SEIIRS Epidemic Models and Their Equilibrium Points and Stability.

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SEIIRS Model:

$$\dot{S} = -\frac{\beta S(I_c + \omega I_{sc})}{N} - \delta R$$

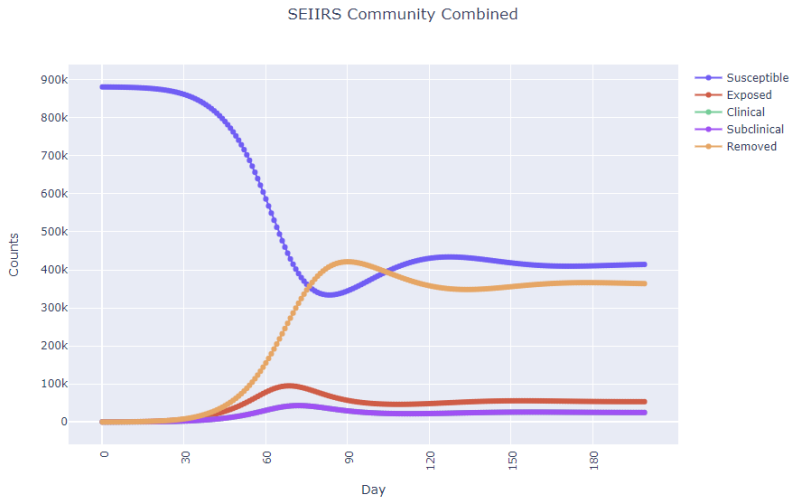
$$\dot{E} = \frac{\beta S(I_c + \omega I_{sc})}{N} - \kappa E$$

$$\dot{I_c} = \rho\kappa E - \gamma I_c$$

$$\dot{I_{sc}} = (1-\rho)E - \gamma I_{sc}$$

$$\dot{R} = \gamma(I_c + I_{sc}) - \delta R$$

$$N = S + E + I_c + I_{sc} + R$$



SEIIRS Simulation with Time-independent Parameters

Nondimensionalization Parameters:

$$\alpha = \frac{\beta}{\gamma}, \qquad \varepsilon = \frac{\kappa}{\gamma}, \qquad \eta = \frac{\delta}{\gamma},$$

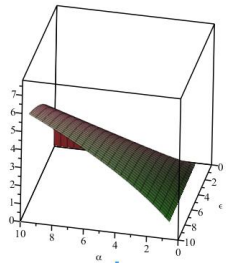
$$\rho = \rho, \qquad \omega = \omega, \qquad t^* = \frac{t}{\gamma}, \qquad \mathrm{N} = \mathrm{N}/\mathrm{N}$$

Objectives:

- Analyze equilibrium and stability characteristics of SEIIRS models.
- Develop a spatial SEIIRS model to better represent the effect of inter-community activities/lockdown.
- Analyze equilibrium and stability characteristics of the spatial model.

Results:

I. Analysis:



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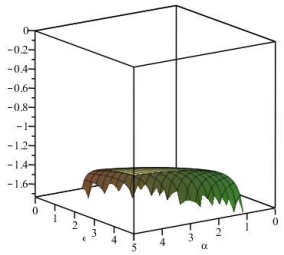
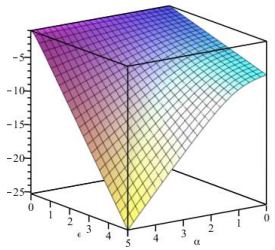
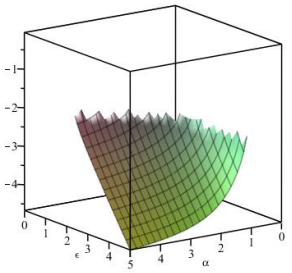




* Showing Fixed Eta



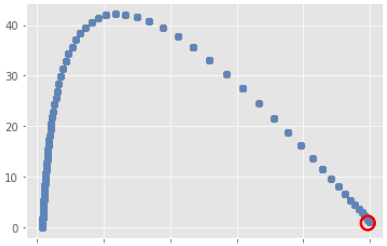
Alpha < 1
Stable **DFE**

$\lambda_{EE}1 = 0$ $\lambda_{EE}\ 2 < 0$ $\lambda_{EE}\ 3 < 0$ $\lambda_{EE}\ 4 < 0$

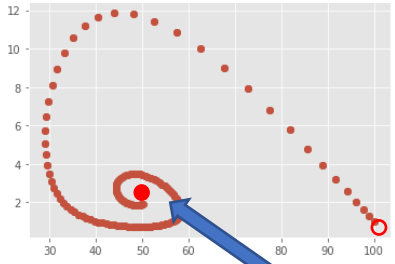
No positive λ for EE. in over 10,000 realistic combinations of parameters tested.

- EP$(S^*, E^*, I_c{}^*, I_{sc}{}^*, R^*)$: (1,0,0,0,0), $\left(-\frac{1}{\alpha(\omega\rho-\omega-\rho)}, E^*, \rho\varepsilon E^*, -\rho\varepsilon E^* + \varepsilon E^*, \frac{\varepsilon}{\eta}E^*\right)$
- E.E. is always stable and DFE is unstable when α > 1. (Realistic parameter ranges)
- Behavior Near the E.E.: Spiral Sink (Exists in all 2-D hyperplanes of SEIIRS)

SIR → SEIIRS

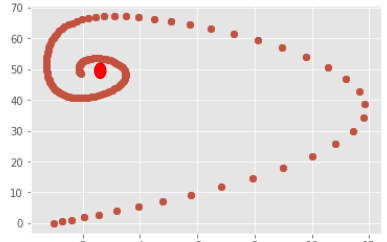


SI Plane

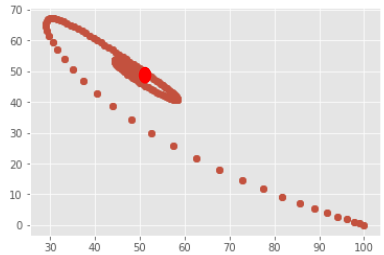


SI Orbit

Stable Attractor



IR Orbit

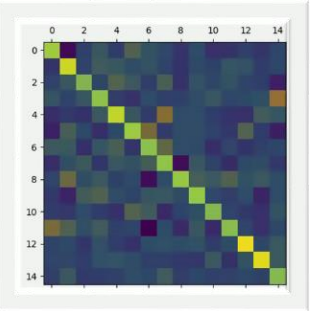


SR Orbit

II. Spatial Model:

$$\frac{S(I_c + \omega I_{sc})}{N}$$

$$\sum_j \tau_{ij} S_i \frac{\sum_k \tau_{kj} (I_k^c + \omega I_k^{sc})}{\sum_k \tau_{kj} N_{kj}}$$



τ_{ij} Activity Matrix

III. Spatial Model Analysis and Application in Covid-19:

Ongoing collaboration with Prof. Muller.