Phase-shifting interferometry in the presence of nonlinear phase steps, harmonics, and noise

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A phase-shifting piezo device commonly employed in phase-shifting interferometry exhibits a nonlinear response to applied voltage. Hence, a method for estimation of phase distribution in the presence of nonlinear phase steps is presented. The proposed method compensates for the harmonics present in the intensity fringe, allows the use of arbitrary phase-step values between 0 and π rad, and does not impose constraints on the selection of particular phase-step values for minimizing nonlinearity and compensating for the harmonics. The comparison of the proposed method with other well-known benchmarking algorithms shows that our method is highly efficient and also works well in the presence of noise. © 2006 Optical Society of America

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Phase determination using temporal techniques commonly involves the use of a piezoelectric device (PZT) for changing the relative phase between two interfering beams. However, the phase computation is sensitive to the nonlinear response of the PZTs to the applied voltage. The error in phase measurement is further aggravated by the nonsinusoidal intensity profile and noise. Few algorithms[1–3] have attempted to address these concerns and have done so at the cost of imposing conditions such as the use of only specific phase-step values to minimize the influence of particular order harmonics and nonlinearity of the phase shift.

While retaining salient features such as compensating harmonics and the use of arbitrary phase steps, the Fourier transform method[4] based on a pseudospectrum presupposes linear phase steps between the acquired data frames. In this method the phase steps are estimated from the pseudospectrum of the intensity data at a pixel (x,y), for $n=1$ to $n=N$ data frames, given by

$$I_n = I_{\mathrm{dc}}\left\{1 + \sum_{k=1}^{\kappa} \gamma_k \cos[k(\varphi + \alpha'_n)]\right\} + \eta_n, \tag{1}$$

where, for the nth data frame, I_{dc} and γ represent the local background intensity and fringe visibility, respectively; φ and α'_n represent the phase difference and phase shift, respectively; and η is additive white Gaussian noise with a mean of zero and variance σ^2. For N data frames, the true phase shift α'_n due to nonlinear characteristics of the PZT can be represented as[1] $\alpha'_n=\alpha_n+\epsilon_1\alpha_n+\epsilon_2\alpha_n^2/\pi$, where α'_n is the polynomial representation of the phase step α_n and ϵ_1 and ϵ_2 are the linear and nonlinear error coefficients in the PZT response, respectively.

The method proposed in Ref. 4, based on estimation of phase steps by locating peaks in the pseudospectrum, is effective only for linear phase steps between the acquired data frames. In the case of nonlinear phase shift α'_n, a phenomenon of spectral broadening can be observed in the pseudospectrum plot. Similarly, the method proposed in Ref. 5, based on the design of a log-likelihood function, is unfortunately effective primarily in compensating for linear phase steps. Hence, the objective of this Letter is to present a modified log-likelihood function such that the errors arising due to the nonlinear response of the PZT can be minimized. The proposed method requires only six data frames for first-order harmonics and retains all other features of the maximum likelihood method,[5] such as compensation for nonsinusoidal waveforms, the use of arbitrary phase steps, and the use of noncollimated waveforms. Although the method that we propose is also effective in compensating for the nonlinearity of phase shifts in configurations consisting of multiple PZTs, the present Letter focuses on studying the performance of the method in the presence of noise and harmonics and on its comparison with other well-known algorithms such as those proposed by Hibino *et al.*[1] The algorithms proposed by Hibino *et al.* are, however, effective primarily for configurations containing single PZTs. The robustness of the proposed method in the estimation of phases is studied in the presence of various magnitudes of nonlinearity and different orders of harmonics.

We first write Eq. (1) in the following form:

$$\bar{\mathbf{I}} = \mathbf{I} + \boldsymbol{\eta} = \mathbf{S}(\boldsymbol{\xi})\mathbf{C} + \boldsymbol{\eta}, \tag{2}$$

where $\mathbf{I}=[I_1\, I_2 \ldots I_N]^{\mathrm{T}}$, $\boldsymbol{\eta}=[\eta_1\ \eta_2 \ldots \eta_N]^{\mathrm{T}}$, T is the transpose operator, and matrix $\mathbf{S}$ is expressed as $\mathbf{S}(\boldsymbol{\xi})=[1\ \mathbf{s}_1\ \mathbf{s}_1^* \ldots \mathbf{s}_\kappa^*]$; $\boldsymbol{\xi}=\{\alpha,\epsilon_1,\epsilon_2\}$ is the parameter set, signal vector $\mathbf{s}_k$ is given by $\mathbf{s}_k(n)=\exp(jk\alpha'_n)$, and $j=\sqrt{-1}$. Matrix $\mathbf{C}$ in Eq. (2) is the coefficient matrix given by $\mathbf{C}=[c_0\ c_1\ c_1^* \ldots c_\kappa^*]^{\mathrm{T}}$ with $c_0=I_{\mathrm{dc}}$, and element $c_k=I_{\mathrm{dc}}\gamma\exp(jk\varphi)$; since the noise is assumed to be additive white Gaussian with variance σ^2, the probabil-

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ity density function of the data vector $\bar{\mathbf{I}}$, parameterized by $\boldsymbol{\xi}$, is given by

$$p(\bar{\mathbf{I}};\boldsymbol{\xi}) = \frac{1}{\pi^N \sigma^N} \exp\left[-\frac{1}{\sigma^2}(\bar{\mathbf{I}} - \mathbf{SC})^{\mathrm{H}}(\bar{\mathbf{I}} - \mathbf{SC}) \right], \quad (3)$$

where H is the Hermitian transpose. The likelihood function of the data $L(\bar{\mathbf{I}},\boldsymbol{\xi})$ is proportional to $p(\bar{\mathbf{I}};\boldsymbol{\xi})$, thus the maximum likelihood estimate of $\boldsymbol{\xi}$ is obtained by maximizing $L(\bar{\mathbf{I}},\boldsymbol{\xi})$. From Eq. (3) we observe that this maximization is equivalent to minimizing the product $\Upsilon = [\bar{\mathbf{I}} - \mathbf{S}(\boldsymbol{\xi})\mathbf{C}]^{\mathrm{H}}[\bar{\mathbf{I}} - \mathbf{S}(\boldsymbol{\xi})\mathbf{C}]$. Taking the complex gradient of Υ and equating it to zero we obtain

$$\frac{\partial \Upsilon}{\partial \mathbf{C}} = \frac{\partial}{\partial \mathbf{C}}[\bar{\mathbf{I}}^{\mathrm{H}}\bar{\mathbf{I}} - \bar{\mathbf{I}}^{\mathrm{H}}\mathbf{SC} - \mathbf{C}^{\mathrm{H}}\mathbf{S}^{\mathrm{H}}\bar{\mathbf{I}} + \mathbf{C}^{\mathrm{H}}\mathbf{S}^{\mathrm{H}}\mathbf{SC}]$$

$$= 2\mathbf{S}^{\mathrm{H}}[\bar{\mathbf{I}} - \mathbf{SC}]. \quad (4)$$

Equating Eq. (4) with zero yields $\mathbf{C} = (\mathbf{S}^{\mathrm{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\bar{\mathbf{I}}$. Substituting $\mathbf{C}$ in Υ, we get

$$\Upsilon = \bar{\mathbf{I}}^{\mathrm{H}}\bar{\mathbf{I}} - \bar{\mathbf{I}}^{\mathrm{H}}\mathbf{S}(\mathbf{S}^{\mathrm{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\bar{\mathbf{I}}. \quad (5)$$

So, minimizing Eq. (5) is equivalent to maximizing the expression

$$\boldsymbol{\xi} = \max \bar{\mathbf{I}}^{\mathrm{H}}[\mathbf{S}(\mathbf{S}^{\mathrm{H}}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}]\bar{\mathbf{I}}. \quad (6)$$

Note that this likelihood function is different from the one that we have proposed in Ref. 5. It can be observed from Eq. (6) that obtaining the optimum $\boldsymbol{\xi}$ will require a multidimensional search over the parameter space to determine the values of α_n, ϵ_1, and ϵ_2. In the present study we have applied the probabilistic global search Lausanne algorithm[6] for global optimization.

Simulations were carried out to test the performance of our method with benchmarking algorithms proposed by Hibino *et al.* for $\kappa=1$ and $\kappa=2$, where κ is the number of harmonics. For $\kappa=1$, the six-sample algorithm by Hibino *et al.* is optimized for phase steps of $\alpha_n = \pi(n-3.5)/3$ and for $\kappa=2$, the nine-sample algorithm is optimized for phase steps of $\alpha_n = \pi(n-5)/2$, to minimize the second-order nonlinearity of the phase shift. To be able to make an objective comparison of the performances of the two algorithms, we have been careful in selecting exactly the same phase-step values for the method proposed in this Letter. The five-sample algorithm by Schmitt and Creath[3] and the seven-sample algorithm by de Groot[2] were not considered by us, since it has already been shown by Hibino *et al.* that their algorithm outperformed both these algorithms. Table 1 shows the values of peak-to-valley errors obtained in the computation of phase φ by our and Hibino's algorithms for $\kappa=1$ and $\kappa=2$ and for various values of error coefficients ϵ_1 and ϵ_2. The results show that an infinitesimally small error is obtained in the computation of phase by our method compared with that obtained by Hibino's algorithm. Since we used a stochastic algorithm to minimize the likelihood function, the error is sometimes even less when the nonlinearity increases; however, the order of error remains substantially small. An important point to note here is that for $\kappa=2$, even in the case of small values of ϵ_2, we observe significantly higher values of error in the computation of phase φ by Hibino's algorithm. Hence, the algorithm proposed by Hibino seems to become more sensitive to nonlinearity with an increase in the number of harmonics. In contrast, our method yields a similar order of error even with an increase in the number of harmonics. Another salient feature of our method is that it does not impose any restriction on the selection of phase-step values to minimize the influence of the second-order nonlinearity and harmonics.

Finally, the measurement being invariably sensitive to noise, it is important to study the robustness of our and Hibino's algorithms in the presence of noise. For doing this, we chose $\kappa=1$, $\epsilon_1=0.1$, and different values of ϵ_2. We performed 500 Monte Carlo simulations at each signal-to-noise ratio (SNR) to compute the mean square error in the estimation of phase φ at each pixel. Figure 1 shows that as nonlinearity increases, a constant error bias is seen to appear beyond a certain SNR in the estimation of phase by Hibino's algorithm. This is in contrast with our method, in which the error decreases with the increase of the SNR. In the presence of noise, our method still performs better than Hibino's algorithm.

Since Table 1 shows that our method is better than Hibino's nine-frame algorithm, we studied the performance of our nine-frame method of compensating for the influence of second-order harmonics and various orders of nonlinearities in the presence of noise. During the analysis we selected phase step α as 0.6283 rad (36°) and phase φ as 0.7854 rad (45°) in Eq. (1) and varied the SNR from 10 to 60 dB. Note

Table 1. Phase Error (in Radians) in Computation of Phase φ

		$N=6$, $K=1$				$N=9$, $K=2$	
ϵ_1	ϵ_2	Our Method	Hibino Algorithm	ϵ_1	ϵ_2	Our Method	Hibino Algorithm
0.1	0.00	7.49×10^{-8}	3.48×10^{-4}	0.1	0.01	2.12×10^{-4}	2.17×10^{-2}
0.0	0.20	6.58×10^{-8}	9.62×10^{-3}	0.0	0.05	1.32×10^{-4}	3.16×10^{-2}
0.1	0.20	1.26×10^{-8}	1.43×10^{-2}	0.1	0.05	2.94×10^{-6}	1.33×10^{-1}
0.0	0.40	3.35×10^{-8}	3.72×10^{-2}	0.0	0.10	9.97×10^{-6}	1.97×10^{-1}
0.1	0.40	3.34×10^{-8}	3.20×10^{-2}	0.1	0.10	1.46×10^{-5}	4.05×10^{-1}

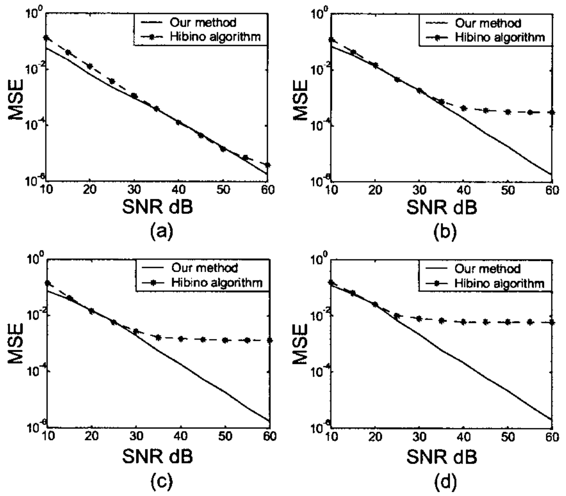

Fig. 1. Plots of mean square error (in rad^2) versus SNR for different values of nonlinearity coefficient ϵ_2: (a) 0.01, (b) 0.1, (c) 0.2, and (d) 0.4.

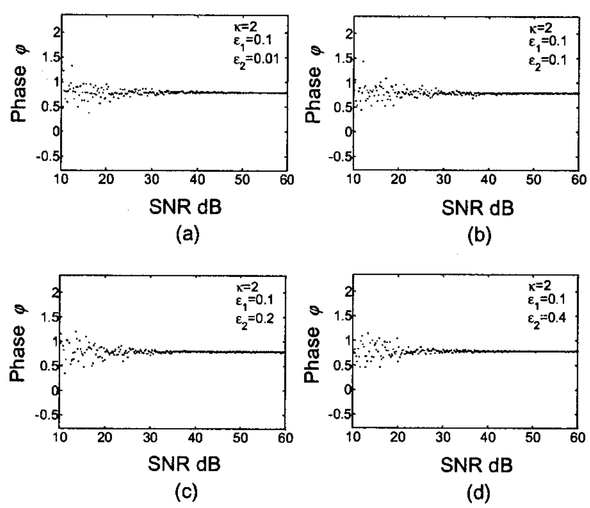

Fig. 2. Plots of the estimated phase (in radians) versus SNR for different values of nonlinearity coefficient ϵ_2 and $\kappa=2$ in Eq. (1).

that our method does not pose any restriction on the symmetricity of the selection of phase steps and hence is not sensitive to commonly occurring errors such as hysteresis. However, most of the nonlinear compensating algorithms[1–3] rely on the symmetricity of the phase steps. We thus estimate phase step α first, using the likelihood function defined in Eq. (6). Once the phase step is computed, the phase is evaluated using the well-known least-squares fit approach.[7] Figure 2 shows the scatterplots for computation of phase for $\kappa=2$, $N=9$, $\epsilon_1=0.1$, and $\epsilon_2=0.01, 0.1, 0.2, 0.4$ by use of our method. This analysis shows the feasibility of our method for working with arbitrary phase steps and higher-order harmonics in the presence of noise. The proposed method requires at least $2\kappa+4$ data frames for minimizing the κth-order harmonic. The time required for estimating the phase step at a pixel is approximately 50 ms on a desktop PC with a Pentium IV 3.2 GHz processor.

To conclude, we have proposed a novel method for estimating phase information pixelwise in an interferogram in the presence of harmonics and nonlinear phase steps. A salient feature of the method is that it does not impose restrictions on the choice of phase steps. Comparison with other well-known benchmarking algorithms has shown that our method by far outperforms these algorithms. In addition, our method does not impose any conditions on the symmetricity of the phase steps. Hence, the hysteresis effect does not play any significant role in the phase estimation. Although it is outside the scope of the present contribution, we believe that the proposed method has the potential to minimize errors in computing phase distributions in the presence of second-order phase step nonlinearity in configurations involving multiple PZTs.

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