# Robust Self Tuning algorithm for phase shifting interferometry; an improvement to the advanced iterative algorithm

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In this work, we develop a regularization technique to demodulate a phase-shifting interferometry sequence with arbitrary inter-frame phase shifts. With this method, we can recover the modulating phase and inter-frame phase shifts in the same process. As all phase-shifting algorithms, the assumption is that the wave front under test does not change over time, but in this case the introduction of phase-shifts can vary in a non constant way. Another feature of this demodulation method is that it is able to find the modulated phase from interferograms with poor visibility since they have non constant background illumination in their fringes. This advantage is a significant improvement to the already published method advanced iterative algorithm (AIA), which is sensible to variations in the background illumination. To see the performance of the demodulation technique developed herein, we will show numerical experimental results and comparisons with the AIA method.


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## 1. Introduction

Nowadays, Phase Shifting Interferometry (PSI) techniques are some of the most used techniques in optical metrology [4]. In PSI, one obtains an small sequence of at least 3 interferograms with a phase-shift among them [4]. To recover the modulating phase, there are standard demodulation PSI methods, the well known 3-, 4-, and 5- step phase-shifting algorithms. Knowing the inter-frame phase-shifts (or temporal carrier), the standard methods recover the $2\pi$ modulus phase map with the minimum possible error [4, 5, 13]. If we do not know the phase-shifts exactly, we obtain a phase map with an unavoidable detuning error whose magnitude depends on the number of interferograms employed and on how far we are from the actual phase-shifts [1, 7, 12, 13]. This unfortunate case can occur when the optical interferometer setup is uncalibrated or when perturbations from the environment affect the interferometer's optical path. For example, for most phase shifters, such as a piezoelectric (PZT), there is a repeatability problem arising from hysteresis, non linearity and temperature linear drift [1,3]; curiously, the first phase-shifting algorithms were self-tuning nonlinear algorithms [2, 15]. To reduce detuning errors, other approaches propose error compensating algorithms that basically use redundant data, such as the Schwider-Hariharan 5- step algorithm, [7,14,16] and more recently, the 9-step algorithm shown in Ref. [6] has been used to do it by constructing a wide-band frequency response of the phase-shifting algorithm. Further methods use the Fourier transform in order to estimate the inter-frame phase-shifts, and others are based on the least-squares scheme, iteratively estimating the inter-frame phase-shifts and phase [10, 11]. Ref. [8] presentes an approach that estimates the local temporal carrier (the phase-shift) as the average of the phase difference between two consecutive phase maps obtained from two realizations of the tunable 3-step algorithm. What we are going to show in this work is a regularized self-tuning demodulation technique that obtains the analytical image (complex interferogram) and inter-frame phase-shifts from an interferogram sequence. Thus, we can recover the modulating phase $2\pi$ modulus and the inter-frame phase shifts in the same process. Here, it is not necessary to know the inter-frame phase-shifts. These inter-frame phase-shifts can vary arbitrarily.

The main difference between the demodulation method presented here and the ones reported in [10], [17] and [9] is that our demodulation method is based on a regularization technique that is robust to non constant modulation variations and non constant background illumination, which is an issue that introduces errors in the methods in works [10] and [17]. Besides, we do not require to estimate the fringe orientation, as the method in work [9].

## 2. Method

In general, an interferogram sequence with arbitrary inter-frame phase-shifts can be modeled as:

$$I^k_{x,y} = a_{x,y} + b_{x,y}cos(\phi_{x,y} + \alpha_k),\ k = 0, 1, 2, ..., L - 1, \tag{1}$$

where $I^k_{x,y}$ is the intensity at the site $x, y$ of the $k$-interferogram in a sequence of $L - 1$ interferograms, being $a_{x,y}$ its background illumination, $b_{x,y}$ its contrast, $\phi_{x,y}$ the modulating phase under test and $\alpha_k$ the phase-shift of the $k$-interferogram.

*2.A. Least-squares method*

As we can see from Eq. (1), conventional phase-shifting algorithms assumes that the background illumination and the modulation amplitude do not have frame-to-frame variation; i.e., they are functions of pixels only. Defining a new set of variables as $\varphi_{x,y} = b_{x,y}cos(\phi_{x,y})$, $\psi_{x,y} = b_{x,y}sin(\phi_{x,y})$, $C_k = cos(\alpha_k)$, $S_k = sin(\alpha_k)$ we can express Eq. (1) as

$$I^k_{x,y} = a_{x,y} + \varphi_{x,y}C_k - \psi_{x,y}S_k,\ k = 0, 1, 2, ..., L-1. \tag{2}$$

If we known $\alpha_k$, there are $3MN$ unknowns (where $M$ and $N$ are the interferogram dimension). These unknowns can be solved using least-squares method. An energy cost function that described above can be written as

$$U(a_{x,y}, \varphi_{x,y}, \psi_{x,y}) = \sum_{k=0}^{L-1} [a_{x,y} + \varphi_{x,y}C_k - \psi_{x,y}S_k - I'_{x,y,k}]^2, \tag{3}$$

where $I'_{x,y,k}$ is the $k$-th experimentally measured intensity of the interferogram sequence. For the known $\alpha_k$, the least-squares criteria require to make zero the gradient of Eq. (3) as

$$\nabla U(a_{x,y}, \varphi_{x,y}, \psi_{x,y}) = 0. \tag{4}$$

Eq. (4) yield

$$X = A^{-1}B, \tag{5}$$

where

$$A = \begin{bmatrix} M \times N & \sum C_k & \sum S_k \\ \sum C_k & \sum C_k^2 & \sum C_k S_k \\ \sum S_k & \sum S_k C_k & \sum S_k^2 \end{bmatrix}, \tag{6}$$

$$B = \begin{bmatrix} \sum I'_{x,y,k} & \sum I'_{x,y,k}C_k & \sum I'_{x,y,k}S_k \end{bmatrix}^T, \tag{7}$$

$$X = \begin{bmatrix} a_{x,y} & \varphi_{x,y} & \psi_{x,y} \end{bmatrix}^T, \tag{8}$$

where the sum $\sum$ runs over $k = 0, 1, 2, ..., L-1$. To ensure that $A$ is nonsingular Eq. (5) requires at least three different phase-steps $\alpha_k$. From Eqs. (6)-(8) the phase $\phi$ at the point $x, y$ can be determined from

$$\phi_{x,y} = \arctan(-\psi_{x,y}/\varphi_{x,y}). \tag{9}$$

Note that the inverse of A is performed only once because its components depend only on the steps $\alpha_k$.

*2.B. Robust self-tuning method*

To determine the steps we propose the following regularized cost functional

$$
\begin{aligned}
U(a_{x,y}, C^k_{x,y}, S^k_{x,y}) \;&=\; \sum_{x,y}\sum_{k=0}^{L-1}\sum_{m=-1}^{1}\sum_{n=-1}^{1}\Big[a_{x,y} + \varphi_{x+m,y+n}C^k_{x,y} - \psi_{x+m,y+n}S^k_{x,y} - I'_{x+m,y+n,k}\Big]^2 \\
&\quad + \sum_{x,y}\sum_{k=0}^{L-1}\left[\lambda\frac{\nabla[a_{x,y}]}{L} + \mu\nabla[C^k_{x,y}] + \mu\nabla[S^k_{x,y}]\right]^2, \qquad (10)
\end{aligned}
$$

where $\lambda$ and $\mu$ are the regularization parameters that controls the smoothness of $a_{x,y}$, $C^k_{x,y}$ and $S^k_{x,y}$. Operator $\nabla[*]$ takes the first order differences along $x$ and $y$ direction, as follows:

$$
\nabla[f_{x,y}] = [a_{x,y} - a_{x-1,y}, a_{x,y} - a_{x,y-1}]^T. \qquad (11)
$$

To find our unkown steps we need to minimize the cost functional in Eq 10. Equating to zero the partial gradient with respect to $a_{x,y}$, $C^k_{x,y}$ and $S^k_{x,y}$ and solving, we obtain a closed formula for iterative computing the background illumination and the phase-shift steps, as follows:

$$
a_{x,y} = \frac{\sum_{k=0}^{L-1}\sum_{m=-1}^{1}\sum_{n=-1}^{1}\Big[-\varphi_{x+m,y+n}C^k_{x,y} + \psi_{x+m,y+n}S^k_{x,y} + I'_{x+m,y+n,k}\Big]}{9L - a_{x-1,y} - a_{x,y-1}}, \qquad (12)
$$

$$
C^k_{x,y} = \frac{\sum_{m=-1}^{1}\sum_{n=-1}^{1}\Big[-a_{x,y}\varphi_{x+m,y+n} + \psi_{x+m,y+n}\varphi_{x+m,y+n}S^k_{x,y} + I'_{x+m,y+n,k}\varphi_{x+m,y+n}\Big]}{\sum_{m=-1}^{1}\sum_{n=-1}^{1}\varphi^2_{x+m,y+n} - C^k_{x-1,y} - C^k_{x,y-1}}, \qquad (13)
$$

$$
S^k_{x,y} = \frac{\sum_{m=-1}^{1}\sum_{n=-1}^{1}\Big[a_{x,y}\psi_{x+m,y+n} + \psi_{x+m,y+n}\varphi_{x+m,y+n}C^k_{x,y} - I'_{x+m,y+n,k}\psi_{x+m,y+n}\Big]}{\sum_{m=-1}^{1}\sum_{n=-1}^{1}\psi^2_{x+m,y+n} - S^k_{x-1,y} - S^k_{x,y-1}}. \qquad (14)
$$

As we see from Eqs 13 and 14 we can comput the phase-shift in every site of each interferogram. Then to obtain the phase-shift step in each interferogram

$$
\alpha_k = \arctan(-S^k_m / C^k_m), \qquad (15)
$$

where $S^k_m$ and $C^k_m$ are the mode, most frequent value, calculated for each interferogram. Now, to determine the modulated phase we use the least squares method and the proposed method alternately until the values of the phase-shifts converge to a desired value.


Fig. 1. Interferogram sequence used for testing the algorithm

## 3. Numerical Experiments and Results

To test the robust self tuning algorithm developed here, we simulated 50 interferogram sequences with different temporal frequecies. Each sequence has five interferograms. An example of these interferogram sequences is shown in Fig.1 . These simulated interferograms have poor fringe visibility since we have added a non constant background illumination and contrast. To reproduce the interferogram sequences that we used in these tests, the reader can use the Eq. 1 wiht the following parameters:

$$a_{x,y} = -\frac{(x-256)^2+(y-256)^2}{1.5259x10^{-5}}, \tag{16}$$

$$b_{x,y} = e^{-\frac{(x-256)^2+(y-256)^2}{100^2}}, \tag{17}$$

$$\phi_{x,y} = 30(-1+\frac{x}{2}-x^5-y^3)e^{(-x^2-y^2)}+\frac{4\pi x}{512}+\frac{4\pi y}{512} \tag{18}$$

and

$$\alpha_k = k\frac{\pi}{3}+\eta_k,\ k=0,1,..,5, \tag{19}$$

where $\eta_k$ is a random number whit mean zero and variance of 0.5 that chages for each interferogram. In our numerical experiments we start always the temporal carrier $\alpha_k = 0,1,..,5$. Regularization parameters $\lambda$ and $\mu$ were set at 100 and 500 respectively (see Eq. 10). Using the Gauss-Seidel method to find the phase-steps (Eqs. 14 15 and 16) were required 50 iterations. Finally, for estimating the phase in the AIA and our proposed method were required 20 iterations.

In Table. 2 , we show the errors values of the estimated phase-shifts using our regularized method and the estimated using the AIA method. This errors are calculated as $|\alpha_k - \hat{\alpha}_k|$, where $\alpha_k$ and $\hat{\alpha}_k$ are the values of the actual and estimated phase-shifts for the $k-$frame, respectively. On the other hand, in Fig. 2 , we show the recovered phase. In Fig. 2(a) is shown the recovered phase using the regularized self-tuning demodulation method presented here, while in Fig. 2(b) is shown the recovered phase using the AIA method. We can see in this figure that our proposed regularized self-tuning demodulation method recovers the phase with less error than with the AIA method. The errors shown in Fig. 2(a) and Fig. 2(b) are calculated as the variance between the recovered phase map and the true phase map used to generate the interferograms.

| Steps | Real | AIA | RST |
|---|---|---|---|
| $\alpha_0$ | 0 | 0 | 0 |
| $\alpha_1$ | 1.6953 | 2.2511 | 1.6994 |
| $\alpha_2$ | 0.6961 | 0.6284 | 0.6769 |
| $\alpha_3$ | 3.3038 | 3.2427 | 3.3031 |
| $\alpha_4$ | 4.0793 | 4.2983 | 4.0678 |
| $e^{ave}$ | | 0.1807 | 0.0071 |

Table 1. Numerical results. Comparison between the phase-shift estimation and the actual phase-shift for the AIA and the proposed method, and the phase-shift average error.

Finally, Table 2 shows the average error of 50 iteration, with the same parameters listed above. As seen from these results, the proposed method is better than the AIA.

| | AIA | RST |
|---|---|---|
| Step-error | 0.1902 | 0.0153 |
| Phase-error | $4.57\text{x}10^{-2}$ | $5.11\text{x}10^{-5}$ |

Table 2. This table compares the average error obtained in 50 random tests for the proposed method and the AIA method. The first row shows the average error of estimated steps, while the second shows the variance between the estimated phase map and the used to generate the sequence of interferograms.

## 4. Conclusions

We have presented a regularized self-tuning phase-shifting demodulation method for interferogram sequences having arbitrary variations of the inter-frame phase-shifts. This method


(a) RST Phase error = $7.0570\text{x}10^{-6}$.

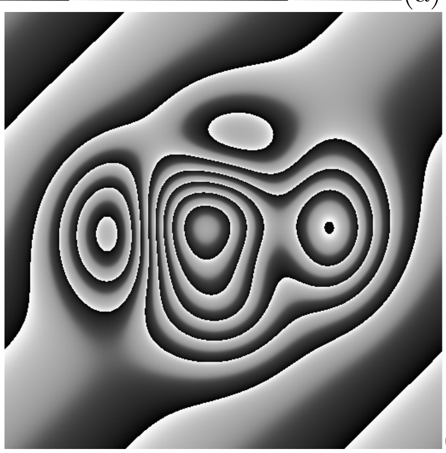
(b) AIA Phase error = 0.0608.

Fig. 2. Phase Comparison.(a) shows the recovered phase and error using the regularized self-tuning (RST) method proposed here. (b) shows the recovered phase and error using the AIA method. The error shown (in radians) is the variance respecting the true phase map. The interferogram frames has a size of 512 x 512.

is robust to non constant background illumination. As shown in the results, our demodulation method is able to recover the modulating phase and the inter-frame phase-shifts with a minimum error. The demodulation method presented here provides stable convergence and accurate phase demodulation with as few as three interferograms, even when the phase-shifts are completely random.

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