes left subtree of Ch's parent Par;
  - Node Ch aquires Par as its right child

Maybe a picture will help



More of the DSW

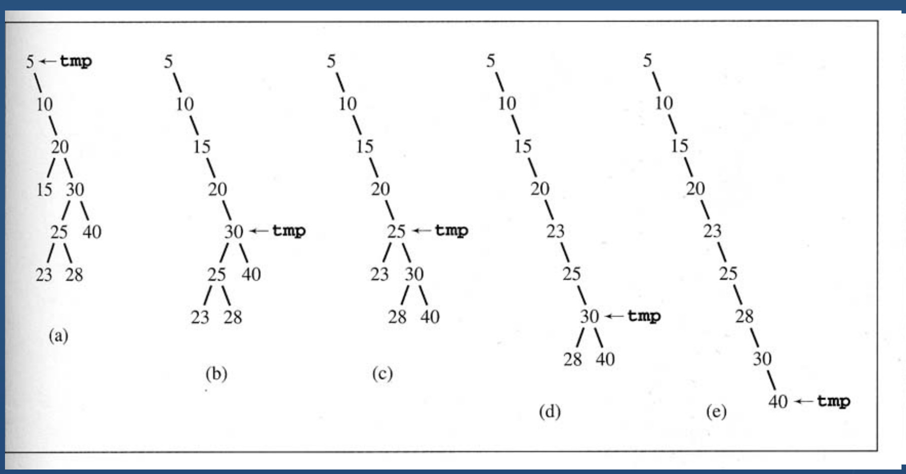
- So the idea is to take a tree and perform some rotations to it to make it balanced.
- First you create a backbone or a vine
- Then you transform the backbone into a nicely balanced tree

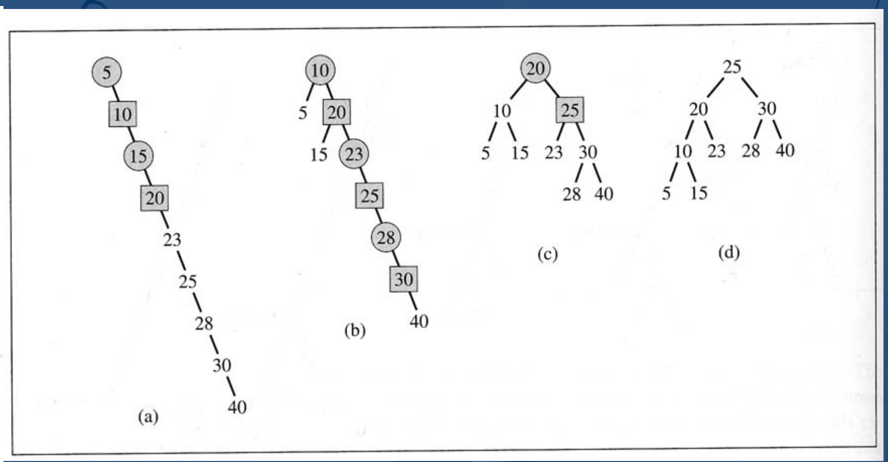
Algorithms

- createBackbone(root, n)
  - Tmp = root
  - While (Tmp != 0)
    - If Tmp has a left child
      - Rotate this child about Tmp
      - Set Tmp to the child which just became parent
    - Else set Tmp to its right child

- createPerfectTree(n)
  - M = $2^{floor[lg(n+1)]}-1$;
  - Make n-M rotations starting from the top of the backbone;
  - While (M > 1)
    - M = M/2;
    - Make M rotations starting from the top of the backbone;

Maybe some more pictures





Wrap-up

- The DSW algorithm is good if you can take the time to get all the nodes and then create the tree
- What if you want to balance the tree as you go?
- You use an AVL Tree

AVL Trees

- Named for Adel’son-Vel’skii and Landis, hence AVL
- The heights of any subtree can only differ by at most one.
- Each nodes will indicate balance factors.
- Worst case for an AVL tree is 44% worst then a perfect tree.
- In practice, it is closer to a perfect tree.

What does an AVL do?

- Each time the tree structure is changed, the balance factors are checked and if an imbalance is recognized, then the tree is restructured.
- For insertion there are four cases to be concerned with.
- Deletion is a little trickier.

AVL Insertion

- Case 1: Insertion into a right subtree of a right child.
  - Requires a left rotation about the child
- Case 2: Insertion into a left subtree of a right child.
  - Requires two rotations
    - First a right rotation about the root of the subtree
    - Second a left rotation about the subtree's parent

Some more pictures

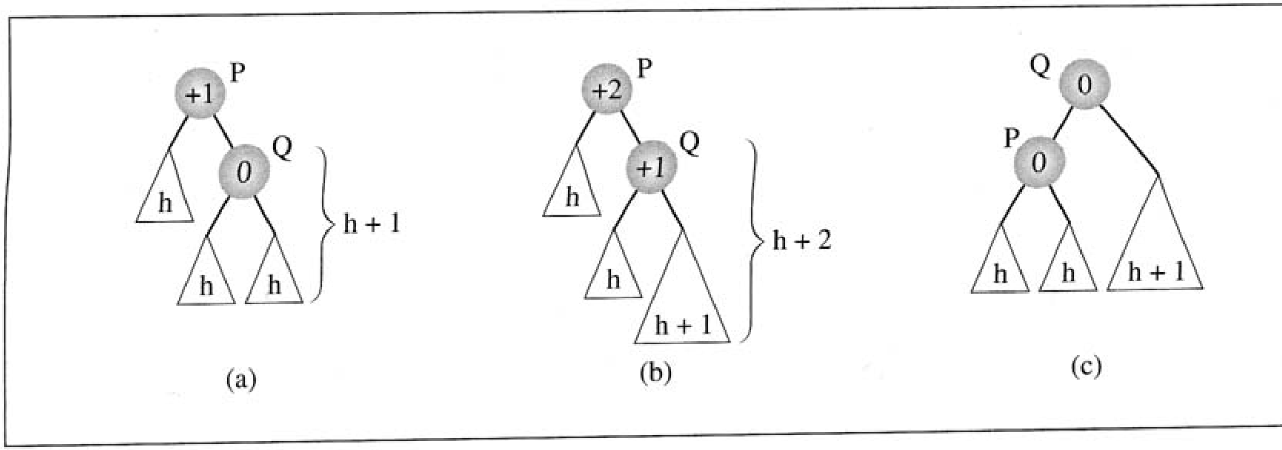
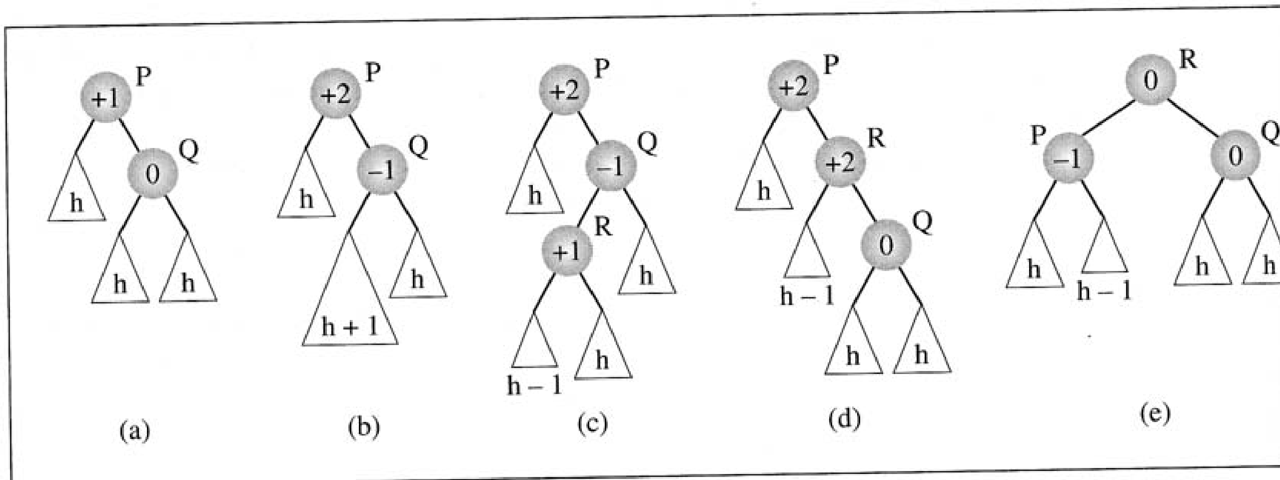


FIGURE 6.42 Balancing a tree after insertion of a node in the left subtree of node Q.



Deletion

- Deletion is a bit trickier.
- With insertion after the rotation we were done.
- Not so with deletion.
- We need to continue checking balance factors as we travel up the tree

Deletion Specifics

- Go ahead and delete the node just like in a BST.
- There are 4 cases after the deletion:

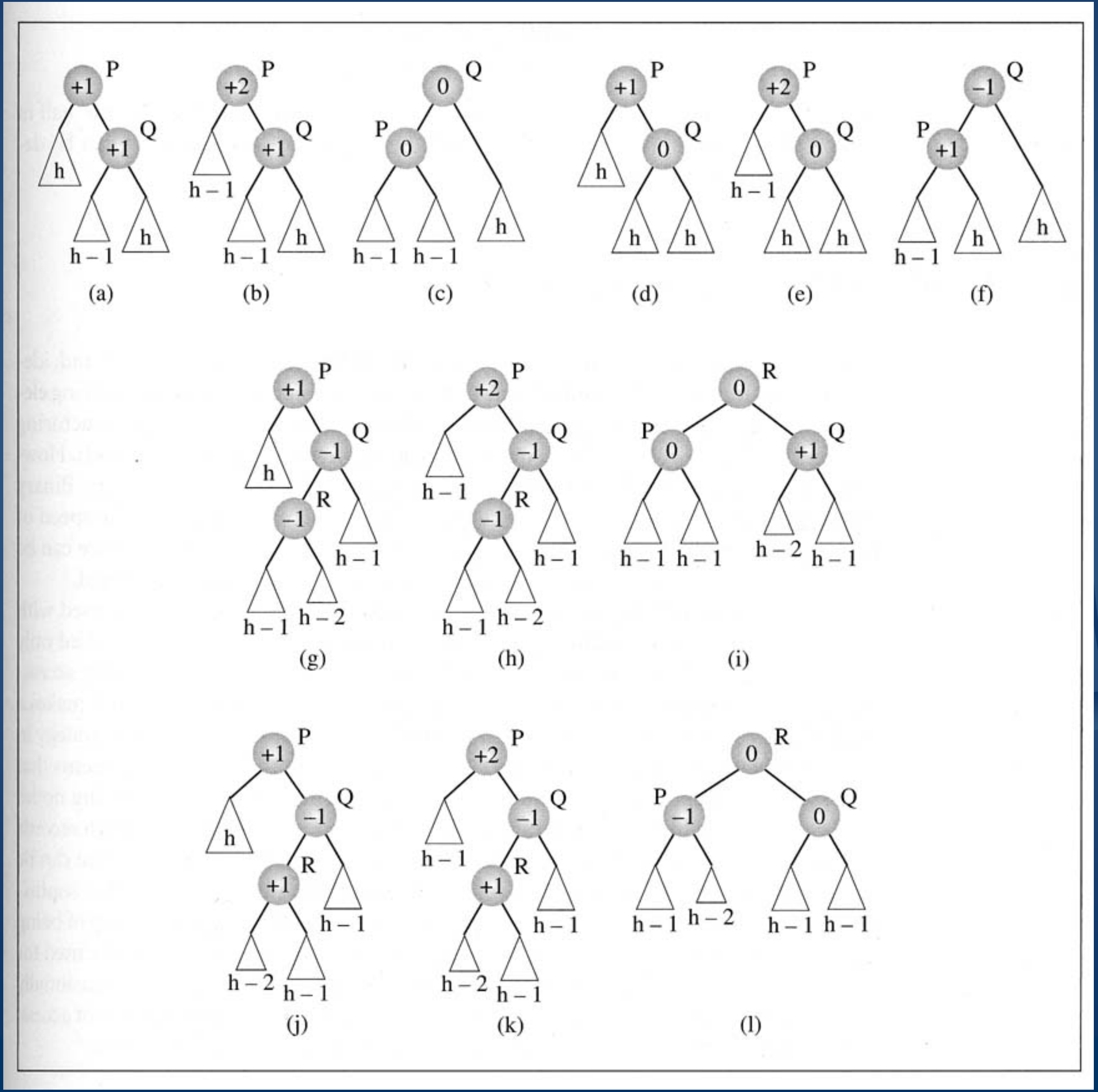
Cases

- Case 1: Deletion from a left subtree from a tree with a right high root and a right high right subtree.
  - Requires one left rotation about the root
- Case 2: Deletion from a left subtree from a tree with a right high root and a balanced right subtree.
  - Requires one left rotation about the root

Cases continued

- Case 3: Deletion from a left subtree from a tree with a right high root and a left high right subtree with a left high left subtree.
  - Requires a right rotation around the right subtree root and then a left rotation about the root
- Case 4: Deletion from a left subtree from a tree with a right high root and a left high right subtree with a right high left subtree
  - Requires a right rotation around the right subtree root and then a left rotation about the root

Definitely some pictures



Self-adjusting Trees

- The previous sections discussed ways to balance the tree after the tree was changed due to an insert or a delete.
- There is another option.
- You can alter the structure of the tree after you access an element
  - Think of this as a self-organizing tree

Splay Trees

- You have some options
- When you access a node, you can move it to the root
- You can also swap it with its parent
- When you modify the move to root strategy with a pair of swaps you get a splay tree

Splay Cases

- Depending on the configuration of the tree you get three cases
- Case 1: Node R's parent is the root
- Case 2: Node R is the left child of its parent Q and Q is the left child of its parent R
- Case 3: Node R is the right child of its parent Q and Q is the left child of its parent R

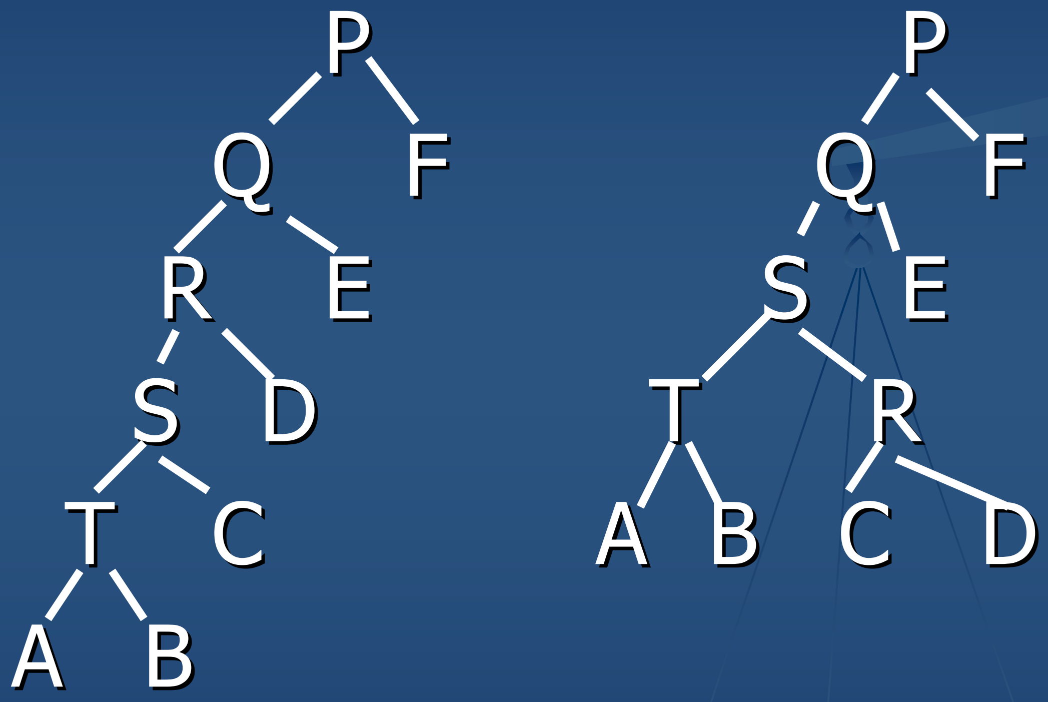
Splay Algorithm

- Splaying(P, Q, R)
  - While R is not the root
    - If R's parent is the root
      - Perform a singular splay, rotate R about its parent
    - If R is in a homogenous configuration
      - Perform a homogenous splay, first rotate Q about P and then R about Q
    - Else
      - Perform a heterogeneous splay, first rotate R about Q and then about P

Semisplaying

- You can modify the traditional splay techniques for homogenous splays
- When a homogenous splay is made instead of the second rotation taking place with R, you continue to splay with the node that was previously splayed

Example



Last Step