

# Phase-shifting algorithms for nonlinear and spatially nonuniform phase shifts: reply to comment

**Kenichi Hibino**

*Mechanical Engineering Laboratory, Namiki, Tsukuba 305, Japan*

**Kieran G. Larkin**

*Canon Information Systems Research Australia, Pty Ltd, 1 Thomas Holt Drive, North Ryde, NSW 2113, Australia,  
and Department of Physical Optics, School of Physics, University of Sydney, NSW 2006, Australia*

**Bob F. Oreb and David I. Farrant**

*Commonwealth Scientific and Industrial Research Organisation, Division of Telecommunications and Industrial Physics, Lindfield, NSW 2070, Australia*

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Two complementary methods have now been proposed for deriving phase-shifting algorithms that correct for both nonlinear and spatially nonuniform phase shifts. Some advantages of symmetrical algorithms are outlined, and a simple method for ensuring algorithmic symmetry is presented. Noise susceptibility of the algorithms and the phase-shifter calibration are also discussed briefly. © 1998 Optical Society of America [S0740-3232(98)01505-1]

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## INTRODUCTION

We comment on a number of statements made in a recent paper by Surrel.<sup>1</sup> It is our belief that Surrel, in his paper on characteristic polynomials,<sup>2</sup> has made a substantial contribution by developing an elegant method for algorithm design. We welcome the extensions and observations he has made in his current comment because it offers an opportunity for both him and us to clarify certain points in his and our previous papers and to present some useful additional information.

### Comment 1: Error in Our References

There is an error in the numbering of the references in our paper under discussion.<sup>3</sup> Only the first reference is correctly numbered: all other references are misnumbered by one. The references appearing on page 930 should be renumbered 2–20 instead of 3–21. This error may lead one to conclude, incorrectly, that we have not referred to a number of significant results in important papers.

### Comment 2: Algorithm Coefficient Symmetry

In Section 4.A of Surrel's paper<sup>1</sup> the general solution to the nonuniform linear miscalibration error is derived in Eq. (22) and described as being equivalent to setting the reference phase-shifter position in the middle of the range. This is equivalent to our explanation,<sup>3</sup> on page 928 following Eq. (66) where we state, "Therefore an algorithm with the symmetries always satisfies property 1."

Algorithm symmetry can be advantageous for the efficient computation of phase.<sup>1</sup> Another consideration is that the symmetric algorithm can be considered the canonical form (in that there is only one symmetric form and an infinite number of asymmetric forms in principle). The original exposition of rules for generating algorithms from the characteristic polynomial<sup>2</sup> omitted a symmetry rule. Since then an additional Hermitian condition<sup>1</sup> has been presented along with the corresponding procedure for ensuring such symmetry in the coefficients. However, the form presented is directly applicable only to the generation of coefficients that use the polynomial characteristic method. A more general form of the complementary rule required for symmetrizing preexisting algorithms is presented here. Although this rule is relatively simple to derive, it has not, to our knowledge, appeared in the literature and is unlikely to be well known. The particular form presented here is equivalent to the Hermitian condition, and its application to any given algorithm is quite straightforward.

The general method can be stated for an asymmetric algorithm with phase  $\phi$  of the following form:

$$\tan[\phi(x, y)] = \frac{\sum_{n=-N}^N s_n I(x, y, \alpha - n\delta)}{\sum_{n=-N}^N c_n I(x, y, \alpha - n\delta)}. \quad (1)$$

Here the interferogram intensity is  $I(x, y, \alpha)$ , where the spatial coordinates are  $(x, y)$  and the phase-shift param-

eter is  $\alpha$ . We have assumed the interferogram to be sampled at intervals determined by  $\delta$ . The algorithm coefficients ( $c$  and  $s$ ) can be made symmetrical functions of the index  $n$  by a phase shift  $\theta$  such that

$$\tan[\phi(x, y) + \theta] = \frac{\sum_{n=-N}^N s_n' I(x, y, \alpha - n\delta)}{\sum_{n=-N}^N c_n' I(x, y, \alpha - n\delta)}. \quad (2)$$

The index parameter  $n$  is conveniently chosen to exist over a symmetric range, so, for example, a five-sample algorithm would utilize  $n = -2, -1, 0, +1, +2$ , whereas a four-sample algorithm would have  $n = -3, -1, +1, +3$ . If the numerator and denominator satisfy the usual condition (which ensures algorithm phase quadrature and magnitude matching)

$$s_n^2 + c_n^2 = s_{-n}^2 + c_{-n}^2, \quad (3)$$

then the required phase shift can be calculated from the following equation:

$$\tan \theta = -\frac{s_n + s_{-n}}{c_n + c_{-n}} = \frac{c_n - c_{-n}}{s_n - s_{-n}}. \quad (4)$$

Finally, the required symmetrical algorithm coefficients can be simply written as:

$$s_n' = \{c_n - c_{-n}\}c_n + \{s_n - s_{-n}\}s_n \quad (5a)$$

$$c_n' = \{s_n - s_{-n}\}c_n - \{c_n - c_{-n}\}s_n. \quad (5b)$$

#### Comment 3: Random Noise of the Nonuniform Algorithm

It is often the case that as the compensation capability (for systematic phase-shift errors) of an algorithm improves, the susceptibility of the algorithm to random noise increases.<sup>4</sup> Our six-frame algorithm [Eq. (39) in Ref. 3] yields a relatively poor signal-to-noise ratio, partly because it has an additional symmetry (over and above that required for the quadratic nonlinearity) for nonuniform phase shift and partly because the total phase shift of the algorithm is limited to less than  $2\pi$  for applications such as liquid-crystal phase modulation. However, if we increase the number of frames while keeping the compen-

sation capability constant, we can decrease the random noise as much as we need to. It will be shown in our next paper that the random noise of an  $N$ -frame nonuniform algorithm can be designed to decrease in proportion to  $3.8/\sqrt{N}$ , while the synchronous algorithm yields  $2/\sqrt{N}$ .

The relative "efficiency," or signal-to-noise parameter  $\eta$ , defined by Surrel<sup>1</sup> should be viewed in this context, although it must be conceded that  $\eta < 0.5$  for the nonuniform algorithms we have presented.

#### Comment 4: Calibration of Nonuniform Phase Shift

Spatially nonuniform phase shifts occur in several cases, such as a piezo-electric transducer-driven mirror in which the mirror rotates or distorts during the phase shift, liquid-crystal phase modulators, or phase-shifting speckle interferometry in which the visibility of fringes varies spatially. It is important to note here that the variation of fringe visibility is approximately equivalent to a nonlinear phase shift.

Measuring and correcting the phase shift is an alternative and complementary procedure to the error-compensating algorithms in phase-shifting interferometry. However, measuring the phase-shift error can be very difficult, especially when we need to detect it with an accuracy greater than the random noise or when the repeatability of the phase shifter is poor. In some cases in which we measure the spatial derivatives of a surface curvature or the elastic strains of deformation by interferometry, the systematic errors of the fringe signal should be much less than the random noise since the systematic errors are accumulated in the successive spatial integration process to yield the final object shape or deformation.

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