Robust Methods for Optical Interferometry Images

Ph.D Thesis

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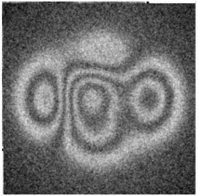
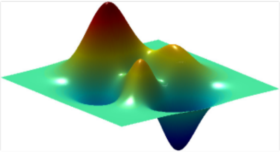
Algoritos de Cuadratura

Patrón de franjas:

$$I(x,y) = a(x,y) + b(x,y)cos[\phi(x,y)] \tag{1}$$



$= a + b\ cos[$

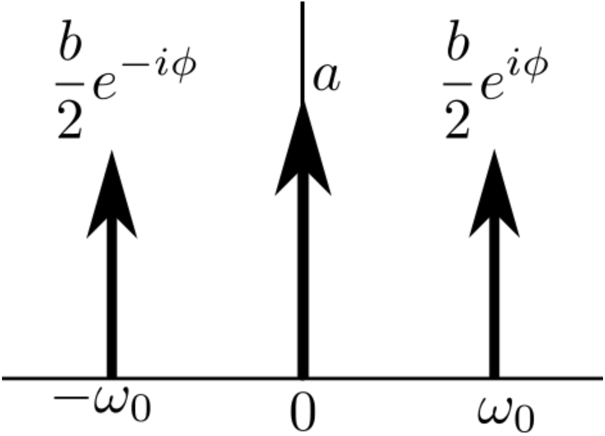


$]$

Algoritos de Cuadratura

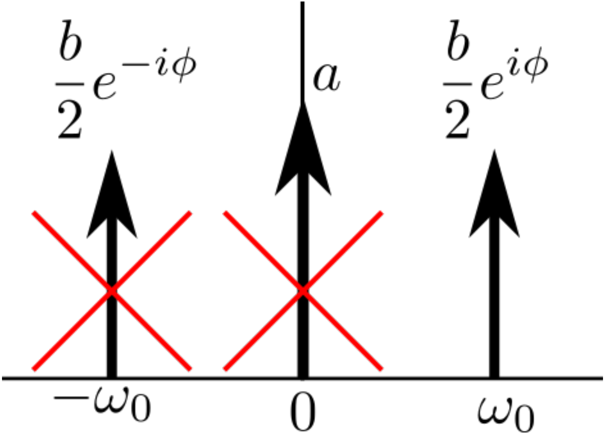
Algoritos de Cuadratura

$$\begin{aligned} \mathcal{F}[I(x,y)] &= I(\omega) \\ &= a\delta(\omega) + \frac{b}{2}e^{-i\phi}\delta(\omega-\omega_0) + \frac{b}{2}e^{i\phi}\delta(\omega+\omega_0) \end{aligned} \quad (2)$$



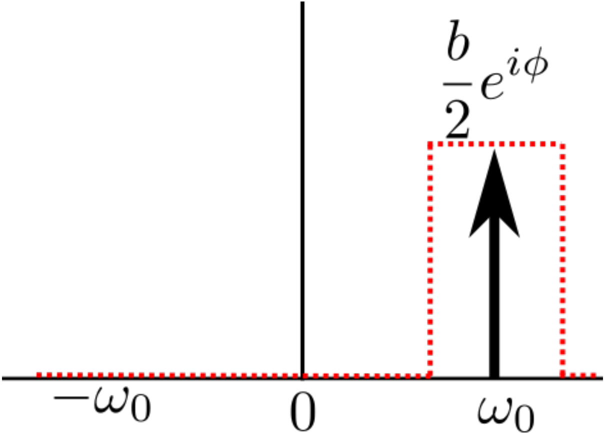
Algoritos de Cuadratura

$$H(-\omega_0) = H(0) = 0, H(\omega_0) \neq 0 \tag{3}$$



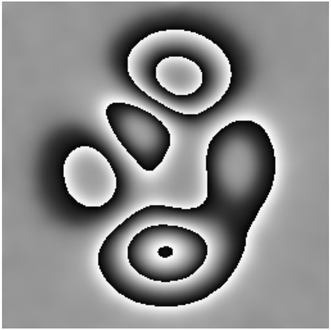
Algoritos de Cuadratura

$$I(\omega)H(\omega) = \frac{b}{2}exp[i\phi] \tag{4}$$



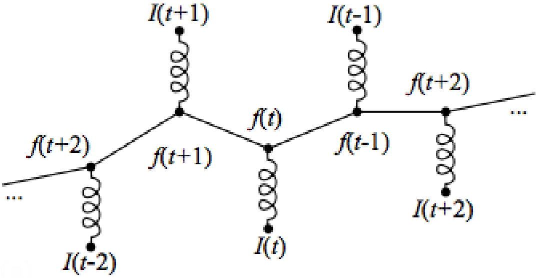
Algoritmos de Cuadratura

$$\hat{\phi} = atan\left[\frac{Im\{\frac{b}{2}exp[i\phi]\}}{Re\{\frac{b}{2}exp[i\phi]\}}\right] \qquad (5)$$



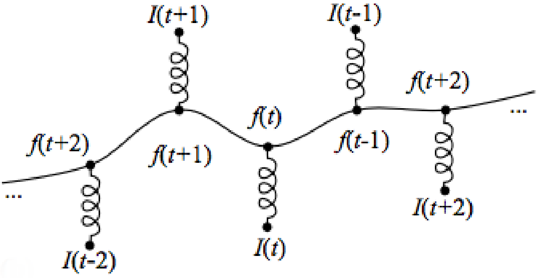
Filtros Regularizados

$$U[f(x,y)] = \iint_{(x,y)\in S} \left\{ [f(x,y) - I(x,y)]^2 + \eta\left[\frac{\partial f(x,y)}{\partial x}\right]^2 + \eta\left[\frac{\partial f(x,y)}{\partial y}\right]^2 \right\} dxdy \qquad (6)$$



Filtros Regularizados

$$U[f(x,y)] = \iint_{(x,y)\in S} \left\{ [f(x,y) - I(x,y)]^2 + \eta \left[\frac{\partial^2 f(x,y)}{\partial x^2} \right]^2 + \eta \left[\frac{\partial^2 f(x,y)}{\partial y^2} \right]^2 + \eta \left[\frac{\partial^2 f(x,y)}{\partial x \partial y} \right]^2 \right\} dxdy \quad (7)$$



Filtros Regularizados

$$U[f(x,y)] = \sum_{(x,y)\in S} \left\{ [f(x,y) - I(x,y)]^2 + \eta R[f(x,y)] \right\} \quad (8)$$

Resorte:

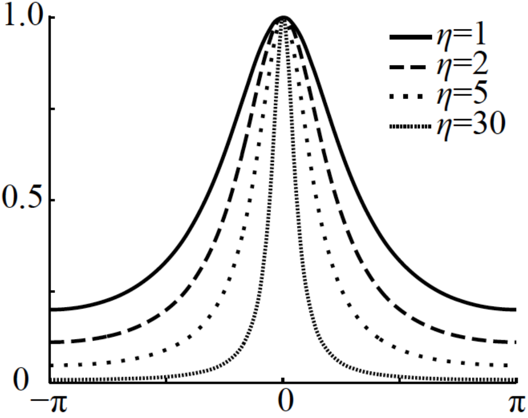
$$R_r[f(x,y)] = [f(x,y) - f(x-1,y)]^2 + [f(x,y) - f(x,y-1)]^2 \quad (9)$$

Placa:

$$\begin{aligned} R_p[f(x,y)] \quad = \quad & [f(x+1,y) - 2f(x,y) - f(x-1,y)]^2 \\ & +[f(x,y+1) - 2f(x,y) - f(x,y-1)]^2 \\ & +[f(x+1,y+1) - f(x-1,y-1) \\ & +f(x-1,y+1) - f(x+1,y-1)]^2 \end{aligned} \quad (10)$$

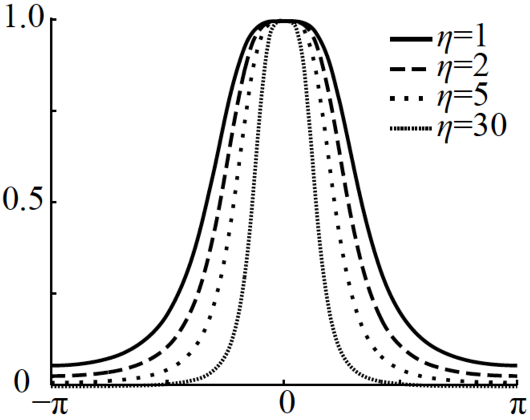
Filtros Regularizados

Resorte:



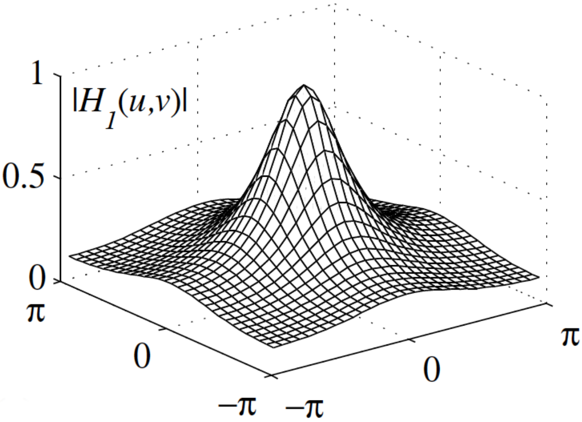
Filtros Regularizados

Placa:



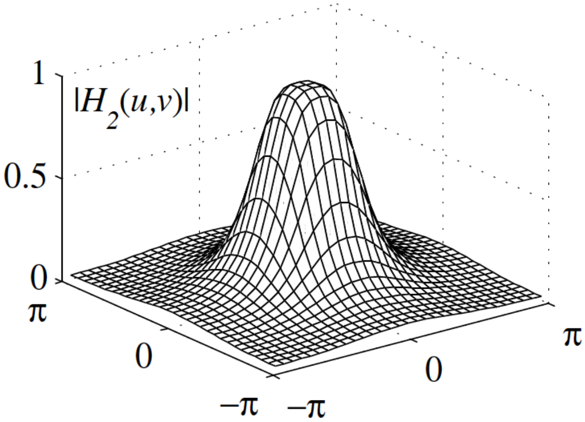
Filtros Regularizados

Resorte 3D:



Filtros Regularizados

Placa 3D:



Minimos Cuadrados Regularizados

$$
\begin{aligned}
I(x,y,k) &= a(x,y) + b(x,y)\cos[\phi(x,y) + \alpha k] \\
&= a(x,y) + C(x,y)\cos[\alpha k] - S(x,y)\sin[\alpha k], \quad (11)
\end{aligned}
$$

donde:

$$C(x,y) = b(x,y)\cos[\phi(x,y)]$$
$$S(x,y) = b(x,y)\sin[\phi(x,y)]$$

Funcional de minimos cuadrados:

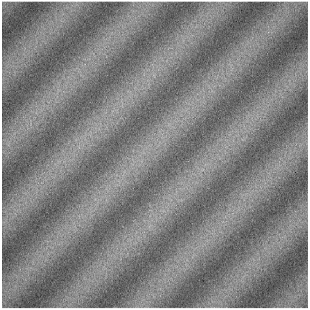
$$U[a,c,s] = \sum_{k=0}^{N-1} \left[a + C\cos(\alpha k) - S\sin(\alpha k) - I(k)\right]^2, \qquad (12)$$

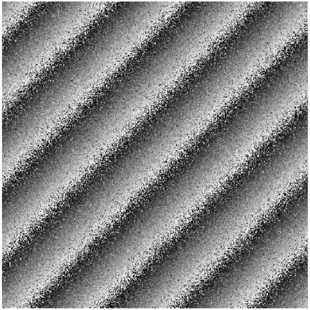
Minimos Cuadrados Regularizados

Funcional de minimos cuadrados regularizados:

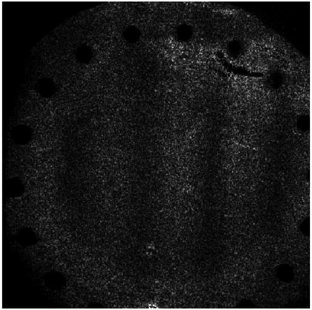
$$
\begin{aligned}
U(\mathbf{a}, \mathbf{C}, \mathbf{S}) = & \\
& \sum_{k=0}^{N-1} \sum_{x,y \in L} \left[a + C\cos(\alpha k) - S\sin(\alpha k) - I_k\right]^2 M_{x,y} \\
& + \lambda \sum_{x,y \in L} \left[(C_{x,y} - C_{x-1,y})^2 + (S_{x,y} - S_{x,y-1})^2\right] \\
& + \mu \sum_{x,y \in L} (a_{x,y} - a_{x-1,y})^2, \qquad (13)
\end{aligned}
$$

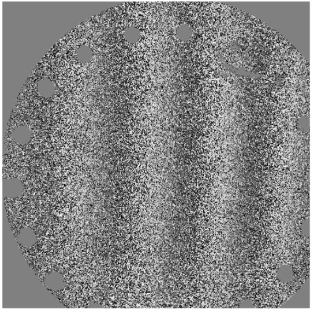
Minimos Cuadrados Regularizados

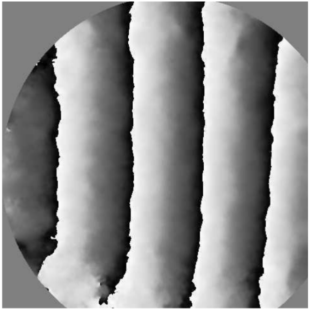




Minimos Cuadrados Regularizados







Minimos Cuadrados Regularizados

Conclusiones!!

Self-tuning Regularizados

$$I_k^{'}(x,y) = b^{'}(x,y)cos(\phi(x,y)+\alpha_k),\ k=0,1,2,...,N-1. \quad (14)$$

$$\begin{aligned} U(\mathbf{f},\alpha) &= \sum_{k=0}^{N-1}\sum_{(x,y)}[\frac{1}{2}[f(x,y)e^{i\alpha_k}+f^*(x,y)e^{-i\alpha_k}]-I_k^{'}(x,y)]^2 \\ &+\lambda\sum_{(x,y)}[||D_x[f(x,y)]||^2+||D_y[f(x,y)]||^2], \end{aligned}$$

Adaptive phase-shifting

$$I_k(x,y) = a(x,y) + b(x,y)cos[\phi_0(x,y) + \eta_k(x,y) + \omega_0 k], \quad (15)$$

renombrando $\beta_k(x,y) = \eta_k(x,y) + \omega_0 k$

$$I_k(x,y) = a(x,y) + b(x,y)cos[\phi_0(x,y) + \beta_k(x,y)], \quad (16)$$

Adaptive phase-shifting

$$E[a(x,y), f(x,y)] = \sum_{k=0}^{K-1} [a(x,y) + Re\{f(x,y)e^{i\beta_k(x,y)}\} - I_k(x,y)]^2$$

$$\begin{pmatrix} K & \sum c_k(x,y) & \sum s_k(x,y) \\ \sum c_k(x,y) & \sum c_k(x,y)^2 & \sum c_k(x,y)s_k(x,y) \\ \sum s_k(x,y) & \sum c_k(x,y)s_k(x,y) & \sum s_k(x,y)^2 \end{pmatrix} \begin{pmatrix} \hat{a}(x,y) \\ \hat{\phi}(x,y) \\ \hat{\psi}(x,y) \end{pmatrix} = \begin{pmatrix} \sum I_k(x,y) \\ \sum I_k(x,y)C_k(x,y) \\ \sum I_k(x,y)S_k(x,y) \end{pmatrix}$$

Adaptive phase-shifting

$$E[a(x,y), g_k(x,y)] =$$
$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[\{a(m,n)+Re\{g_k(m,n)e^{i\phi_0(x,y)}\}-I_k(m,n)\}h(x-m,y-n)\right]^2$$

$$\begin{pmatrix} [1s*h](x,y) & [\phi*h](x,y) & [\psi*h](x,y) \\ [\phi*h](x,y) & [\phi*h]^2(x,y) & [\phi\psi*h](x,y) \\ [\psi*h](x,y) & [\phi\psi*h](x,y) & [\psi*h]^2(x,y) \end{pmatrix} \begin{pmatrix} \hat{a}(x,y) \\ \hat{c}_k(x,y) \\ \hat{s}_k(x,y) \end{pmatrix} = \begin{pmatrix} [I_k*h](x,y) \\ [I_k\phi*h](x,y) \\ [I_k\psi*h](x,y) \end{pmatrix}$$

Phase Detuning Correcting Method

$$
\begin{aligned}
U(f) &= \sum_{x,y \in L} \mid f(x,y) - g(x,y) \mid^2 \\
&\quad + \lambda \sum_{x,y \in L} \mid f(x,y) - f(x-1,y) e^{i u_{x,y}} \mid^2 \\
&\quad + \lambda \sum_{x,y \in L} \mid f(x,y) - f(x,y-1) e^{i v_{x,y}} \mid^2
\end{aligned}
$$

$$\text{Para } g(x,y) = e^{i \phi^{\varepsilon}_{x,y}}$$

Phase Detuning Correcting Method

$$\nabla \phi_{x,y} = \nabla \left[\arctan \left(\frac{\sin \phi_{x,y}}{\cos \phi_{x,y}} \right) \right]$$

$$u_{x,y} = \frac{\sin \phi \frac{\partial}{\partial x} \cos \phi - \cos \phi \frac{\partial}{\partial x} \sin \phi}{\cos^2 \phi + \sin^2 \phi}$$

$$\frac{\partial}{\partial x} \cos \phi_{x,y} = \cos \phi_{x,y} - \cos \phi_{x+1,y},$$

Phase Detuning Correcting Method

$$\nabla \phi_{x,y} = \nabla \left[\arctan \left(\frac{\sin \phi_{x,y}}{\cos \phi_{x,y}} \right) \right]$$

$$u_{x,y} = \frac{\sin \phi \frac{\partial}{\partial x} \cos \phi - \cos \phi \frac{\partial}{\partial x} \sin \phi}{\cos^2 \phi + \sin^2 \phi}$$

$$\frac{\partial}{\partial y} \sin \phi_{x,y} = \sin \phi_{x,y} - \sin \phi_{x,y+1}.$$