A Joint-chirp-rate-time-frequency Transform for Binary Black Hole Merger Signal Detection using Neural Network

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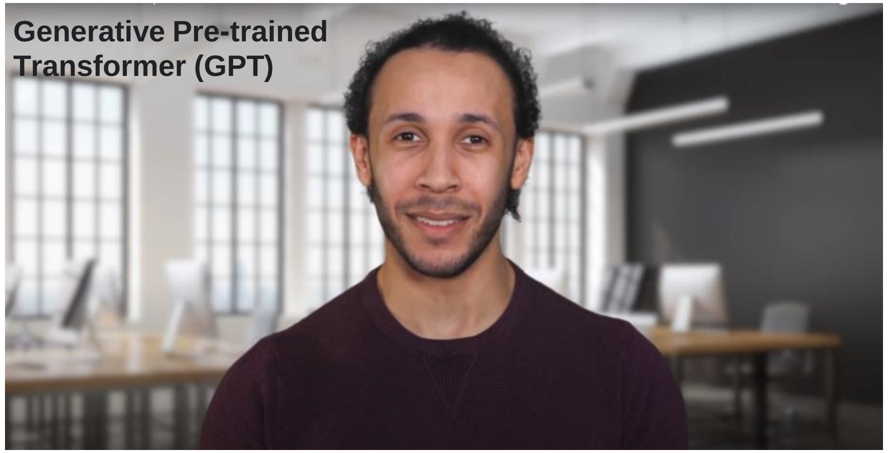
Motivation

- Machine Learning (ML) algorithms have transformed the methods of data analysis, image pattern recognition, and feature detection.
- Artificial Neural Networks (ANNs) are among the most talked about techniques in the ML family with a wide range of applications.
  - Applications of ANNs

Self-driving Cars*

*Side-by-side camera view and ANN annotated LIDAR data (Waymo)



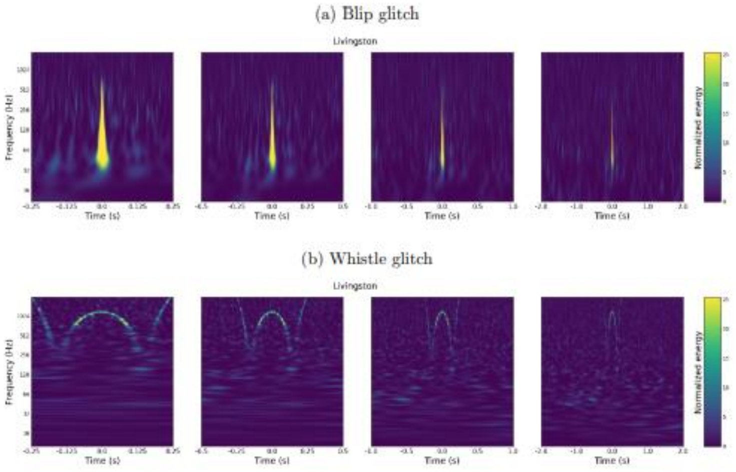
Natural Language Processing^

^AI chat robot with facial expressions, movements, and voice generated using GPT-3 (OpenAI)

What can ANNs do to accelerate Gravitational Wave (GW) research?

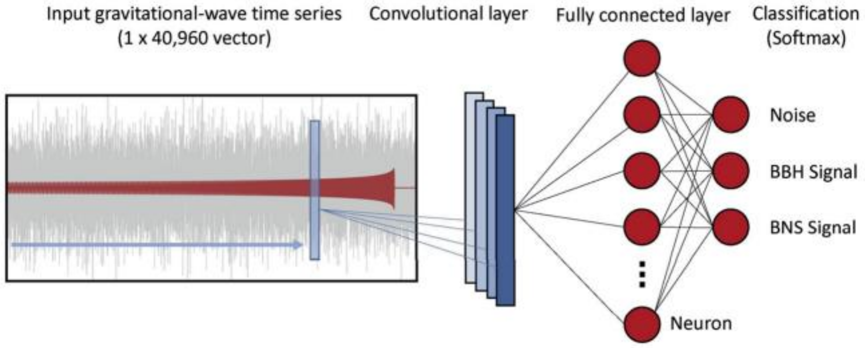
Motivation

- Active research areas:
  - Detector transient noise (glitch) classification.
  - Real-time Binary Black Hole (BBH) Binary Neutron Star (BNS) merger event detection.
  - BBH/BNS merger event forecasting.



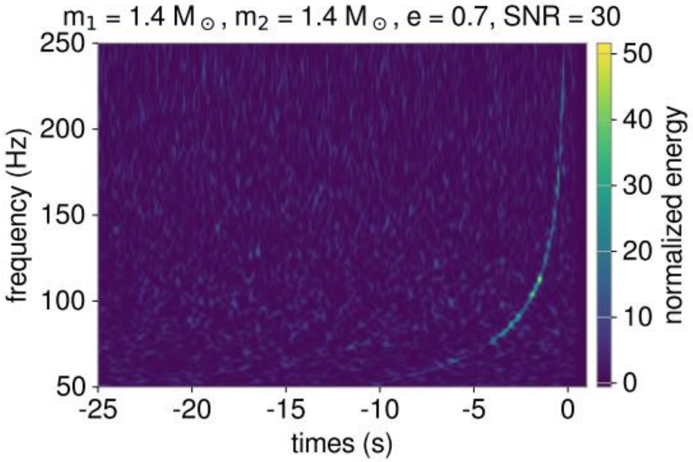
Glitch Classification*

***Spectrograms** of two types of glitches (Gravity Spy)



BBH/BNS Merger Detection^

^ Convolutional Neural Network (CNN) in merger event detection, classification. (Plamen G. Krastev)



BBH/BNS Merger Forecasting◦

◦ **Spectrogram** of a simulated BNS merger signal (W. Wei and et al.)

M. Zevin and et al., “Gravity spy: integrating advanced ligo detector characterization, machine learning, and citizen science,” Classical and Quantum Gravity, vol. 34, no. 6, p. 064003, 2017

Krastev, P. G. (2020). Real-time detection of gravitational waves from binary neutron stars using Artificial Neural Networks. *Physics Letters B*, *803*, 135330.

W. Wei and et al., “Deep learning with quantized neural networks for gravitational-wave forecasting of eccentric compact binary coalescence,” The Astrophysical Journal, vol. 919, no. 2, p. 82, 2021

Motivation

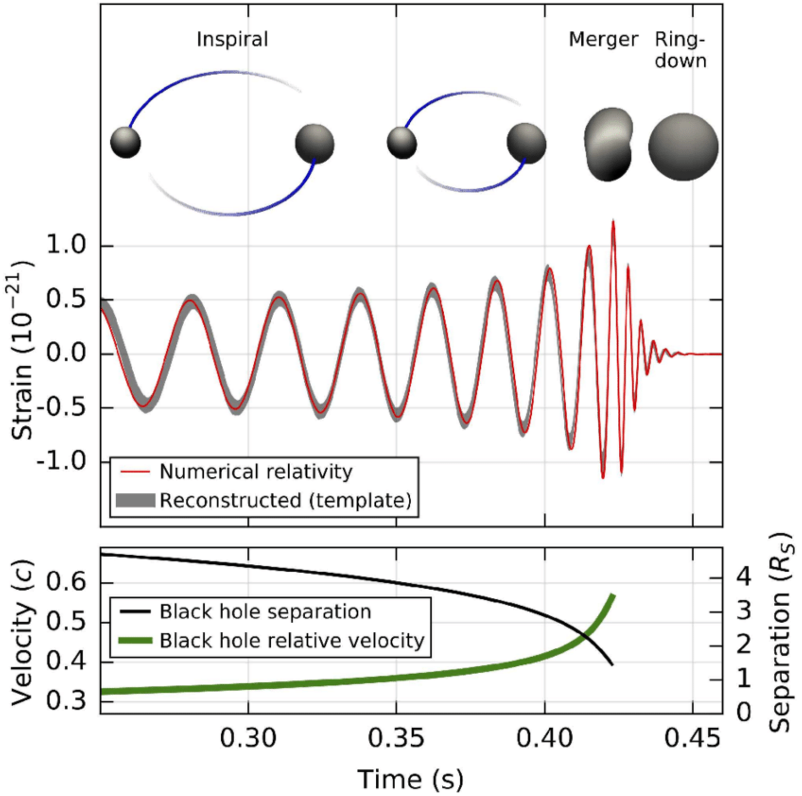
- Design a transform method that produces **chirp-rate enhanced spectrograms** to potentially improve spectrogram ANNs' performance in BBH, BNS merger signal **detection** and **forecasting**.

Current Detection Techniques

BBH Chirp Signal Detection

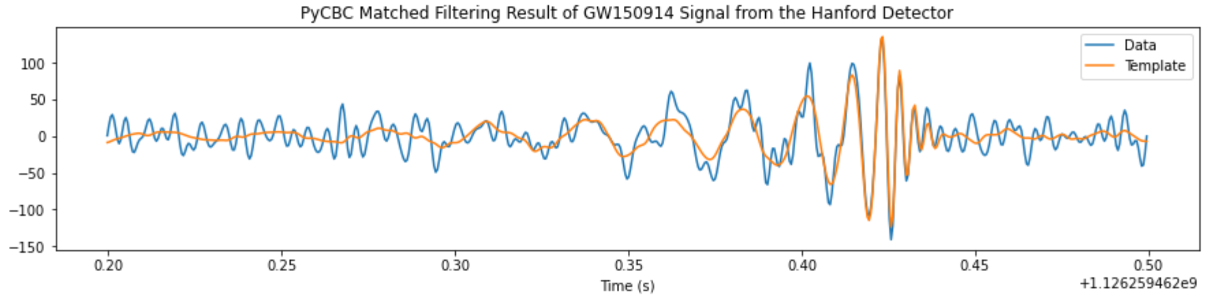
Chirp signal: ~ changing frequency

BBH Merger Process and Waveform



Francisco R. Villatoro (2018)

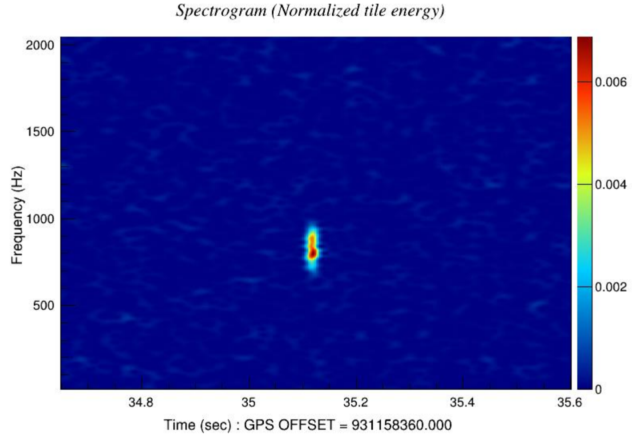
Technique 1: **Templated Search** – Matched Filtering



Hanford detector signal of BBH merger event GW150914 (September 14 2015, 09:50:45 UTC) plotted against the matched waveform template in PyCBC.

Technique 2: **Non-templated Search** – Burst Search

Spectrogram Generation



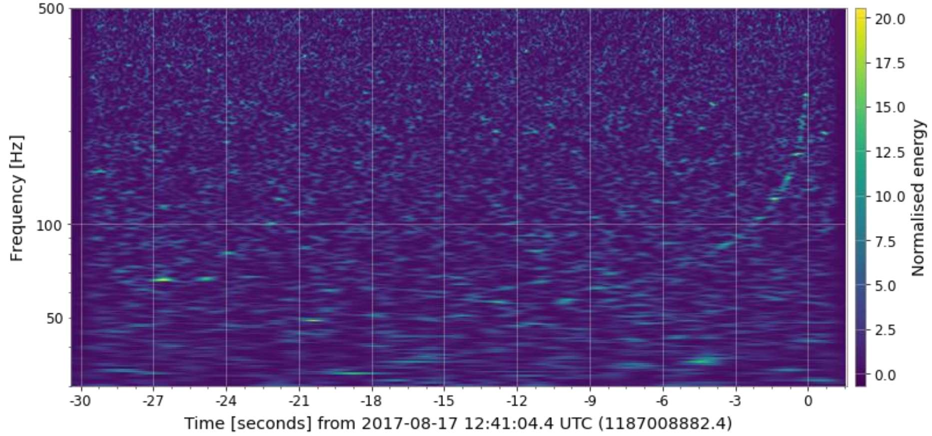
The spectrogram of an unknown event recorded by the Livingston detector at GPS time 931158360 (July 8 2009, 07:05:45 UTC), generated by the coherence waveBurst pipeline.

Existing Spectrogram Generation Methods

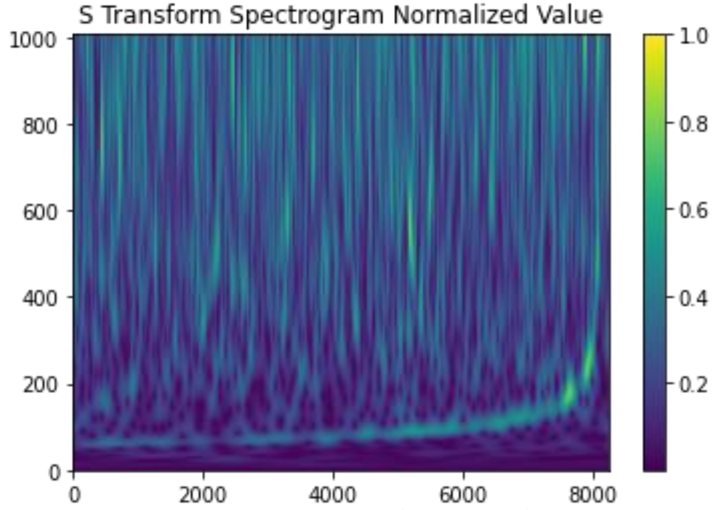
- Short-time Fourier Transform (STFT)
- Gabor Transform (GT)
- Constant Q Transform (CQT)
- S (Stockwell) Transform (ST)
- …

- All use the Fourier transform as the foundation.
- Only decompose the relationship between time and frequency.
- The defining characteristic of a BBH merger signal, **the chirp**, is abandoned.

Constant Q Transform of GW 150914



S Transform of a simulated BBH merger



Simulated merger m1 = m2 =, normalized amplitude 1, injected to Gaussian noise of amplitude 10.

Obtaining the Chirp-rate Information

Fourier Transform (FT)

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt,$$

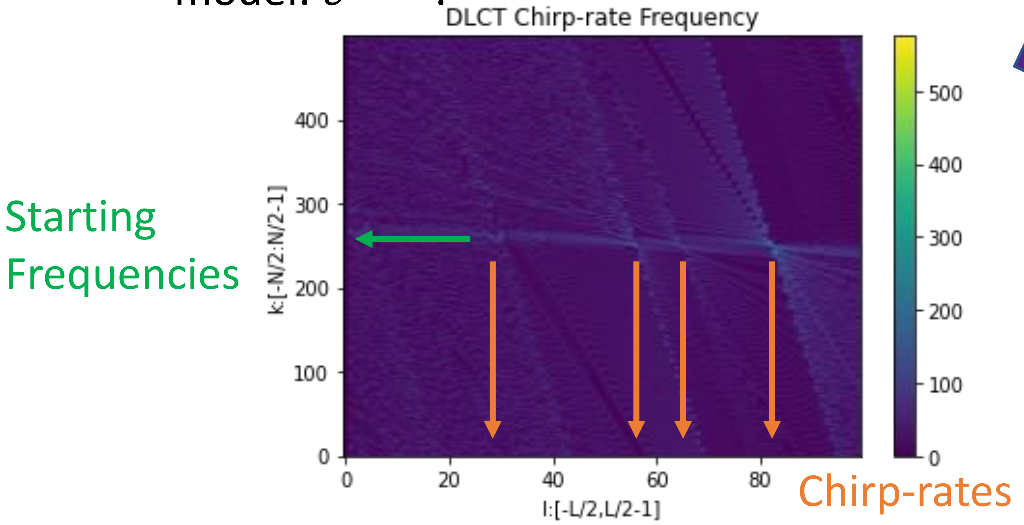
Linear Chirp Transform (LCT)

$$X(\Omega, \gamma) = \int_{-\infty}^{\infty} x(t) e^{-j(\Omega t + \gamma t^2)} dt,$$

O, A, Alkaishriwo & L.F. Chaparro (2012)

Matching the input signal x(t) to a **constant frequency** signal model: $e^{-j\Omega t}$.

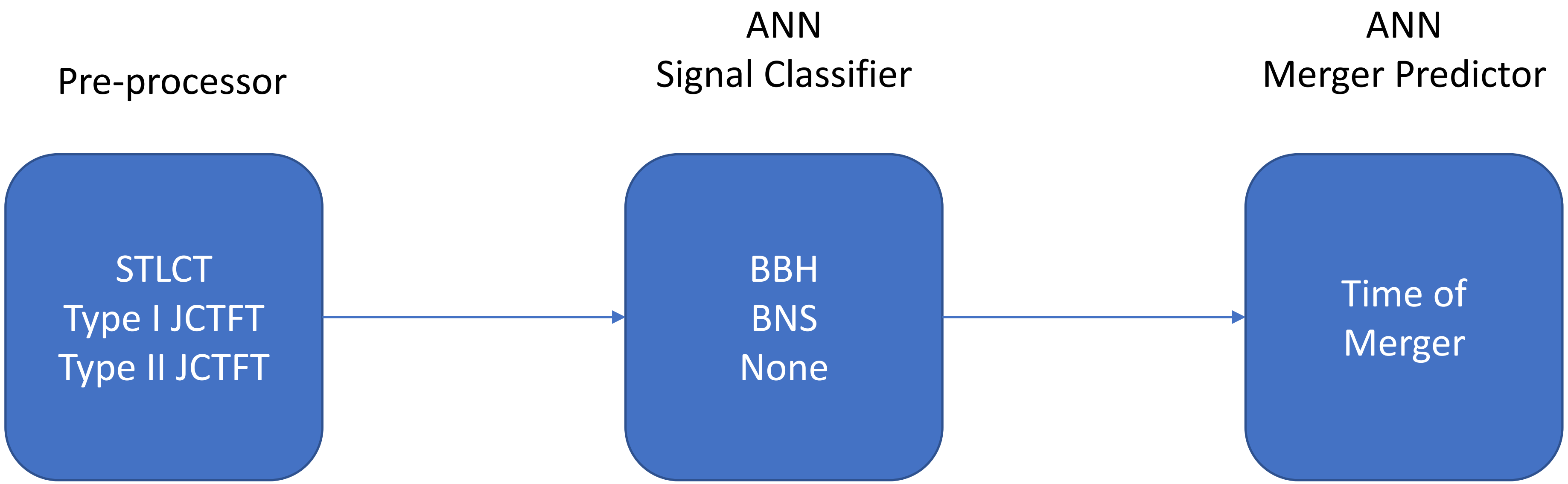
Matching the input signal x(t) to a **linear chirp** signal model: $e^{-j(\Omega t+\gamma t^2)}$.



Discrete Linear Chirp Transform (DLCT) of a 4-component linear chirp signal.

One can obtain chirp rates and the starting frequency of each chirp rate by further processing the Linear Chirp Transform frequency-chirp-rate diagram.

Obtaining the Chirp-rate Information



The Short-time Linear Chirp Transform (STLCT)

The continuous STLCT:

$$X(\Omega,\tau)=\int_{-L/2}^{L/2}\int_{-\infty}^{\infty}x(t)g(t-\tau)e^{-j(\Omega t+\gamma t^2)}dt\,d\gamma,$$

$$\gamma = 0 \longrightarrow$$

The continuous STFT:

$$X(\Omega,\tau)=\int_{-\infty}^{\infty}x(t)g(t-\tau)e^{-j\Omega t}dt,$$

The discrete STLCT:

$$X(k,t)=\sum_{l=-L/2}^{L/2-1}\sum_{n=0}^{N-1}h[n]g[n-t]e^{-i\frac{2\pi}{N}(kn+Cln^2)},$$

The Type I Joint-Chirp-Rate-Time-Frequency Transform

- The continuous Type I JCTFT:

ST

$C_s($

$\times$

- The discrete Type I JCTFT:

interpolated STLCT

C_s

where

- The enhancement function:

$$\boxed{f}(x) = \cot(\frac{\pi}{2}a)\tan(a\pi x) + 1,$$

where x ∈[−0.5,0.5], a ∈(0,1), f(x) ∈[0, 2].

The Type I Joint-Chirp-Rate-Time-Frequency Transform

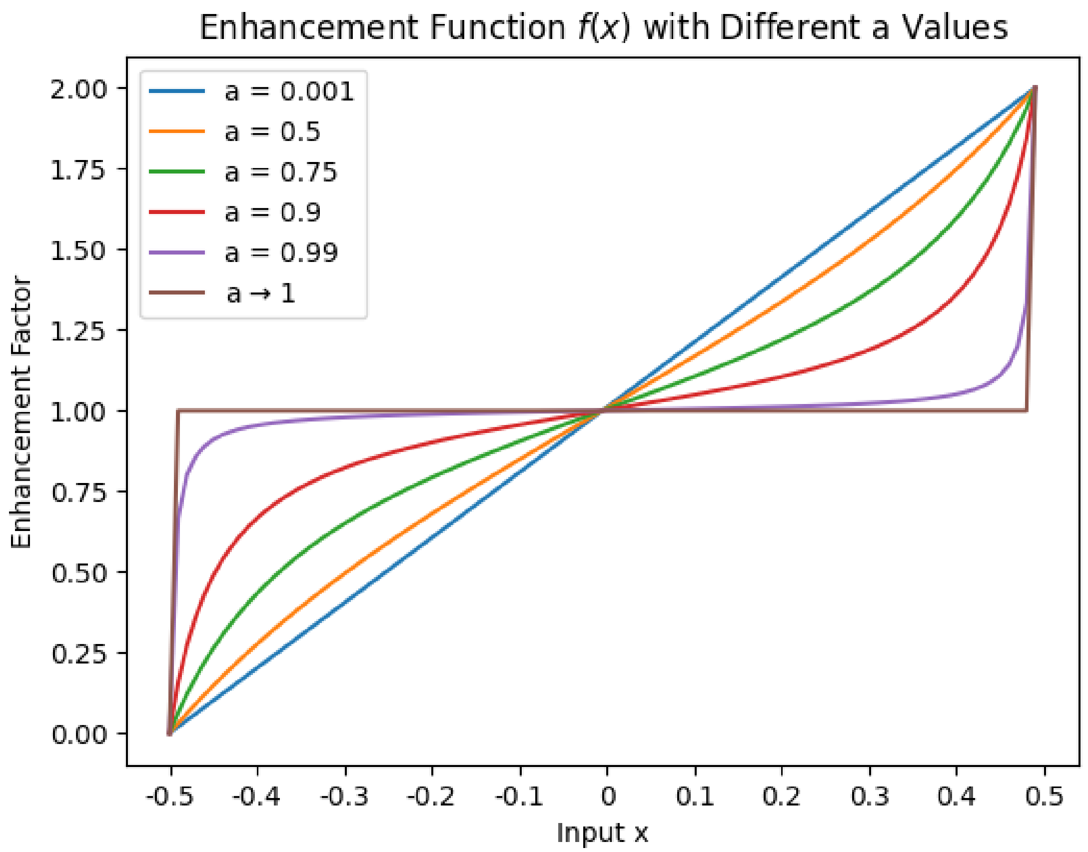
- The enhancement function:

$$f(x) = \cot\left(\frac{\pi}{2}a\right)\tan(a\pi x) + 1,$$

where x is the normalized input value with shifted origin at x = 0 and range [−0.5,0.5], a is the control parameter and a ∈(0,1).

- When **a →0**, f(x) → f(x) = 2x +1, a linear function centered at x = 0, y = 1.

- When **a →1**, $\lim_{a\to 1} \cot\left(\frac{\pi}{2}a\right) \to 0$, thus f(x) →1, all x values are equally shifted to 1, except at the two ends.



Enhancement function values with different control parameter a ranging from 0.001 to 1. The a →1 case is computed using numerical approximations.

The Type II Joint-Chirp-Rate-Time-Frequency Transform

$$w(t,f) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2}} e^{-i2\pi f t},$$

Constant Frequency Mother Wavelet

- The continuous Type II JCTFT:

where $\sum_0^q e^{-i2\pi\Gamma_p t^2}$, is the sum of chirp signal bases with chirp-rate γ in the set

$$\Gamma_p = \{\gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_q\}.$$

Linear Chirp Mother Wavelet

- The discrete Type II JCTFT:

where

The Type II Joint-Chirp-Rate-Time-Frequency Transform

- The continuous Type II JCTFT:

where Σ … t

In the cas …),

$$h_{linear}(t) = Ae^{i(\Omega_0 t + \gamma_0 t^2)},$$

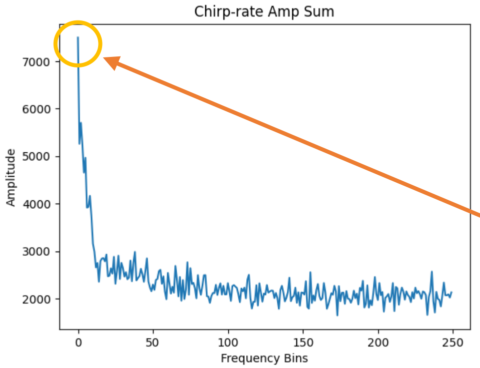
(2.23)

such that … ng:

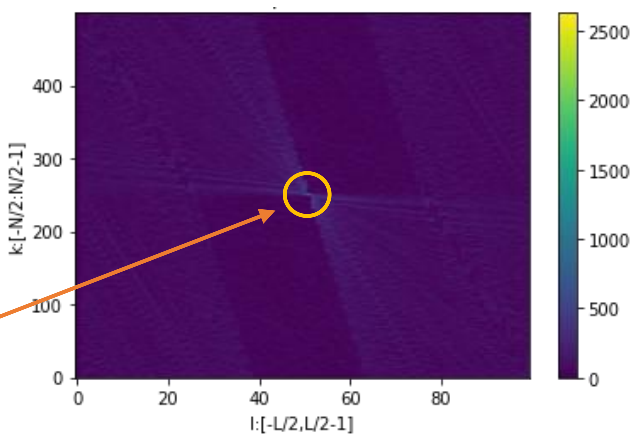
(2.24)

resulting

$X($ … (2.25)



Integrated (w.r.t. γ) DLCT diagram of signal (2.10



A DLCT diagram of signal (2.10), the peak is located at the center of the shifted axis.

The Short-time Linear Chirp Transform (STLCT)

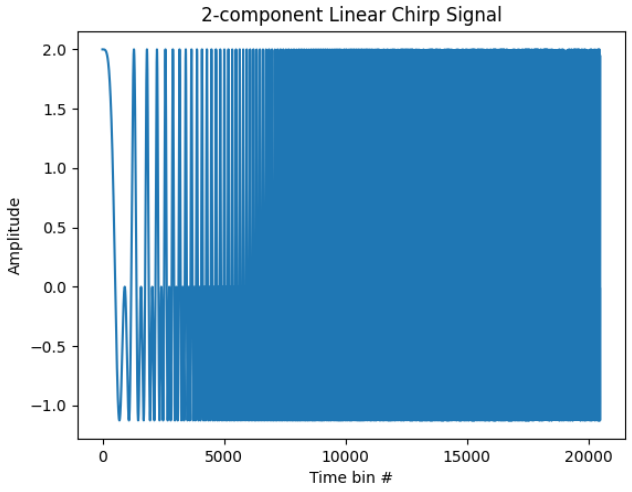
Test signal:

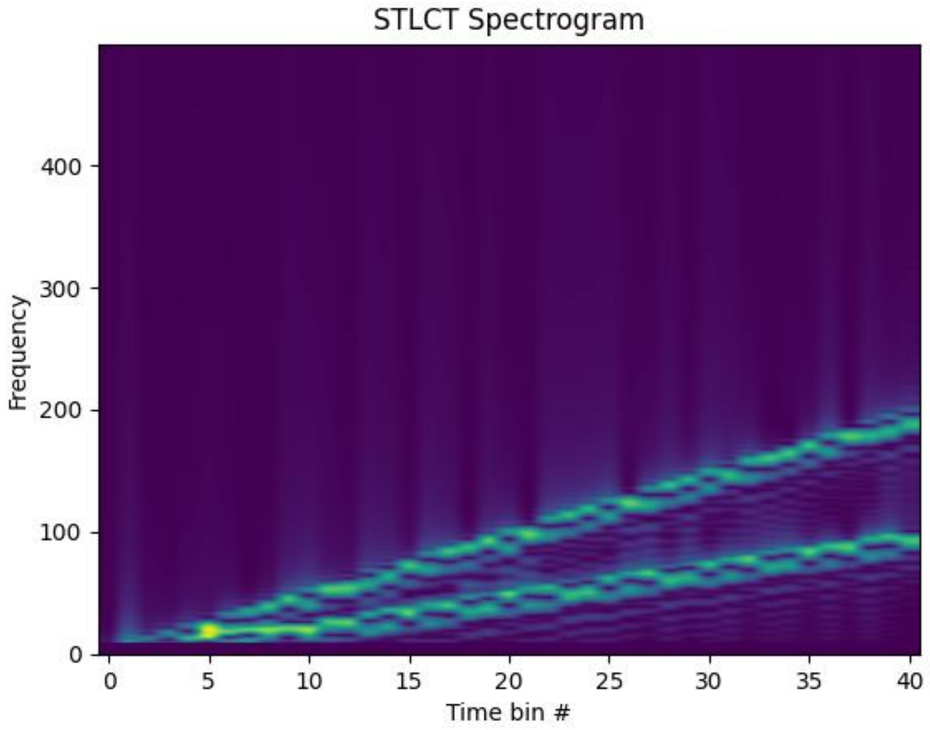
$$h(t) = e^{j2\pi 20t^2} + e^{j2\pi 10t^2}$$

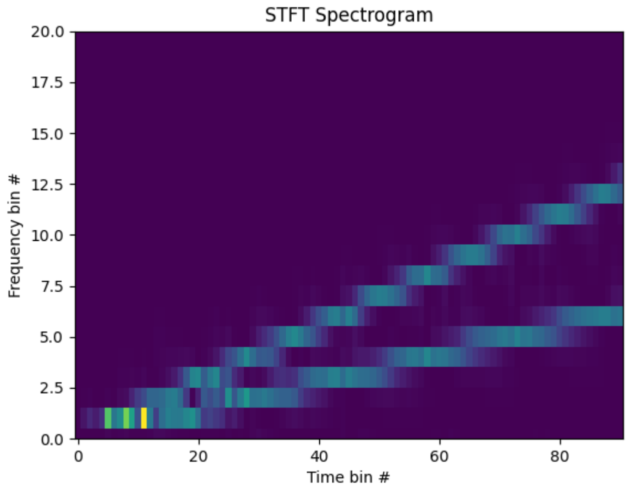
The **discrete STLCT**:

The discrete STFT (for validation):

$$X(k,t) = \sum_{n=0}^{N-1} h[n]g[n-t]e^{-i\frac{2\pi}{N}kn}$$







The Type I Joint-Chirp-Rate-Time-Frequency Transform

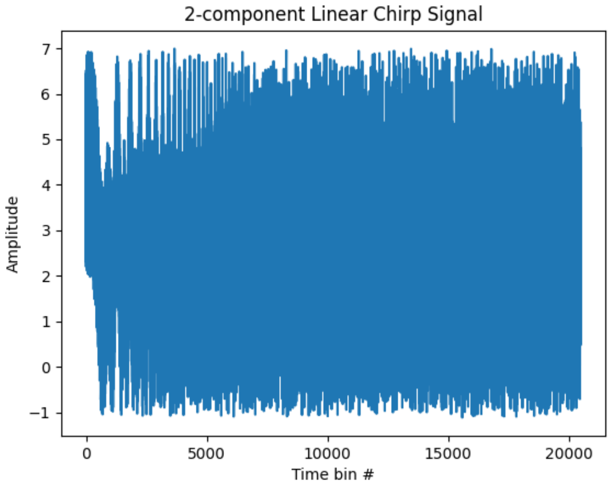
Test signal:

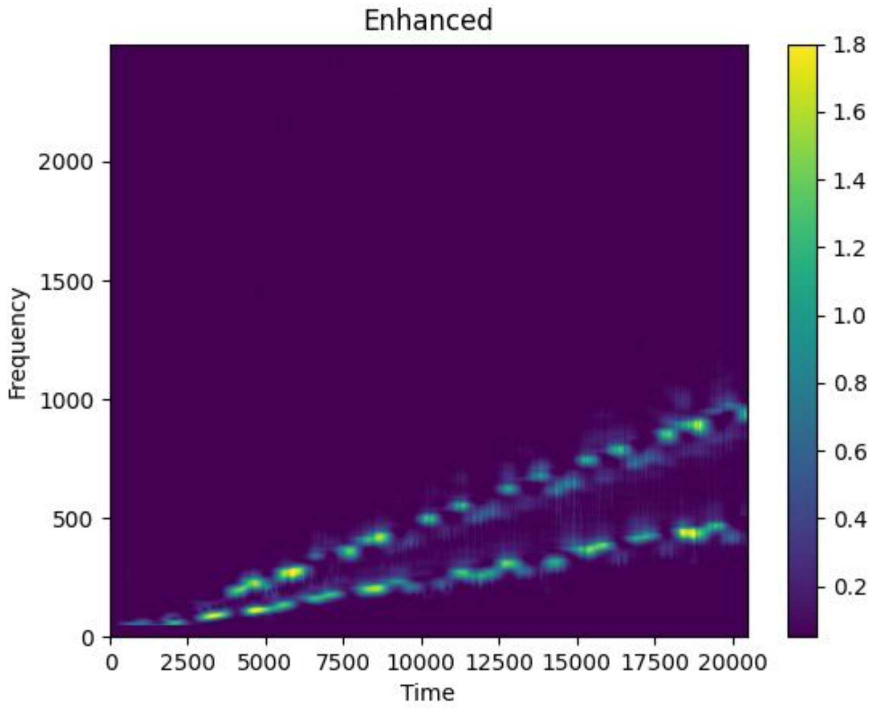
$h(t) = e^{j2\pi 20t^2} + e^{j2\pi 10t^2} +$ Gaussian*5

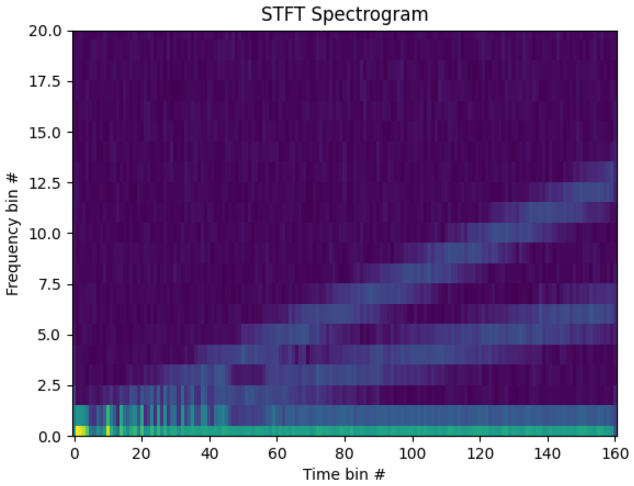
The **Type I JCTFT**:

The discrete STFT (for comparison):

$$X(k,t) = \sum_{n=0}^{N-1} h[n]g[n-t]e^{-i\frac{2\pi}{N}kn}$$







Current Progress and Next Steps

Currently, some transform methods are available as python packages

- constantQ (0.0.1): `pip install constantQ`
  a stripped-down version of the constant Q transform from Gwpy.
- TFchirp (0.0.2): `pip install TFchirp`
  accurate S transform and inverse S transform.

Purely written in Python, no additional compiler, interpreter, or language installation other than basic scientific computing packages for Python.

Next,

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  py

Additional Information

Linear Chirp Base for Merger Signal Representation

A complex linear chirp signal can be represented as

$$h_{linear}(t) = Ae^{i(\Omega_0 t + \gamma_0 t^2)}, \tag{2.10}$$

where A is the amplitude, Ω_0 is the starting frequency, and γ_0 is the chirp-rate. A simple differentiation of the power term $\Omega_0 t + \gamma_0 t^2$ gives the instantaneous frequency (IF)

$$IF_{linear}(t, \Omega_0) = 2\gamma_0 t + \Omega_0. \tag{2.11}$$

Suppose $h_{nonlinear}(t)$ is a geometric chirp signal given by

$$h_{nonlinear}(t) = Ae^{i\Omega_0[(\gamma_0^t - 1)/\ln \gamma_0]}, \gamma_0 \in (1, \infty), \tag{2.12}$$

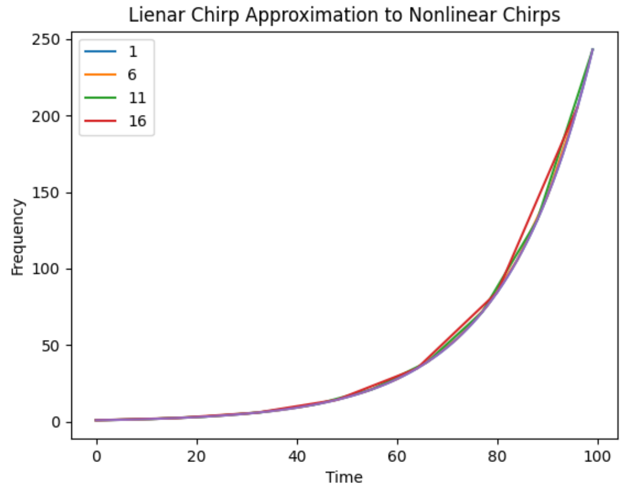
then its IF is

$$IF_{nonlinear}(t, \Omega_0) = \Omega_0 \gamma_0^t. \tag{2.13}$$

It is easy to show that $IF_{nonlinear}(t, \Omega_0)$ is an entire function (continuous and differentiable everywhere) and it has a linear approximation

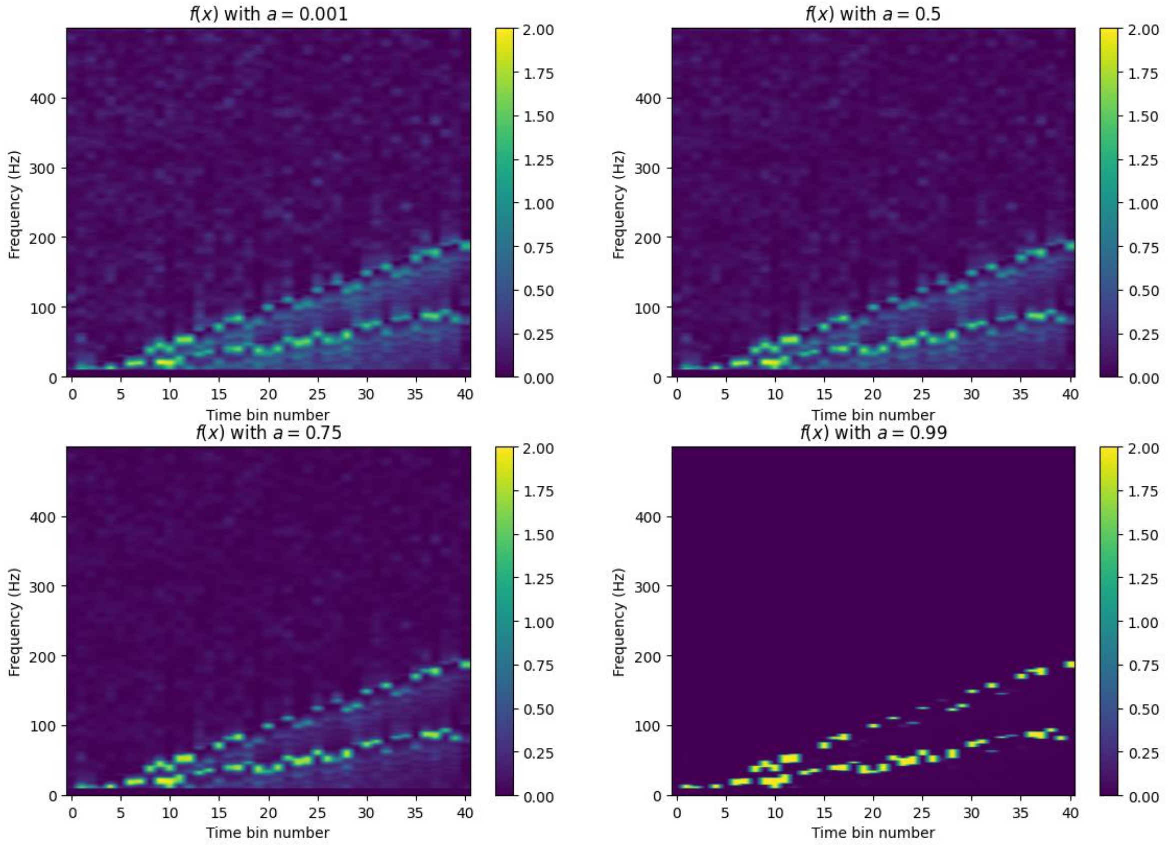
$$L_{approx}(t, \Omega_0) \approx IF_{nonlinear}(t + \delta t, \Omega_0) - \delta t \frac{\partial}{\partial t} IF_{nonlinear}(t + \delta t, \Omega_0), \tag{2.14}$$

by taking the first two terms of the Taylor series expansion at a point $t = t + \delta t$, which can be accurately represented by $IF_{linear}(t, \Omega_0)$. This approximation can be extended to other families of nonlinear chirp signals with appropriate choices of δt.

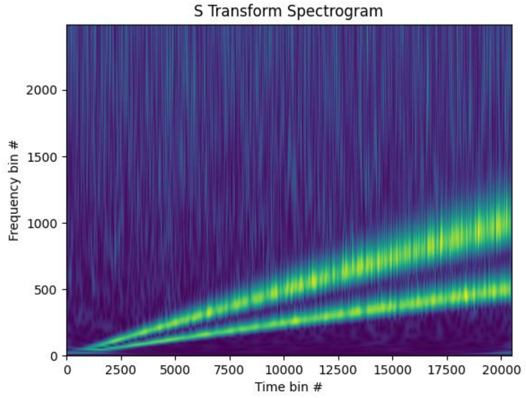


The Type I Joint-Chirp-Rate-Time-Frequency Transform

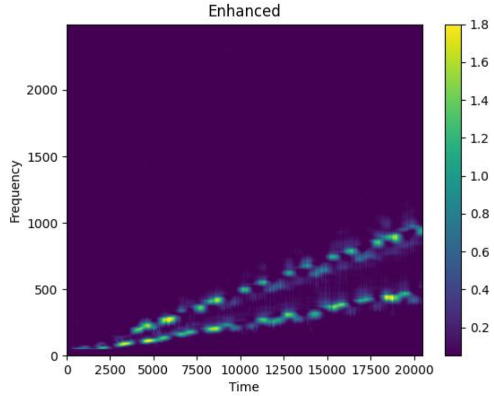
The Effect of Enhancement Function f(x)



The **ST** portion:



The **convolution** of the ST and the interpolated STLCT:



a=0.5