A Robust and Fast Active Contour Model for Image Segmentation with Intensity Inhomogeneity

Keyan Ding and Guirong Weng
School of Mechanical and Electric Engineering, Soochow University, China

ABSTRACT

In this paper, a robust and fast active contour model is proposed for image segmentation in the presence of intensity inhomogeneity. By introducing the local image intensities fitting functions before the evolution of curve, the proposed model can effectively segment images with intensity inhomogeneity. And the computation cost is low because the fitting functions do not need to be updated in each iteration. Experiments have shown that the proposed model has a higher segmentation efficiency compared to some well-known active contour models based on local region fitting energy. In addition, the proposed model is robust to initialization, which allows the initial level set function to be a small constant function.

Keywords: Image segmentation, intensity inhomogeneity, active contour model, level set method, robust initialization.

1. INTRODUCTION

Image segmentation is a fundamental task in image analysis and recognition. Active contour models have been widely applied in image segmentation since the presentation by Kass *et al.* [1]. They can achieve sub-pixel accuracy of object boundaries and obtain a smooth and closed contour. Existing active contour models can be roughly categorized into two basic classes: *edge-based* models [2–4] and *region-based* models [5–11]. Edge-based models often use an edge indicator to drive the curve towards the object boundaries, such as GAC model [2, 3]. Chan–Vese model [6] is a famous region-based model and has been widely applied in the practice application. But Chan–Vese model cannot work well for images with intensity inhomogeneity due to the assumption that the intensities of image are statistically homogeneous.

In order to handle the intensity inhomogeneity which is often occurred in medical and natural images, Li *et al.* [7] presented an active contour model based on region-scalable fitting (RSF) energy. The RSF model draws upon the local image information by a kernel function. With the information of local image intensities, the RSF model can effectively segment images with intensity inhomogeneity. But when the initial contour is set inappropriately, the RSF model will be stuck in local minima because the its energy functional is non-convex [12]. In addition, the segmentation efficiency of RSF model is relatively low due to the computation of four convolutions in each iteration. Zhang *et al.* [8] presented an active contour model driven by local image fitting (LIF) energy. Compared to the RSF model, the computational cost of LIF model is smaller because only two convolutions need to be computed in each iteration. However, the problem of initialization has not been solved. Liu *et al.* [9] proposed a local region-based Chan–Vese (LRCV) model. Similarly, the segmentation efficiency is higher than RSF model. But it is also sensitive to initialization. Wang *et al.* [10] presented an active contour model based on local Gaussian distribution fitting (LGDF) energy. It defines a local Gaussian distribution fitting energy by using local means and variances as variables. The LGDF model is able to distinguish regions with similar intensity means but different variances. But the problem of initialization is still unsolved and the segmentation efficiency is relatively low due to the extra computation of variances.

In this paper, we present a robust and fast active contour model based on local intensities fitting energy, which is more robust to the choice of initial contour and has a lower computational cost compared to traditional local fitting-based models, such as the RSF model. We first define two local fitting functions according to the average intensities in a local region. And an energy based on these fitting functions is proposed to drive the curve towards the boundaries of objects. Then, we incorporate this energy into a variational level set formulation, and add a distance regularized term to avoid reinitialization and a length term to smooth and shorten the contours. Finally, we use the steepest descent method to minimize the entire energy functional. Experiments on synthetic and real images have shown that the proposed model can achieve accurate and efficient segementation in the presence of intensity inhomogeneity.

2. THE RELATED WORKS

2.1 Region-Scalable Fitting Model

Li *et al.* [7] proposed a region-scalable fitting (RSF) model for segmenting images with intensity inhomogeneity. They define the following energy functional:

$$\begin{aligned} E^{RSF}(\phi, f_1, f_2) = \lambda_1 \int_\Omega \left(\int_\Omega K_\sigma(x-y)\,|I(y)-f_1(x)|^2\, H_\varepsilon(\phi(y))dy\right)dx \\ +\lambda_2 \int_\Omega \left(\int_\Omega K_\sigma(x-y)\,|I(y)-f_2(x)|^2\,[1-H_\varepsilon(\phi(y))]dy\right)dx \\ +\upsilon \int_\Omega \delta_\varepsilon(\phi(x))\left|\nabla\phi(x)\right|dx + \mu\int_\Omega \frac{1}{2}(\left|\nabla\phi(x)\right|-1)^2 dx \end{aligned} \tag{1}$$

where, $x, y \in \Omega$, λ_1, λ_2, v and μ are positive constants. K_σ is a Gaussian kernel function with standard deviation σ. $f_1(x)$ and $f_2(x)$ are two smooth functions that approximate the intensities of image outside and inside the contour C in a local region, respectively. $H_\varepsilon(x)$ and $\delta_\varepsilon(x)$ are regularized Heaviside and Dirac function defined by

$$\begin{cases} H_\varepsilon(x) = \dfrac{1}{2}(1+\dfrac{2}{\pi}\arctan(\dfrac{x}{\varepsilon})) \\ \delta_\varepsilon(x) = \dfrac{\varepsilon}{\pi(\varepsilon^2+x^2)} \end{cases} \tag{2}$$

The method of steepest descent is used to minimize the energy functional (1). Keeping level set function ϕ fixed and minimizing E^{RSF} with respect to f_1 and f_2, the following formulations can be obtained:

$$\begin{cases} f_1(x) = \dfrac{\int_\Omega K_\sigma(x-y)[H_\varepsilon(\phi(y))\cdot I(y)]dy}{\int_\Omega K_\sigma(x-y)H_\varepsilon(\phi(y)dy} \\ f_2(x) = \dfrac{\int_\Omega K_\sigma(x-y)[(1-H_\varepsilon(\phi(y))\cdot I(y)]dy}{\int_\Omega K_\sigma(x-y)[1-H_\varepsilon(\phi(x)]dy} \end{cases} \tag{3}$$

Keeping f_1 and f_2 fixed and minimizing E^{RSF} with respect to the level set function ϕ, the following gradient descent flow can be obtained:

$$\frac{\partial\phi}{\partial t} = -\delta_\varepsilon(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + v\delta_\varepsilon(\phi)div(\frac{\nabla\phi}{|\nabla\phi|}) + \mu(\nabla^2\phi - div(\frac{\nabla\phi}{|\nabla\phi|})) \tag{4}$$

where e_1 and e_2 are

$$\begin{cases} e_1(x) = \int_\Omega K_\sigma(y-x)\,|I(x)-f_1(y)|^2\, dy \\ e_2(x) = \int_\Omega K_\sigma(y-x)\,|I(x)-f_2(y)|^2\, dy \end{cases} \tag{5}$$

2.2 Local Image Fitting Model

Zhang *et al.* [8] presented an active contour model driven by local image fitting (LIF) energy. This energy functional is defined by minimizing the difference between the fitted image and the original image:

$$E^{LIF}(\phi, m_1, m_2) = \frac{1}{2}\int_\Omega \left|I(x) - I^{fit}(x)\right|^2 dx \tag{6}$$

where I^{fit} is the local fitting image defined as follows:

$$I^{fit}(x) = m_1(x)H_\varepsilon(\phi(x)) + m_2(x)[1-H_\varepsilon(\phi(x))] \tag{7}$$

with

$$\begin{cases} m_1(x) = mean(I(x): x \in \{\phi(x) < 0\} \cap \Omega_w(x)) \\ m_2(x) = mean(I(x): x \in \{\phi(x) > 0\} \cap \Omega_w(x)) \end{cases} \tag{8}$$

where Ω_w is a truncated Gaussian window whose size is $w \times w$ and the standard deviation is σ. The means $m_1(x)$ and $m_2(x)$ can be seen as the weighted averages of the image intensities in a Gaussian window outside and inside the contour, respectively. Thus, $m_1(x)$ and $m_2(x)$ can be expressed with a level set formulation:

$$\begin{cases} m_1(x) = \dfrac{\int_\Omega K_\sigma(x-y)[H_\varepsilon(\phi(y)) \cdot I(y)]dy}{\int_\Omega K_\sigma(x-y) H_\varepsilon(\phi(y)dy} \\ m_2(x) = \dfrac{\int_\Omega K_\sigma(x-y)[(1-H_\varepsilon(\phi(y)) \cdot I(y)]dy}{\int_\Omega K_\sigma(x-y)[1-H_\varepsilon(\phi(x)]dy} \end{cases} \tag{9}$$

Minimizing the above energy functional by using the steepest descent method, the following variational formulation can be obtained:

$$\frac{\partial \phi}{\partial t} = (I - I_f)(m_1 - m_2)\delta_\varepsilon(\phi) \tag{10}$$

In [8], Gaussian filtering is used to regularize the level set function. It can free the procedure of reinitialization to some extent. In addition, $m_1(x)$ and $m_2(x)$ are totally the same to $f_1(x)$ and $f_2(x)$ in the RSF model.

3. THE PROPOSED MODEL

The In this section, we will present a robust and fast active contour model based on local intensities fitting energy. First, let $\Omega \subset R^2$ and $I(x)$: $\Omega \rightarrow R$ be the image domain and a given image, respectively. We define the following functions:

$$\begin{cases} f_m(x) = mean(I(y) \mid y \in \Omega_x) \\ f_a(x) = mean(I(y) \mid y \in \Omega_a) \\ f_b(x) = mean(I(y) \mid y \in \Omega_b) \end{cases} \tag{11}$$

where Ω_x represents a small neighborhood centered at x with radius r on the image domain Ω. Ω_a and Ω_b are defined by

$$\begin{cases} \Omega_a = \{y \mid I(y) < f_m(x)\} \cap \Omega_x \\ \Omega_b = \{y \mid I(y) > f_m(x)\} \cap \Omega_x \end{cases} \tag{12}$$

That means Ω_a is the region where the intensities smaller than the average intensities in Ω_x, and Ω_b is the region where the intensities larger than the average intensities in Ω_x. $f_m(x)$, $f_a(x)$ and $f_b(x)$ are the average intensities in Ω_x, Ω_a and Ω_b, respectively.

Next, we propose the following energy functional:

$$E(C) = \iint_\Omega | I(x) - f_1(C) - f_2(C) |^2 \, dx + |C| \tag{13}$$

where, C is a closed curve. $|C|$ represents the length of curve C. $f_1(C)$ and $f_2(C)$ are

$$\begin{cases} f_1(C) = f_a(x), \ x \in \{Outside(C) \cap \Omega\} \\ f_2(C) = f_b(x), \ x \in \{Inside(C) \cap \Omega\} \end{cases} \tag{14}$$

where, $f_a(x)$ and $f_b(x)$ are defined in (11).

The energy $E(C)$ in (13) can be represented by a level set formulation. Using the zero-level set of a Lipschitz function ϕ to represent the curve C, the energy functional (13) can be rewritten as:

$$E(\phi,b)=\iint_{\Omega}|I-f_a(x)\times H(\phi)-f_b(x)\times(1-H(\phi))|^2\,dxdy+\iint_{\Omega}\delta(\phi)\left|\nabla\phi\right|dxdy \tag{15}$$

where, $H_\varepsilon(x)$ and $\delta_\varepsilon(x)$ are defined in (2). In addition, we add a regularized distance term [4] to avoid re-initialization in the proposed model. Thus, the entire energy functional of proposed model can be expressed as follows:

$$\begin{aligned}E^*(\phi,b)=&\iint_{\Omega}|I-f_a(x)\times H_\varepsilon(\phi)-f_b(x)\times(1-H_\varepsilon(\phi))|^2\,dxdy\\&+\upsilon\iint_{\Omega}\delta_\varepsilon(\phi)\left|\nabla\phi\right|dxdy+\mu\iint_{\Omega}\frac{1}{2}(\left|\nabla\phi\right|-1)^2\,dxdy\end{aligned} \tag{16}$$

where, the parameters v and μ are the weight of length term and distance regularized term, respectively.

Using the steepest descent method to minimize the energy functional (16) with respect to the level set function ϕ, the following gradient descent flow equation can be obtained:

$$\begin{aligned}\frac{\partial\phi}{\partial t}=&2\delta_\varepsilon(\phi)\left(I-f_a(x)\times H_\varepsilon(\phi)-f_b(x)\times(1-H_\varepsilon(\phi))\right)\times\left(f_a(x)-f_b(x)\right)\\&+v\delta_\varepsilon(\phi)div(\frac{\nabla\phi}{|\nabla\phi|})+\mu(\Delta\phi-div(\frac{\nabla\phi}{|\nabla\phi|}))\end{aligned} \tag{17}$$

Remark 1. According to (11) and (12), for a given Ω_x, the functions $f_a(x)$ and $f_b(x)$ can be calculated directly. For instance, Fig. 1 shows the region Ω_x, Ω_a and Ω_b, and the values of $f_a(x)$ and $f_b(x)$ on an edge point x.

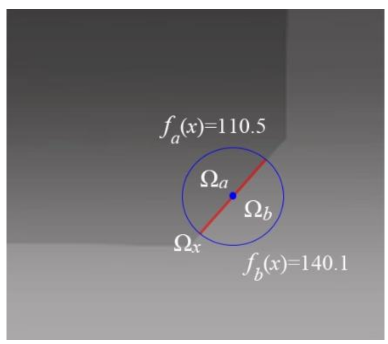

Figure 1. Illustration of the region Ω_x, Ω_a and Ω_b, and the values of $f_a(x)$ and $f_b(x)$ on an edge point x. The bule line represents the region Ω_x with center point x. The red line represents the boundary of Ω_a and Ω_b.

In the traditional local fitting-based active contour models, such as the RSF model, the fitting functions are related to the level set function ϕ. Thus, they need to be updated in each iteration, which is a time-consuming process. But in the proposed model, because the functions $f_a(x)$ and $f_b(x)$ are not involved with ϕ, they do not need to update in each evolution and be calculated only once at the beginning of curve evolution. In addition, it is because $f_a(x)$ and $f_b(x)$ are invariant functions that the proposed energy is not easy to fall into local minima. Therefore, the proposed model can achieve fast segmentation and be robust to initialization.

4. EXPERIMENTAL RESULTS

4.1 Implementation

The evolution equation of level set in (17) is implemented by a simple finite difference method. The spatial partial derivatives $\partial\phi/\partial x$ and $\partial\phi/\partial y$ are discretized as central difference and the temporal partial derivative $\partial\phi/\partial t$ is discretized as forward difference. Thus, Equation (17) can be rewritten as

$$\phi_{i,j}^{k+1} = \phi_{i,j}^{k} + \Delta t \times L(\phi_{i,j}^{k}) \tag{18}$$

where (i, j) is the spatial index, k is the temporal index, Δt is the time step, and $L(\phi_{i,j}^{k})$ is the approximation of the right-hand side in (17) by using the above spatial difference scheme. In the proposed model, the initial level set function ϕ_0 can be set to a small constant function $\phi_0(x) = c$. Unless otherwise specified, the following parameters are used in the proposed model: c=1, ε=1, μ=10, υ=0.01×255^2, Δt=0.01 and Ω_x is a circle window with radius r = 7. The proposed model is implemented in Matlab R2012b on a 2.0-GHz Inter Core i5 computer.

4.2 Segmentation Results of Proposed Model

Fig. 2 shows the segmentation results of proposed model for some synthetic and medical images. Note, the initial contours are invisible because ϕ_0 is a constant function. Every row of Fig. 2 represents the process of curve evolution from initial contour to final contour. Fig. 2A−C are a synthetic image with typical intensity inhomogeneity. Fig. 2D is a vessel image with weak adges and inhomogeneous background. Fig. 2E is a tubal angiography image with weak edges. Fig. 2F−J are natural images, which exist intensity inhomogeneity, texture or strong noises. For this image, the parameter υ = 0.02×255^2. The last column of Fig. 2 demonstrates that our model can obtain the satisfactory segmentation results for these images. (The values of some parameters are shown in Table 1, unlisted parameters are defaults)

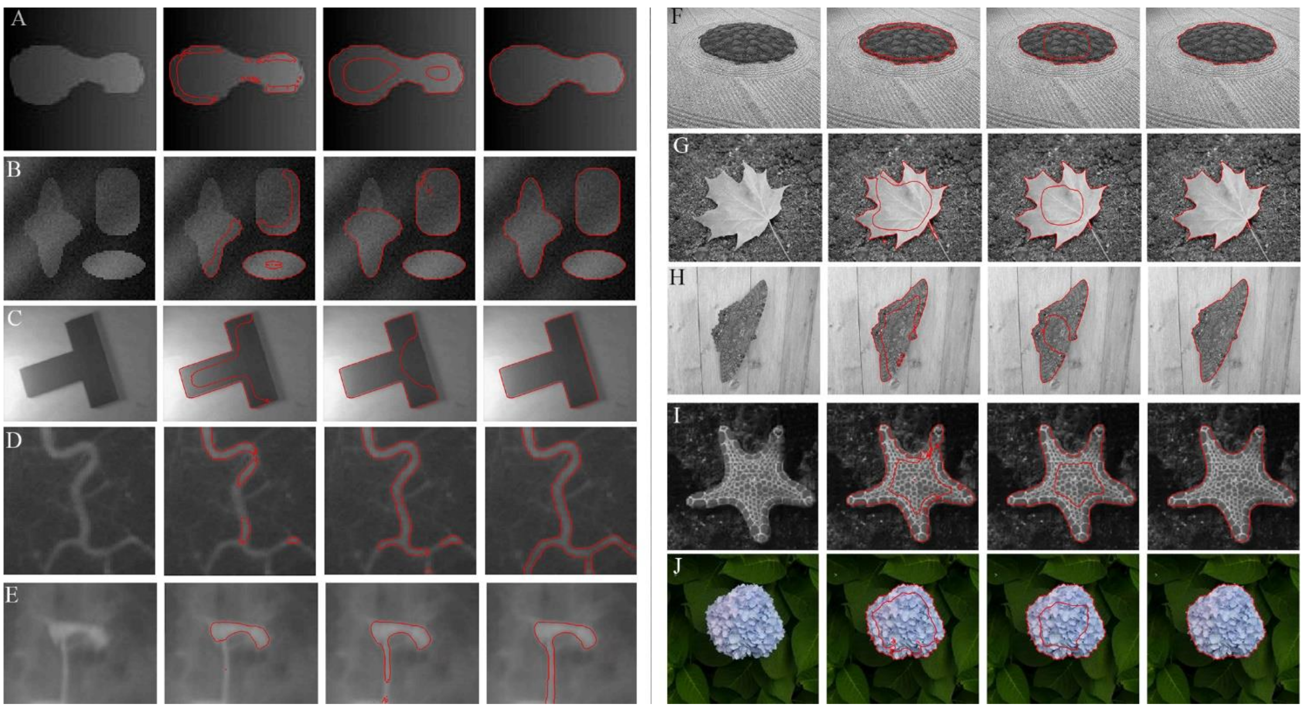

Figure 2. Segmentation results of the proposed method for some images. First column: original image. Second and third columns: intermediate segmentation results. Last column: final segmentation results.

Table 1: The values of parameters used in Fig. 2

Image	A	B	C	D	E	F	G	H	I	J
υ (×255^2)	0.01	0.01	0.01	0.005	0.01	0.03	0.1	0.05	0.1	0.05
r	7	7	7	7	7	7	10	7	15	7

4.3 Analysis of Robustness against Initial Contour

The proposed model is robust to initialization. In Fig. 3, we set four different initial contours in each image. From Fig. 3, we notice that all of the initial contours obtain a desired result in the proposed model. It demonstrates that the position and shape change of initial contour has no influence on the segmentation results in the proposed model. But in the local fitting-based models, i.e., the RSF model, the change of initial contour may cause an undesired result. Therefore, our model is more robust against the choice of initial contour. In fact, the initial level set function ϕ_0 can be set as a constant function in our model, as used in Fig. 2. In this way, the segmentation result can be obtained more conveniently because there is no need to consider the problem of initial contour.

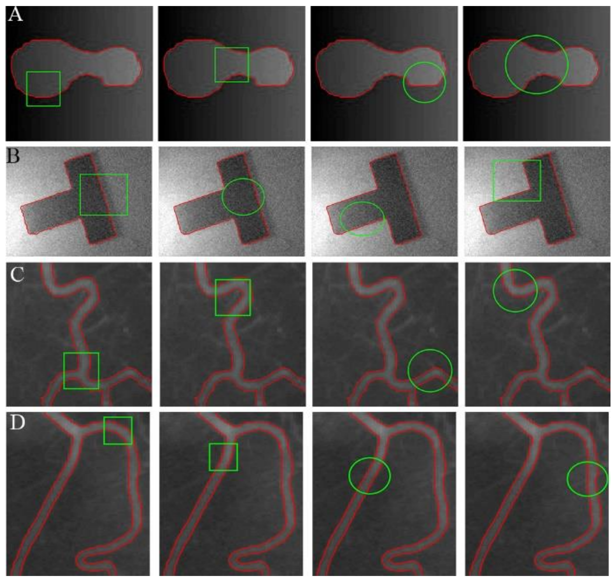

Figure 3. Segmentation results of proposed method with different initial contours for four different images.

4.4 Analysis of Computational Cost

The proposed model has a lower computational cost compared to traditional local fitting-based active contour models, such as the RSF model [7], the LIF model [8], the LRCV model [9] and the LGDF model [10]. Taking the images in Fig. 3 for example, we set the same initial contour. Table 2 shows the corresponding number of iterations and the time spent of RSF, LIF, LRCV, LGDF and proposed model. From Table 2, we notice that the time spent of our model is the least, and the number of iterations is the least in most cases. It means the proposed model has a higher segmentation efficiency than above models.

Table 2. The number of iterations and the time spent of RSF, LIF, LRCV, LGDF and proposed model for the images in Fig. 3. (Iterations / Time(s))

Image	RSF	LIF	LRCV	LGDF	Proposed
Fig. 3 A	80 / 0.734	100 / 0.649	100 / 0.613	145 / 1.321	60 / 0.408
Fig. 3 B	260 / 1.748	300 / 1.423	280 / 1.445	310 / 2.594	100 / 0.745
Fig. 3 C	100 / 0.692	130 / 0.701	140 / 0.606	130 / 1.091	35 / 0.204
Fig. 3 D	200 / 1.589	280 / 1.341	240 / 1.247	300 / 2.678	60 / 0.411

5. CONCLUSIONS

In this paper, we have presented a robust and fast active contour model for image segmentation in the presence of intensity inhomogeneity. By introducing the local image intensities fitting functions before the evolution of curve, the proposed model can effectively segment images with intensity inhomogeneity. The experimental results for some images have shown that the proposed model has a lower computational cost and a stronger robustness against initial contour than some traditional local fitting-based active contour models.

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