Reply to comment on "Phase-shift extraction and wave-front reconstruction in phase-shifting interferometry with arbitrary phase steps"

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In answer to the questions raised by Ferrari and Garbusi [Opt. Lett. **29,** 1257 (2004)] we indicate that, even if the intensities of the object and reference waves can be measured, the phase-shifting procedure is still necessary for correct reconstruction of the object wave field. We also show that the statistical approximation used in our previous Letter [Opt. Lett. **28,** 1808 (2003)], $\langle|\sin\varphi_o(x,y)|\rangle = 2/\pi$, where $\varphi_o(x,y)$ is the phase of a diffraction field and $\langle\,\rangle$ means average over the whole interferogram frame, is reasonable for most objects, including both smooth and diffusing surfaces, and that the method we proposed for phase-shift extraction is correct.

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In their comments[1] on our previous Letter[2] Ferrari and Garbusi raised three questions. We are pleased to have an opportunity to clarify some related ideas in more detail by answering all the questions in the following.

First, we explain the necessity of using phase-shifting interferometry (PSI). Let us begin with the expression of the intensity distribution of the jth interferogram in PSI,

$$I_j(x,y) = {A_o}^2(x,y) + {A_r}^2 + 2A_o(x,y)A_r \times \cos[\varphi_o(x,y) - \delta_j], \tag{1}$$

where $O(x,y) = A_o(x,y)\exp[i\varphi_o(x,y)]$ is the complex object field in recording plane P_H, and $A_r \exp(i\delta_j)$ is the reference (a plane wave normal to P_H here) field in the jth step. The comments claim that if $I_o = {A_o}^2$ and $I_r = {A_r}^2$ can be measured, one interferogram is enough to reconstruct $\varphi_o(x,y)$ and the whole PSI procedure is "absolutely unnecessary" because there are only three variables in Eq. (1). This is not exactly true. Since in this case what we get is (δ is supposed to be zero here without loss of generality)

$$A_o \cos\varphi_o = [A_o \exp(i\varphi_o) + A_o \exp(-i\varphi_o)]/2\,, \tag{2}$$

which is the combination of $O(x,y)$ and its conjugate $O^*(x,y)$, and the latter is unavoidably presented. On the contrary, in PSI we get only $O(x,y)$. If we take a standard four-frame PSI with phase step $\pi/2$ as an example,[2,3] it is easy to see that

$$\cos\varphi_o = (I_1 - I_3)/(4A_oA_r),$$
$$\sin\varphi_o = (I_2 - I_4)/(4A_oA_r), \tag{3}$$

therefore

$$O(x,y) = A_o \exp(i\varphi_o) = [(I_1 - I_3) + i(I_2 - I_4)]/(4A_r)\,. \tag{4}$$

In other words, in PSI both signs of $\cos\varphi_o$ and $\sin\varphi_o$, instead of only the sign of $\cos\varphi_o$, can be determined, so that the conjugate object wave can be eliminated. This is an important difference and the reason why we cannot reconstruct the object wave correctly with one interferogram even when I_o and I_r are known. For this purpose, PSI with at least two interferograms is still needed. In fact, Guo *et al.*[4] have reported a technique for phase-shift error elimination with measured I_o and I_r and two interferograms I_1 and I_2. However, their method can be used for only pure phase objects, and the approximate values of phase steps must be known, whereas in our Letter this limitation and requirement have been removed.

A related problem is why more interferograms are often used in PSI than the number of unknown variables. The reason is that an algorithm with more frames usually yields a better performance and has better error-compensating ability. This has been studied extensively in many papers[5–8] and is well known.

The second question is the reasonableness of our approximation [Eq. (4) in Ref. 2]

$$\langle|\sin\varphi_o(x,y)|\rangle = 2/\pi\,, \tag{5}$$

where $\langle\,\rangle$ means average over the whole frame. We want to indicate again that this equation is only an approximation for the purpose of further iteration, not an exact expression. Furthermore, Ferrari and Garbusi mistake the "frame" in our Letter [the line after Eq. (4)] for "space" and then say it is "absolutely unrealistic." Of course any practical computation can be taken over only a limited area, not an unlimited space. Here the area we used is the whole interferogram frame, as we said.

The phase distribution of a diffraction field is surely determined by the property of the object surface. In this sense, the phase of any diffraction field, even for a speckle field, is determinate. We can calculate it analytically in principle for a given object with related parameters. For example, the phase resulting from a

Table 1. Computer Simulation Results of Eq. (5) for Different Objects

Objects	$\langle\lvert\sin\,\varphi_o(x,y)\rvert\rangle$	Relative Errors (%)
Point object	0.6368	0.3
Smooth surface of $R = 10$ m	0.6343	-3.6
Smooth surface of $R = 1$ m	0.6382	2.5
Smooth surface of $R = 0.2$ m	0.6377	1.7
Diffusing surfaces of $R = 10$ m	0.6357, 0.6359	-1.4, -1.1
Diffusing surfaces of $R = 1$ m	0.6368, 0.6372	0.3, 0.9
Diffusing surfaces of $R = 0.2$ m	0.6366, 0.6363	0.0, -0.5

point source has a form like Newton rings. What we mean by "spatially random distribution" in our Letter is that $\varphi_o(x,y)$ may take any value in the range of $[-\pi,\pi]$ (as module 2π), and the chance of any special phase value in this range is nearly equal statistically over an area much larger than the average spatial period of phase change. In this case the phase $\varphi_o(x,y)$ undergoes many 2π changes in the whole area, leading to the approximation of Eq. (5). Since the phase of the diffraction field of a real object usually changes fast in space, the CCD area (usually approximately 1 cm^2 and 7.68 mm $\times$ 7.68 mm in our simulations) can be reasonably considered as a large area, and then Eq. (5) is approximately satisfied for most objects.

To test this idea, we calculated $\langle\lvert\sin\,\varphi_o(x,y)\rvert\rangle$ for a variety of object surfaces, including the point object suggested in the comment and both smooth and diffusing spherical surfaces with different radius R, and compared the values with $2/\pi$. Some calculation results are listed in Table 1, where the related parameters are the same as what we used in our previous Letter,[2] and two different samples are tested for each R in the case of diffusing surfaces. From Table 1 we can see that a smaller R and the addition of phase noise to the object surface may lead to a smaller error of Eq. (5) because of stronger phase variation, but the maximum relative error in Table 1 is less than 0.4%. We also made calculations for some more-complex irregular surfaces (not shown here for brevity) and found that this error is less than 1% in most cases and 2–3% for the worst, as long as the diffraction phase does not vary too slowly in the whole frame like a plane wave. It is good enough for an approximate estimation used as a starting point, and actually we indeed can improve the approximate results rapidly by the iterative process described in our Letter[2] until they reach exact results.

The last question in the comment deals with the necessity of using Fresnel diffraction. Of course we can use a lens system to image the object to CCD plane P_H and then record this field. But, if we want to reconstruct the original object field in plane P_o exactly, this setup is often inappropriate, especially for the precise phase measurement of a three-dimensional (3-D) object, since the lens system may introduce additional phase distortions, image aberrations, and 3-D perspective error. Therefore in many applications researchers use Fresnel and inverse Fresnel diffraction to record and retrieve the original object field. These applications include, to name a few, image formations and wave-front reconstructions,[9,10] surface contouring,[11,12] and 3-D object recognition[13] and encryption.[14] If one pays attention to the extensive work in this field, the use of Fresnel transformation in PSI would not be surprising or seem unnecessary.

In summary, from the analysis and explanations above, we can conclude that the basic ideas and principles in our last Letter[2] are correct. Computer simulations have convincingly verified the effectiveness of our proposed method for both smooth and diffusing surfaces.

Finally, we want to indicate that even though the algorithm in our Letter[2] works well, the requirement of measuring the object wave intensity I_o may cause inconvenience sometimes. In another Letter of ours published recently[15] this requirement was removed.

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