Comment on "Phase-shift extraction and wave-front reconstruction in phase-shifting interferometry with arbitrary phase steps"

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We comment on the recent Letter by Cai *et al.* [Opt. Lett. **28,** 1808 (2003)] in which an approach to phase-shifting interferometry with arbitrary phase steps was proposed. Cai *et al.* based their method of phase shifting on the idea that the intensities of the reference and object beams can be measured previously, which actually makes the whole posterior phase-shifting procedure absolutely unnecessary. Their method is also based on the statement that the phase of the Fresnel diffraction pattern of a test object if generally a spatially random distribution, which in most situations is wrong.

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In a recent Letter Cai *et al.*[1] presented an approach to reconstructing the object wave front in phase-shifting interferometry with arbitrary phase steps. They assume that an object is at a certain distance (e.g., a distance z) from the recording plane P_H, and they write the object wave in plane P_H (i.e., the Fresnel pattern at P_H) as $O(x, y) = A_0(x, y)\exp[i\varphi_0(x, y)]$. The reference wave in this plane at the jth exposure (step) is written as $R_j(x, y) = A_r \exp(i\delta_j)$. The intensity distribution of the jth interferogram can be written as

$$I_j(x,y) = A_0^2(x,y) + A_r^2 + 2A_0(x,y)A_r \cos[\varphi_0(x,y) - \delta_j]. \quad (1)$$

Hence, in phase-shifting interferometry, at least three interferograms are required for the reconstruction of $\varphi_0(x, y)$, because there are three variables in Eq. (1).[2]

We have some critical remarks about the Letter by Cai *et al.* First, after Eq. (2), Cai *et al.* remarked that intensities $I_0 = A_0^2$ and $I_r = A_r^2$ can be measured, and afterwards they used the measured values of I_0 and I_r as an essential part of their phase-shifting procedure. It is true that in some situations I_0 and I_r can be measured, but then only one interferogram (and not three interferograms) would be necessary to reconstruct $\varphi_0(x, y)$. In other words, if I_0 and I_r were measured independently of the interferograms (i.e., assuming that I_0 and I_r are time independent), then the whole phase-shifting procedure becomes unnecessary.[2]

Second, after Eq. (3), Cai *et al.* stated that "because $\varphi_0(x, y)$ is generally a spatially random distribution because of Fresnel diffraction, we can reasonably expect

$$\langle|\sin[\varphi_0(x,y) - (\delta_j + \delta_{j+1})/2]|\rangle = \langle|\sin \varphi_0(x,y)|\rangle = 2/\pi \quad (4)$$

regardless of the values of the constants δ_j and δ_{j+1}, where $\langle\ \rangle$ means average over the whole space." In general, this statement is wrong. It is clear that generally the phase in the Fresnel diffraction pattern of an object is not a "spatially random distribution," and the examples are trivial. For example, the phase in the Fresnel diffraction pattern of a pointlike object placed at the origin of coordinates is of the form $\varphi_0(x, y) = \pi(x^2 + y^2)/\lambda z$ (with λ being the wavelength and z being the distance to P_H), and it is not a spatially random distribution. Another similar expression is obtained for the phase of an object with a quadratic phase profile (i.e., a lens is the paraxial approximation) illuminated by a plane wave, and so on. Practically all examples of Fresnel diffraction patterns found in textbooks are not spatially random distributions. Thus the applicability of the statement is restricted to speckle patterns.

Furthermore, their statement that the "average is over the whole space" in order to justify taking $\langle|\sin \varphi_0(x, y)|\rangle = 2/\pi$ is absolutely unrealistic. Actually, the average can be taken over only the area of the interferogram (e.g., the surface of the digital camera used to record the interferogram). In the trivial examples mentioned above, clearly $\langle|\sin \varphi_0(x, y)|\rangle \neq 2/\pi$ for most average regions. Obviously, the exact values of $\langle|\sin \varphi_0(x, y)|\rangle$ depend on the exact analytical form of the phase and on the integration region. In the computer simulations shown by Cai *et al.* a conveniently gradually decreasing Gaussian intensity distribution with a certain fluctuation Δh was added to the real phase object (a spherical surface), and thus perhaps then the proposed procedure works.

Third, at the beginning of the second column on page 1809 [after Eq. (12)] Cai *et al.* said that "when the object field $O(x, y)$ in plane P_H is found this way, the optical phase in the actual object plane, $O'(x', y')$, can be computed with inverse Fresnel diffraction." Thus, according to the method described by Cai *et al.*, after reconstructing the phase $O(x, y)$ with their "phase-shifting procedure," one must also compute the actual phase $O'(x', y')$ by a numerical inverse Fresnel transformation. Everybody who is working in Fourier optics knows that such a procedure is highly inefficient and time consuming for practical applications. The surprising issue is why Cai *et al.* did not use a lens (or a lens system) to image the

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actual phase $O'(x', y')$ onto plane P_H where the interferogram is acquired. In this way the numerical inverse Fresnel transformation would be unnecessary.

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References

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2. E. Greivenkamp and J. H. Bruning, in *Optical Shop Testing*, D. Malacara, ed. (Wiley, New York, 1992), pp. 501–598.