Generalized phase-shifting interferometry by use of a direct stochastic algorithm for global search

Abhijit Patil, Benny Raphael, and Pramod Rastogi

Applied Computing and Mechanics Laboratory, Swiss Federal Institute of Technology, 1015 Lausanne, Switzerland

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A new phase-shifting interferometric technique that uses an unknown phase step is described in which the phase step is determined by use of an algorithm called Probabilistic Global Search Lausanne (PGSL). One of the main sources of error in phase stepping is piezoelectric device (PZT) nonlinearity. The PGSL algorithm identifies the characteristics of the response of the PZT to the applied voltage through matching predicted and measured responses. The unknown phase step is also calculated with 0.097% error. This approach overcomes the limitations of existing techniques to determine unknown phase steps. Linear regression is subsequently applied for interference phase determination. © 2004 Optical Society of America

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In phase-shifting interferometry that uses phase-stepping techniques, the phase difference between an object and the reference beam is varied conventionally either in known phase steps or in unknown but constant phase steps.[1] The generalized data reduction algorithm for the extraction of interference phase distribution by the principle of least-squares estimation exhibits the possibility of using arbitrary phase steps.[2,3] Nevertheless, exact estimation of the phase step is imperative for the error-immune calculation of the interference phase in both conventional and generalized phase shifting. The phase-stepping errors primarily originate from piezoelectric device (PZT) nonlinearity and are one of the main sources of statistical errors in phase-shifting interferometry.[1]

Various self-calibrating algorithms for conventional phase-stepping techniques have been reported to minimize these error sources.[4,5] For generalized phase-stepping techniques the exact estimation of reference phase steps by the generation of parallel Fizeau fringes has limitations in working with fringes that are not straight and equally spaced.[6,7] The methods suggested for extracting arbitrary but constant phase steps utilize computational resources.[8–10] Incidentally, the first algorithm suggested by Carré,[11] although offering the possibility of using arbitrary phase steps, has limitations in the selection of phase steps to overcome various error sources.[12]

We propose a generalized phase-stepping technique that uses a direct stochastic algorithm for a global search called Probabilistic Global Search Lausanne[13] (PGSL). A direct search method is a numerical optimization algorithm that uses the function to be optimized (called the objective function) only for determining which points are better than the others. These methods do not require the objective function to be expressed in an explicit mathematical form, and derivatives are not required. Tests carried out on complex nonlinear objective functions indicate that PGSL performs better than genetic and simulated annealing algorithms. PGSL determines the global minimum of the least-squares error objective function for the interference fringes at randomly selected pixels of the acquired data frames. The errors due to PZT nonlinearity are computed with this algorithm, and the true phase step is estimated, averaged over the selected data points, and subsequently applied in the linear-regression technique for the estimation of interference phase distribution. Although PGSL is a powerful optimization tool for complex objective functions,[13] its use is precluded for the estimation of interference phase distribution since this objective function is linear once the exact phase step is determined and linear regression is more efficient. One optimization is needed for each pixel, and a total of 100 optimizations are carried out. It requires approximately 1 min to carry out all the optimizations and the subsequent linear-regression technique at 512×512 pixels on a desktop PC with a Pentium IV processor.

A simple mathematical representation of the recorded interference fringe intensity at pixel (x, y) of the data frame is given by

$$I_n(x,y) = I_0(x,y)\{1 + \gamma(x,y)\cos[\varphi(x,y) + n\alpha]\}, \quad n = 0, 1, 2 \ldots K - 1, \tag{1}$$

where, after omitting (x, y) for the sake of simplicity, I_n, I_0, γ, φ, and α represent the intensity, local background intensity, fringe visibility, phase that contains the desired information, and phase step, respectively, of the pixel at (x, y) of the nth data frame. For the K sample phase-stepping algorithm the true phase shift $\alpha_n{}^*(x,y)$ due to the nonlinear characteristics of the PZT can be represented as[14]

$$\alpha_n{}^*(x,y) = n\alpha(x,y) + \varepsilon_1 n\alpha(x,y) + \varepsilon_2 \frac{[n\alpha(x,y)]^2}{2\pi} + \varepsilon_3 \frac{[n\alpha(x,y)]^3}{4\pi^2} + \varepsilon_4 \frac{[n\alpha(x,y)]^4}{8\pi^3}, \tag{2}$$

where $\alpha_n{}^*(x,y)$ is a polynomial representation of the fourth order for the phase shift, ε_1 represents the linear error, and ε_2, ε_3, and ε_4 represent the nonlinear errors in the phase shift. The simple equation for interference fringes represented by Eq. (1) is corrupted by the error sources and thus can be represented by

$$I_n(x,y) = I_0(x,y)\{1 + \gamma(x,y)\cos[\varphi(x,y) + \alpha_n{}^*(x,y)]\}, \quad n = 0, 1, 2 \ldots K - 1. \tag{3}$$

With Eq. (3) an expression for residual error $\Pi(x, y)$ at each pixel location (x, y) can be written as

$$\Pi(x,y) = \sum_{n=0}^{K-1} (I_n(x,y) - I_0(x,y)\{1 + \gamma(x,y)\cos[\varphi(x,y) + {\alpha_n}^*(x,y)]\})^2. \quad (4)$$

We employ PGSL to determine exact step ${\alpha_n}^*(x, y)$ by computing the global minimum of the least-squares error objective function $\Pi(x, y)$ defined in Eq. (4). PGSL is a direct search technique that employs random sampling with a probability density function (PDF) to locate the global minimum of a user-defined objective function. At the beginning of the search a uniform PDF is assumed; the PDF is updated dynamically as search progresses, such that more-intensive sampling is performed in regions where good solutions are found. In describing PGSL, "solution point" refers to a point consisting of a set of values for each variable (in our case I_0, γ, φ, α, ε_1, ε_2, ε_3, and ε_4), and "search space" refers to the set of all potential solution points, that is, all possible combinations of values of variables defined in the solution point within permissible ranges.

The PGSL algorithm carefully samples the search space through four nested cycles, namely, the sampling cycle, probability-updating cycle, focusing cycle, and subdomain cycle. Each cycle serves a different purpose in the search for the global optimum. The sampling cycle permits a more uniform search over the entire search space. The probability-updating and focusing cycles refine the search in the neighborhood of good solutions. Convergence to the optimum solution is achieved by means of the subdomain cycle. For more details on PGSL the reader is referred to Ref. 13.

To apply PGSL to estimating variables I_0, γ, φ, α, ε_1, ε_2, ε_3, and ε_4, the bounds for each variable must be defined. Initially, PGSL generates random values for each variable within the bounds. These points are evaluated with residual error function $\Pi(x, y)$. The probability-updating algorithm of PGSL uses the value of the error function to modify the PDF. The probability of each variable is increased in regions where residual errors are small. The probability-updating algorithm ensures that more points are generated in good regions without ignoring other regions altogether. This process continues until convergence to a minimum of $\Pi(x, y)$ is achieved.

To test the PGSL algorithm, it was applied to computer-generated interference fringes. Ten data frames were generated with a step value α of 40°. It was assumed that the phase shift is constant over the entire interferogram. The bounds for the variables in Eq. (3) were defined as

$$0 \le I_0 \le \frac{I_{\max} - I_{\min}}{2}, \qquad 0 \le \gamma \le 1,$$

$$0 \le \varphi \le 2\pi, \qquad 0 \le \alpha \le \pi,$$

where $I_{\max}$ and $I_{\min}$ are the maximum and minimum values, respectively, of the intensity at point (x, y) of the generated interference fringes. For 8-bit data, $I_{\max} = 255$ and $I_{\min} = 0$. Fringe visibility γ was generated randomly between 0 and 1. From the observation for smooth objective functions, the values of the PGSL parameters described in Ref. 13 were selected as NS = 2, NPUC = 1, NFC = $60N$, and NSDC = 60, where N is the number of variables in the function. The results of applications of PGSL for the determination of an unknown phase step at 1000 randomly selected pixel locations are shown in Fig. 1(a) (dashed curve). During this simulation, it was assumed that the PZT has a linear response, and error coefficients ε_1, ε_2, and ε_3 were considered to be zero.

Figure 1(a) shows that optimization over six data frames is sufficient to estimate the phase step. The average step value is estimated to be 40.0008° with 0.002% error. The step value can be estimated with higher precision if more frames are acquired. Simulation results show that when 11 or 12 frames are acquired the error in the calculation of the phase step is 0.00025%. However, a trade-off is made among the number of frames, the computational time required, and the accuracy desired. Acquisition of a large number of frames makes the interference phase measurement prone to air turbulence, temperature fluctuations, and laser drift.

In the second simulation, error coefficients ε_1, ε_2, and ε_3 were considered to have magnitudes as high as 5%, 4%, and 3%, respectively. Figures 1(a) and 1(b) show that the step value and error coefficients can be estimated exactly when approximately nine data frames are acquired. The average step value was estimated with a 0.097% error. Error coefficients were found to be 5.099%, 4.031%, and 3.003%. Hence the simulation results prove that, if the objective function is correctly modeled, PGSL can identify the characteristics of the system. In our case the algorithm characterized the simulated nonlinear behavior of the PZT to the applied voltage.

The subsequent step after estimation of the exact phase step is to apply the least-squares method (linear regression) outlined by Morgan[2] and Greivenkamp[3] for the determination of interference phase distribution. The equation for interference can be written as

$$I_n(x,y) = \sum_{n=0}^{K-1} [I_0(x,y) + \hbar(x,y)\cos {\alpha_{n\mathrm{AVG}}}^* + \lambda\!\!\!^{-}(x,y)\sin {\alpha_{n\mathrm{AVG}}}^*], \quad (5)$$

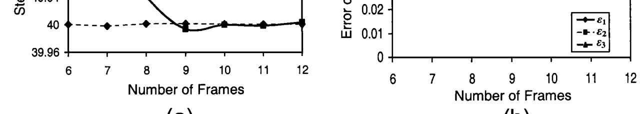

Fig. 1. (a) Phase step (in degrees) versus number of frames. (b) Error coefficients (in percent) versus number of frames. Note that the number of frames is a discrete quantity, and the solid and dashed curves are added only to show the trend of convergence.

where $\hbar(x,y) = I_0(x,y)\gamma(x,y)\cos[\varphi(x,y)]$, $\lambdabar(x,y) = -I_0(x,y)\gamma(x,y)\sin[\varphi(x,y)]$, and $\alpha_{n\mathrm{AVG}}{}^*$ is obtained from Eq. (2) with values of α, ε_1, ε_2, and ε_3 averaged over the nine data frames that were used in the optimization. The interference phase distribution can be determined by $\varphi(x,y) = \tan^{-1}[-\lambdabar(x,y)/\hbar(x,y)]$. Since Eq. (5) is linear with respect to unknown coefficients $\hbar(x,y)$ and $\lambdabar(x,y)$, the linear-regression technique can be used to minimize the error. The maximum error that can occur in the phase step by use of the averaged values of α, ε_1, ε_2, and ε_3 obtained from PGSL is 0.082% in the ninth data frame in the linear-regression technique. Error function $\Pi(x,y)$ can hence be written as

$$\Pi(x,y) = \sum_{n=0}^{K-1}[I_0(x,y) + \hbar(x,y)\cos\alpha_{n\mathrm{AVG}}{}^* + \lambdabar(x,y)\sin\alpha_{n\mathrm{AVG}}{}^* - I_n(x,y)]^2. \tag{6}$$

For the best fit the linear-regression function in Eq. (6) should be minimized [min $\Pi(x,y)$]. This is done by obtaining the first derivative of the residual error function with respect to unknown coefficients $I_0(x,y)$, $\hbar(x,y)$, and $\lambdabar(x,y)$ and setting them equal to zero. The three equations resulting from first-order derivatives can be written in matrix form as

$$\begin{bmatrix} K & \sum_{n=0}^{K-1}\cos\alpha_{n\mathrm{AVG}}{}^* & \sum_{n=0}^{K-1}\sin\alpha_{n\mathrm{AVG}}{}^* \\ \sum_{n=0}^{K-1}\cos\alpha_{n\mathrm{AVG}}{}^* & \sum_{n=0}^{K-1}\cos^2\alpha_{n\mathrm{AVG}}{}^* & \sum_{n=0}^{K-1}\sin\alpha_{n\mathrm{AVG}}{}^*\cos\alpha_{n\mathrm{AVG}}{}^* \\ \sum_{n=0}^{K-1}\sin\alpha_{n\mathrm{AVG}}{}^* & \sum_{n=0}^{K-1}\sin\alpha_{n\mathrm{AVG}}{}^*\cos\alpha_{n\mathrm{AVG}}{}^* & \sum_{n=0}^{K-1}\sin^2\alpha_{n\mathrm{AVG}}{}^* \end{bmatrix} \begin{bmatrix} I_0(x,y) \\ \hbar(x,y) \\ \lambdabar(x,y) \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{K-1} I_K \\ \sum_{n=0}^{K-1} I_K\cos\alpha_{n\mathrm{AVG}}{}^* \\ \sum_{n=0}^{K-1} I_k\sin\alpha_{n\mathrm{AVG}}{}^* \end{bmatrix}. \tag{7}$$

If the above matrix is not ill-conditioned, the solution to the matrix results in a wrapped phase map that is modulo 2π because of the arctangent operator in the calculation of the phase. Phase demodulation techniques can be used to remove the π phase ambiguities between the adjacent pixels in the wrapped phase map to yield interference phase $\varphi(x,y)$. Figure 2(a) shows the wrapped phase function $\varphi(x,y)$ obtained with the linear-regression technique with and without using PGSL for the determination of $\alpha_{n\mathrm{AVG}}{}^*$. The true wrapped phase is not shown in Fig. 2(a) because the map almost coincides with the results obtained with PGSL. Typical errors in the computation of a true unwrapped phase map with and without PZT compensation are shown in Figs. 2(b) and 2(c), respectively.

To conclude, PGSL appears to be a practical tool in phase-shifting interferometry, with its ability to estimate phase step accurately without any additional optical setup. The number of frames acquired is also fewer compared with previously suggested methods. This technique is complemented well by the Gaussian

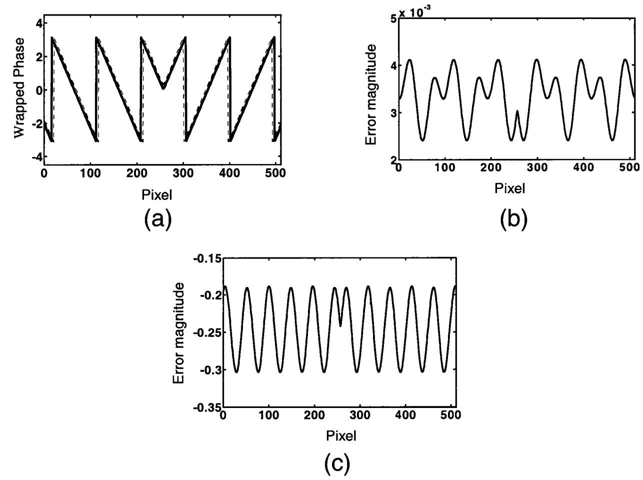

Fig. 2. (a) Wrapped phase map (in radians) with (solid curve) and without (dashed curve) PZT nonlinearity compensation. Error in phase φ (b) with PGSL for the estimation of reference phase steps and (c) without PGSL (phase step 40° and error coefficients equal to zero).

least-squares error fit technique for subsequent measurement of phase in interferometry.

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