SURV625 Applied Sampling

2025-04-17

SM 625: Week 4 Sampling Project Notes

For each of the three variables that will be the focus of the final course project, the Department of Education would like to generate estimates of means and proportions having a coefficient of variation of no more than 0.05. Using the numbers provided to you in the description of the final project, compute estimates of the element variances for each variable. Given these estimates, compute the desired level of precision (the desired sampling variance) for each estimate that corresponds to the desired coefficient of variation.

Now, given the desired levels of precision for each estimate, compute estimates of the necessary sample sizes for each of these three estimates (assuming simple random sampling), ignoring the finite population correction. These will be starting points for the eventual two-stage cluster sample design.

We first build a table to store our results for each week's assignments.

- We also add the expected averages for each outcome variable.

```
# build dataframe with inputs
MI_school_samples <- tibble(
  Outcome = c("smoked_cig", "smoked_mj", "age_approached_to_smoke"),
  type = c("prop", "prop", "mean"),
  desire_cv = rep(.05, 3),
  expect_mean = c(.25, .15, 12),
)
  # calculate element
MI_school_samples |> kable()
```

Outcome	type	desire_cv	expect_mean
smoked_cig	prop	0.05	0.25
smoked_mj	prop	0.05	0.15
age_approached_to_smoke	mean	0.05	12.00

Our process is to:

- 1st, calculate the estimated element variance.
  - For a proportion, to get the element variance we use $\hat{p}(1-\hat{p})$.
  - For a mean, to get the element variance we simply just square the estimated standard deviation $v(\bar{y}) = \sigma^2$.
- 2nd, we calculate the estimated standard error as $se(\hat{p}) = CV \times \hat{p}$.
- 3rd, we compute the desired sampling variance as: $var(\hat{p}) = se(\hat{p})^2$, where $se(\hat{p}) = \sqrt{var(\hat{p})}$

```
MI_school_samples <- MI_school_samples |>
  mutate(
    # compute element variance
    var = if_else(type=="prop", # for proportions
                            expect_mean * (1 - expect_mean),
                            if_else(type=="mean",# for means
                                    1^2, NA)),
    # compute stand dev
    sd = sqrt(var),
    # compute standard error
    se = desire_cv * expect_mean,
    # compute desired sample variance
    V = se^2
    )

MI_school_samples |> select(-type) |> kable()
```

Outcome	desire_cv	expect_mean	var	sd	se	V
smoked_cig	0.05	0.25	0.1875	0.4330127	0.0125	0.0001563

Outcome	desire_cv	expect_mean	var	sd	se	V
smoked_mj	0.05	0.15	0.1275	0.3570714	0.0075	0.0000562
age_approached_to_smoke	0.05	12.00	1.0000	1.0000000	0.6000	0.3600000

We now estimate the desired sample sizes when we desire a CV =.05 as $n = \frac{s^2}{se^2}$

```
MI_school_samples <- MI_school_samples |>
  mutate(SRS_n = var / V)

MI_school_samples |> select(1, SRS_n) |> kable()
```

Outcome	SRS_n
smoked_cig	1200.000000
smoked_mj	2266.666667
age_approached_to_smoke	2.777778

SM 625: Week 5 Sampling Project Notes

For this week, we will consider the information available for stratified sampling of students. Eventually you are going to design a stratified cluster sample of students, where the clusters (or PSUs) are schools, but we aren't there yet.

Recall the regions of interest in the sampling project description:

```
school_frame <- read_xls(
  "~/work/d/SURV625project/data/MI_school_frame_head_counts.xls")
```

Region	County_ID
1	07, 31, 66
2	22, 27, 36, 55
3	02, 21, 52
4	17, 48, 49, 77
5	01, 04, 06, 16, 20, 26, 35, 60, 65, 68, 69, 71, 72
6	05, 10, 15, 18, 24, 28, 40, 43, 45, 51, 53, 57, 67, 83
7	03, 08, 11, 12, 13, 14, 34, 39, 41, 54, 59, 61, 62, 64, 70, 75, 80
8	09, 19, 23, 25, 29, 30, 33, 37, 38, 46, 47, 56, 73, 78, 81
9	32, 44, 50, 58, 63, 74, 76, 79, 82

As "State officials are interested in providing, if at all possible, separate estimates for each of nine education regions in the state, where the regions are defined by groups of counties", we will use these nine regions as strata.

Prepare a table that includes the:

- Overall population counts in each of these nine strata (the total count of students in the target population at each school is in the tot_all column on the sampling frame).
- Given these counts, once you have the working overall sample size (unknown for now and will be decided by your team next week), what is the proportionate allocation plan of that sample of students across these nine strata?

```
# we will use Region, County_ID, and tot_all

# region counts
strata_Prop_allocate <- school_frame |>
  group_by(Region) |>
```

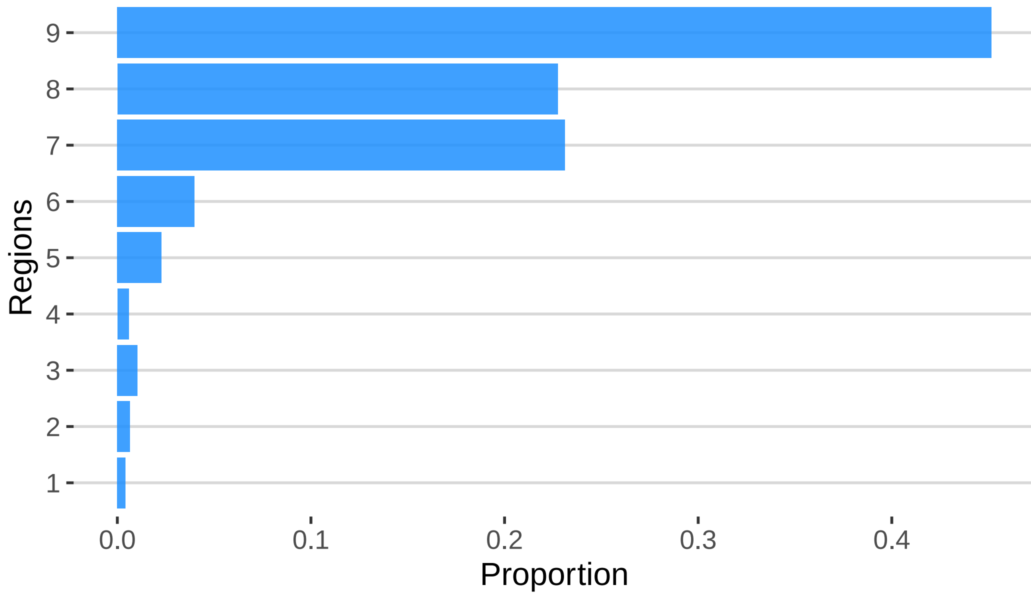
```r
  reframe(M_h = sum(tot_all), # total of students in stratum
          N_h = n()) |> # total of schools in stratum
  mutate(prop_allocation = M_h/sum(M_h))

strata_Prop_allocate |>
  kable()
```

Region	M_h	N_h	prop_allocation
1	3561	20	0.0042896
2	5474	30	0.0065941
3	8631	33	0.0103971
4	4855	31	0.0058484
5	18907	80	0.0227757
6	33133	133	0.0399126
7	191992	644	0.2312772
8	188830	549	0.2274682
9	374755	923	0.4514370

```r
# what is the proportionate allocation plan of that sample
## of students across these nine strata?
```

```r
strata_Prop_allocate |>
  mutate(Region = factor(Region)) |>
  ggplot(aes(x=Region, y=prop_allocation)) +
  geom_col(position="dodge", fill="dodgerblue", alpha=.85) +
  coord_flip() +
  guides(fill=guide_legend(title="", reverse = TRUE)) +
  labs(
    title = "Figure 1. Proportionate Allocation Plan Across Nine Strata",
    x = "Regions",
    y = "Proportion"
  ) +
  theme_hc()
```

Figure 1. Proportionate Allocation Plan Across Nine Strata



SM 625: Week 6 Sampling Project Notes

From a previous study, you obtain estimates of the following design effects for each of these three estimates:

- proportion ever smoked one cigarette = 2.5;
- proportion ever smoked marijuana = 2.0; and
- mean age when first asked to smoke = 1.7.

This previous study featured a sample of size n = 7,500 students between the ages of 13 and 19, selected from a total of a = 150 clusters. Using this information, compute a synthetic estimate of roh for each of the three variables. These synthetic estimates of roh will be used to consider alternative cluster sample designs as you continue with your project work. Finally, budget and cost information is now available. The total budget for data collection for this project will be $500,000. The client and the data collection organization estimate that the data collection will cost $3,000 per primary stage cluster (school), and $50 per completed questionnaire within a cluster. We will use this cost information moving forward for optimal subsample size calculations.

We can estimate the sample ICC or roh from the given design effect estimate as:

$$\hat{roh} = \frac{deff - 1}{m - 1}$$

We now that the sample total is $nm = 7500$ and the sample number of cluster is $n = 150$, which we can take the mean cluster size as $m = nm/n = 7500/150 = 50$ and use it to calculate roh.

```r
nm <- 7500
n <- 150
m <- nm / n

MI_school_samples <- MI_school_samples |>
  # add deff and roh to our table
  mutate(desire_deff = c(2.5, 2.0, 1.7),
         # compute roh
         roh = (desire_deff - 1) / (m - 1),
```

```r
         roh = round(roh, 4)
         )

MI_school_samples |> select(Outcome, desire_deff, roh) |>  kable()
```

Outcome	desire_deff	roh
smoked_cig	2.5	0.0306
smoked_mj	2.0	0.0204
age_approached_to_smoke	1.7	0.0143

SM 625: Week 7 Sampling Project Notes

Recall that the client and the data collection organization estimated that the data collection would cost \$3,000 per primary stage cluster (school), and \$50 per completed questionnaire within a cluster. We will now use this information for optimum subsample size calculations. Recall that the total budget for data collection will be \$500,000.

Given this cost information and your estimates of roh for the three different variables of primary interest from last week, compute the optimum subsample size (and the corresponding optimal number of first stage clusters, given the total budget above) for each of the variables.

- We now have budget constraints and denote the cost per cluster as $c_n = \$3,000$ and cost per element as $c_m = \$50$, with a total budget constraint of $C = \$500,000$. Since we know there are $n = 150$ clusters and a total sample size of $7,500$ students.
- To compute the optimum m size we use the following equation:

$$m_{opt} = \sqrt{\frac{c_n}{c_m}\frac{1 - roh}{roh}}$$

```
c_n = 3000 # cost per cluster
c_m = 50 # cost per element within cluster
C = 500000 # total budget

MI_school_samples <- MI_school_samples |>
  mutate(
    # compute optimum m size
    m_opt = sqrt( (c_n / c_m) * ( (1-roh)/roh) ),
    n_opt = C / (c_n + m_opt * c_m),
    # compute new deff
    deff_new = 1 + (m_opt-1) * roh,
     # compute total SSU
    total_nm = m_opt * n_opt)

MI_school_samples |> select(Outcome, m_opt, n_opt) |>
  kable()
```

Outcome	m_opt	n_opt
smoked_cig	43.59799	96.52697
smoked_mj	53.67659	87.96886
age_approached_to_smoke	64.31022	80.44391

How will you decide on a single overall optimum subsample size to use in your design?

- Above we estimated the new design effects which range from 2.3 to 1.9, which are almost in line with our desired design effects of 2.5, 1.7. Below we print the new design effects, optimum number of cluster and cluster size, total sample size `total_nm`for our projected $500,000 budget for all three outcome variables.
  - Finally, we compute the sampling cost as $n \times c_n + n \times m \times c_m$ which we defined these terms above.

```
MI_school_samples |>
  select(Outcome, m_opt, n_opt, deff_new, total_nm) |>
  mutate_at(c(2, 3, 5), floor) |>
  mutate(cost = (c_n * n_opt) + (c_m * n_opt * m_opt),
         cost = scales::dollar(cost),
         Outcome = ifelse(Outcome == "age_approached_to_smoke",
                          "age_smoke", Outcome),
             deff_new = round(deff_new, 4)) |>
  kable()
```

Outcome	m_opt	n_opt	deff_new	total_nm	cost
smoked_cig	43	96	2.3035	4208	$494,400
smoked_mj	53	87	2.0746	4721	$491,550
age_smoke	64	80	1.9053	5173	$496,000

Think about a comparison of alternative cluster sample designs: under a fixed cost constraint, how would we decide which design would be best? What will be your overall sample size (n) under this new optimum subsample size?

As you make progress in writing up what you have done so far, provide some discussion of the rationale for your choices in this regard.

Next, given this optimum subsample size and treating the values of roh as portable, compute the new expected DEFF for each estimate given the new design (this can be specific to each variable / estimate, given the different optimum subsample sizes). In addition, compute a new expected SRS variance for each variable under the new design, using the new "optimum" overall sample size (remember that you can treat the element variances for each variable estimated last week as portable). Finally, compute the new expected sampling variance for each estimate under this new cluster sample design. Are you still meeting the client's precision requirements?

- Given that we have three outcome variables, we also have three optimum number of clusters and cluster size estimates. That is, we can design and examine three options of different optimum number of clusters and cluster sizes.
- We will use the portable roh estimate and calculate new design effects, SRS variance, and complex design variance for each outcome variable.

```
map(seq(1,3), function(x){

  MI_school_samples |>
  select(Outcome, roh, m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[x], # optimum m from first row
         n_opt = n_opt[x],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_new = 1 + (m_opt-1) * roh,
         # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance
         var_srs =  var / (total_nm - 1),
         # calculate complex design variance
         var_crs = var_srs * deff_new,
         #m_opt = round(m_opt)
         ) |>
    select(-roh, -var)  |>
    mutate_at(2:3, floor) |>
    mutate(total_nm = m_opt*n_opt) |>
    mutate_at(5:7, round, 5) |>
  kable()

}) |>
  set_names(str_c(rep("Option ", 3), seq(1,3)))
```

```
$`Option 1`


|Outcome                  | m_opt| n_opt| total_nm| deff_new| var_srs| var_crs|
|:------------------------|-----:|-----:|--------:|--------:|-------:|-------:|
|smoked_cig               |    43|    96|     4128|  2.30350| 0.00006| 0.00013|
|smoked_mj                |    43|    96|     4128|  1.86900| 0.00003| 0.00006|
|age_approached_to_smoke  |    43|    96|     4128|  1.60915| 0.00214| 0.00344|
```

```
$`Option 2`


|Outcome                 | m_opt| n_opt| total_nm| deff_new| var_srs| var_crs|
|:-----------------------|-----:|-----:|--------:|--------:|-------:|-------:|
|smoked_cig              |    53|    87|     4611|  2.61190| 0.00005| 0.00013|
|smoked_mj               |    53|    87|     4611|  2.07460| 0.00003| 0.00006|
|age_approached_to_smoke |    53|    87|     4611|  1.75328| 0.00191| 0.00334|

$`Option 3`


|Outcome                 | m_opt| n_opt| total_nm| deff_new| var_srs| var_crs|
|:-----------------------|-----:|-----:|--------:|--------:|-------:|-------:|
|smoked_cig              |    64|    80|     5120|  2.93729| 0.00005| 0.00014|
|smoked_mj               |    64|    80|     5120|  2.29153| 0.00002| 0.00006|
|age_approached_to_smoke |    64|    80|     5120|  1.90534| 0.00174| 0.00332|
```

We print standard error for the complex design with 95% confidence intervals, and we also flag whether the sampling variance from the clustering is equal or smaller than the desired sampling variance.

```r
map(seq(1,3), function(x){

  MI_school_samples |>
  select(Outcome, expect_mean, V, roh, m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[x], # optimum m from first row
         n_opt = n_opt[x],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_new = 1 + (m_opt-1) * roh,
        # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance
         var_srs =  var / (total_nm - 1) ,
         # calculate complex design variance
         var_crs = var_srs * deff_new,
         # compute confidence intervals
         se = sqrt(var_crs),
         lower = expect_mean  - 1.96*se,
```

```
        upper = expect_mean + 1.96*se,
       # flag if var_crs is lower or = to desired sampling var
        var_ck = ifelse(var_crs <= V, "yes", "no")) |>
  select(Outcome, expect_mean, se, lower, upper, var_ck) |>
  mutate_at(3:5, round, 4) |>
  kable()

}) |>
  set_names(str_c(rep("Option ", 3), seq(1,3)))
```

```
$`Option 1`


|Outcome                 | expect_mean|     se|   lower|   upper|var_ck |
|:-----------------------|-----------:|------:|-------:|-------:|:------|
|smoked_cig              |        0.25| 0.0115|  0.2275|  0.2725|yes    |
|smoked_mj               |        0.15| 0.0075|  0.1352|  0.1648|no     |
|age_approached_to_smoke |       12.00| 0.0587| 11.8850| 12.1150|yes    |

$`Option 2`


|Outcome                 | expect_mean|     se|   lower|   upper|var_ck |
|:-----------------------|-----------:|------:|-------:|-------:|:------|
|smoked_cig              |        0.25| 0.0115|  0.2274|  0.2726|yes    |
|smoked_mj               |        0.15| 0.0075|  0.1353|  0.1647|yes    |
|age_approached_to_smoke |       12.00| 0.0578| 11.8867| 12.1133|yes    |

$`Option 3`


|Outcome                 | expect_mean|     se|   lower|   upper|var_ck |
|:-----------------------|-----------:|------:|-------:|-------:|:------|
|smoked_cig              |        0.25| 0.0117|  0.2271|  0.2729|yes    |
|smoked_mj               |        0.15| 0.0075|  0.1353|  0.1647|no     |
|age_approached_to_smoke |       12.00| 0.0576| 11.8871| 12.1129|yes    |
```

- Option 2 with a number of cluster of 87 and cluster size of 53 is the design we will choose given that the total sample size of 4,611 is within the allocated budget ($491,550).

> We prefer this model because it stays close to the desired design effects we received from the customer. Additionally, the standard errors we estimate for this second option overall are the smallest resulting in tighter 95% confidence intervals for the expected estimates we were provided. This design in close to option 3, yet we prefer having a slightly smaller SSU if we can increase the number of PSUs sampled since this gives us a cost efficiency.

The client has also provided other new information: the estimated size of the target population is N = 830,138. Given this population size and your overall sample size (n) under the new optimum subsample size computed above, what is your overall working sampling fraction (f)? Does it seem like finite population corrections will be necessary in your sampling variances if you choose to perform SRSWOR at some point?

```
# total pop
N <- 830138

# optimum n
total_nm <- MI_school_samples |>
  slice(2) |> # second design option
  summarise(m_opt*n_opt) |>
  pull()


samp_frac <- total_nm / N; samp_frac
```

```
[1] 0.005688052
```

```
MI_school_samples_table <- MI_school_samples |>  select(Outcome, expect_mean, roh,
                             m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[2], # optimum m from first row
         n_opt = n_opt[2],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_new = 1 + (m_opt-1) * roh,
         # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance with sampl_fraction
         var_srs =(1 - samp_frac) * var / (total_nm - 1),
         # calculate complex design variance
         var_crs = var_srs * deff_new,
```

```
         # compute confidence intervals
         se = sqrt(var_crs),
         lower = expect_mean  - 1.96*se,
         upper = expect_mean + 1.96*se )

MI_school_samples_table |>
  select(Outcome, var_crs, se, lower, upper) |>
  mutate_at(2:4, round, 5) |>
  kable()
```

Outcome	var_crs	se	lower	upper
smoked_cig	0.00013	0.01149	0.22748	0.2725212
smoked_mj	0.00006	0.00746	0.13537	0.1646295
age_approached_to_smoke	0.00332	0.05765	11.88701	12.1129933

Our overall sampling fraction is .0057. In examining the complex design variances, and recalculating the expected standard error and 95% confidence interval given the sampling raction, it does not appear that accounting for a population correction makes a huge impact, and we suggest it will not be necessary in our sampling variance for an SRSWOR design

SM 625: Week 8 Sampling Project Notes

Assume that you will decide to allocate your final computed n_{opt} number of clusters to each of the nine project strata based on the proportions of the total number of students in the population in each stratum (i.e., if 20% of the population of students comes from Region 1, you would sample 20% of your clusters from that region). Describe the first-stage sampling fractions for each stratum, where the total number of schools to sample at the first stage in each stratum is defined by your proportionate allocation of the n_{opt} clusters.

Next your team should extend your design to consider stratified PPeS selection of schools from each of the nine strata at the first stage of your sample design.

You have been provided with a sampling frame that lists the schools within each region. Given the information on the sampling frame, how might you sort this list to achieve implicit stratification within the regions? You can treat the overall student count from a previous year (tot_all) as the measure of size for the PPeS sampling. Given this information, compute your zone size for systematic PPeS sampling within each of the nine strata (regions), and proceed with systematic selection based on fractional intervals to select the allocated number of schools within each stratum using PPeS sampling. What is your first-stage sampling fraction within each of the nine strata?

- Using the proportionate allocation by strata computed earlier, we assign and add cluster allocation by stratum by $n_{opt} \times prop - allocation$.
- nonresponse adjustment is achieved by taking our optimum values and adjusting them by the amount of respondents that are likely to complete the survey.
- We also calculate the zone size which we label as k_h as:

$$k_h = \frac{nMOS_i}{\sum_t MOS_i}$$

```
set.seed(9999)

# response rates
school_rr <- .30
student_rr <- .70
# Given values
n_opt <- MI_school_samples |>
  slice(2) |> select(n_opt) |>
  pull() / school_rr

m_opt <- MI_school_samples |>
  slice(2) |> select(m_opt) |>
  pull() / student_rr
```

```r
# Compute proportional allocation of clusters to each stratum
region_summary <- strata_Prop_allocate |>
  # Ensure at least 1 cluster per
  mutate(n_h = round(n_opt * prop_allocation)#,
           # we need to adjust the last n_h to get an exact 290
         #n_h = ifelse(n_h == 131, n_h-1, n_h)
         ) |>
  mutate(N_h = as.double(N_h)) |>
  group_by(Region) |>
  reframe(across(where(is.double), ~ sum(.x))) |>
  mutate(
         f_h = n_h / N_h,     # sampling fraction
         k_h =  round(M_h / n_h)) |>    # zone size
  # create random start values
  rowwise() |>
  mutate(RN = sample(1:k_h, 1)) |>
  ungroup() |>
  mutate_at(vars(prop_allocation, f_h), round, 3)


region_summary |>
  select(Region, prop_allocation, n_h, k_h, RN) |>
  kable()
```

Region	prop_allocation	n_h	k_h	RN
1	0.004	1	3561	3168
2	0.007	2	2737	2310
3	0.010	3	2877	1321
4	0.006	2	2428	131
5	0.023	7	2701	2122
6	0.040	12	2761	2114
7	0.231	68	2823	374
8	0.227	67	2818	380
9	0.451	132	2839	1673

- To achieve implicit stratification we order the school list sorted by size of student in each region. To compute zone size we use

```
# sort list of schools by student size
school_frame_sorted <- school_frame |>
  arrange(g7_totl, g8_totl, g9_totl, g10_totl, g11_totl, g12_totl)

min_MOS <- m_opt

# create vectors of selection values for each stratum
RN_sample <- map(1:nrow(region_summary), function(x){

  # pass table created in last code chunk
  round(seq(region_summary$RN[x], # random start
      region_summary$M_h[x], # total number of students
      region_summary$k_h[x])) # k sampling interval
})
```

```
# we link the selected blocks
dat <- school_frame_sorted |>
  group_by(Region) |>
  mutate(
    # assing ids
    id = row_number(),
    # flag if minimum MOS not met
    min_m_req = ifelse(tot_all >= min_MOS, 1 , 0),
    # create links and convert to clusters
    linking = lead(min_m_req, default=1),
    # assign clustering
    cluster = cumsum(lag(linking, default=1)),
    # add cumulative counts
    cumulative_max = cumsum(tot_all),
    cumulative_min = 1 + lag(cumulative_max, default = 0) )
```

```
# for each region loop through RN_sample & assign selection to schools
dat_selected <- map_dfr(1:9, function(x){

  dat |>
    filter(Region %in% x) |>
    add_column(RN_sample[[x]] |> tibble() |> data.table::transpose()) |>
    # create flag for blocks that are selected
    mutate(selected =
              as.numeric(if_any(starts_with("V"), ~
                          between(.x, cumulative_min, cumulative_max))) |>
    # drop select population elements
```

```
    select(-starts_with("V"))

})
```

```
# this is where schools are linked
dat_linked <- dat_selected |>
  group_by(Region) |>
  mutate(
    # flag if minimum MOS not met
    min_n_req = ifelse(tot_all >= min_MOS, 1 , 0),
    # create links and convert to clusters
    linking = lead(min_n_req, default=1),
    # assign clustering
    cluster = cumsum(lag(linking, default=1))) |>
  ungroup()
```

```
# show cluster of blocks selected, total HUs
sample_selected <- map_dfr(1:9, function(x){

  linkage = dat_linked |>
    filter(Region %in% x, selected == 1) |>
    select(Region, cluster) |>
    mutate(Selection = RN_sample[[x]]) |>
    pull(cluster)

  dat = dat_linked |>
    filter(Region %in% x,
           cluster %in% linkage) |>
    mutate(MOS = as.numeric(tot_all)) |>
    group_by(cluster) |>
    mutate(
      cluster = cur_group_id(),
      total_MOS = sum(MOS, na.rm = TRUE)
    ) |>
    arrange(desc(id))  # optional: sort within cluster

  if (x == 1) {
    # get unique cluster id from Region 1
    first_cluster_id <- dat |>
      filter(Region == 1) |>
      pull(cluster) |>
      unique() |>
```

```
      min()

    # filter the first cluster
    first_cluster <- dat |> filter(cluster == first_cluster_id)

    # split it into two halves (or roughly)
    n <- nrow(first_cluster)
    first_half <- first_cluster[1:floor(n/2), ] |>
      mutate(
        SECU = "1A",
        SECU_MOS = sum(MOS/2, na.rm = TRUE)
      )
    second_half <- first_cluster[(floor(n/2) + 1):n, ] |>
      mutate(
        SECU = "1B",
        SECU_MOS = sum(MOS/2, na.rm = TRUE)
      )

    # everything else from Region 1
    remaining <- dat |> filter(cluster != first_cluster_id) |>
      mutate(SECU = as.character(cluster),
             SECU_MOS = total_MOS)

    # combine all Region 1 units
    dat <- bind_rows(first_half, second_half, remaining)
  } else {
    dat <- dat |>
      mutate(
        SECU = as.character(cluster),
        SECU_MOS = total_MOS
      )
  }

  return(dat)
}) |>
  ungroup()

# save data to github
write_xlsx(sample_selected,
           "~/work/d/SURV625project/data/sample_selected.xlsx")
```

```
# Form Pseudo-Strata for Paired Selection Model
pseudo_strata_df <- sample_selected |>
  group_by(Region) |>
  arrange(desc(MOS)) |>   # optionally sort by size for pairing
  mutate(row_in_group = row_number(),
         pseudo_stratum_id = paste0("R", Region, "_P", ceiling(row_in_group / 2))) |>
  ungroup()

# how many pseudo strata in each region?
pseudo_strata_df |>
  group_by(Region) |>
  distinct(pseudo_stratum_id) |>
  count() |>
  kable()
```

Region	n
1	1
2	3
3	3
4	4
5	4
6	9
7	61
8	51
9	77

SM 625: Week 10 Sampling Project Notes

There are four primary tasks for your team to consider over the next week:

1. Given your overall m_{opt} n_opt and N (based on the sampling frame), you've already computed the overall sampling fraction, . For each of the nine strata, compute the required number of students to subsample from each sampled school based on the stratified PPeS design in order to maintain epsem across all strata.

- Within strata, retain epsem for stratified PPS sampling across strata $f = f_h$ for all h.

$$f_h = \frac{n_h MOS_{hi}}{\sum_{,i \in h} MOShi} \frac{m_h^*}{MOS_{hi}}$$

```r
# Required Students per School (m_h_star) to Maintain EPSEM:
region_summary<- region_summary |>
  mutate(m_h_star=c(samp_frac*k_h))

region_summary |>
  select(Region, k_h, RN, m_h_star) |>
  kable()
```

Region	k_h	RN	m_h_star
1	3561	3168	20.25515
2	2737	2310	15.56820
3	2877	1321	16.36453
4	2428	131	13.81059
5	2701	2122	15.36343
6	2761	2114	15.70471
7	2823	374	16.05737
8	2818	380	16.02893
9	2839	1673	16.14838

2. Do each of the schools that you sampled in a given region have the minimum sufficient size, given the stratum-specific subsample sizes computed in Task #1? Do subsequent schools on the list have the minimum sufficient size? If not, what will you do?

```r
region_min_MOS <- region_summary %>%
  group_by(Region) %>%
  mutate(
    min_MOS2 = ceiling(m_h_star /  0.7)  # Total response rate = 0.21, expanded sample size
```

```
  )

# Processing schools by region and generating clusters of links
linked_schools <- sample_selected %>%
  left_join(region_min_MOS, by = "Region") %>%  # Combined Minimum MOS
  group_by(Region) %>%
  arrange(desc(tot_all)) %>%  # Listed in descending order of MOS (prioritizing large schools
  mutate(
    # Initialize cumulative MOS and link tags
    cumulative_mos = cumsum(tot_all),
    need_link = if_else(tot_all < min_MOS2, 1, 0),
    # Dynamic generation of cluster IDs: linking when cumulative MOS is insufficient
    cluster_id = cumsum(
      if_else(
        cumulative_mos - lag(cumulative_mos, default = 0) >= min_MOS2 | row_number() == 1,
        1, 0
      )
    )
  ) %>%
  ungroup()

# how many linked clusters by region
linked_schools |>
  group_by(Region) |>
  count(cluster_id) |>
  count() |>
  kable()
```

Region	n
1	2
2	3
3	3
4	5
5	7
6	17
7	100
8	90
9	141

```r
# Summarize the total MOS for each cluster and check for compliance
cluster_summary <- linked_schools %>%
  group_by(Region, cluster_id) %>%
  summarise(
    total_mos = sum(tot_all),
    schools = toString(BCODE),
    min_MOS2 = first(min_MOS2),
    .groups = "drop"
  ) %>%
  mutate(
    sufficient = if_else(total_mos >= min_MOS2, "Yes", "No")
  )
# Output clusters that need to be relinked (total MOS still insufficient)
clusters_to_relink <- cluster_summary %>% filter(sufficient == "No")

# Recursive linking until all clusters are up to standard
while (nrow(clusters_to_relink) > 0) {
  linked_schools <- linked_schools %>%
    group_by(Region) %>%
    mutate(
      cluster_id = if_else(
        cluster_id %in% clusters_to_relink$cluster_id,
        cluster_id + 1,  # Merge to the next cluster
        cluster_id
      )
    ) %>%
    ungroup()

  # Summary of recomputation clusters
  cluster_summary <- linked_schools %>%
    group_by(Region, cluster_id) %>%
    summarise(
      total_mos = sum(tot_all),
      schools = toString(BCODE),
      min_MOS2 = first(min_MOS2),
      .groups = "drop"
    ) %>%
    mutate(sufficient = if_else(total_mos >= min_MOS2, "Yes", "No"))

  clusters_to_relink <- cluster_summary %>% filter(sufficient == "No")
}
```

```r
final_clusters <- linked_schools %>%
  group_by(Region, cluster_id) %>%
  summarise(
    linked_schools = paste(BCODE, collapse = ", "),
    total_mos = sum(tot_all),
    min_MOS2 = first(min_MOS2),
    .groups = "drop"
  ) %>%
  mutate(
    status = if_else(total_mos >= min_MOS2, "Valid", "Invalid")
  )


linked_schools <- linked_schools %>%
  left_join(
    cluster_summary %>% select(Region, cluster_id, total_mos),
    by = c("Region", "cluster_id")
  )

# Print results
final_clusters |>
  select(Region:total_mos) |>
  kable(col.names = c("h", "id", "n", "m"))
```

h	id	n	m
1	1	08558	190
1	2	08558	190
2	1	02039	675
2	2	02040	258
2	3	08877, 01762, 08944	42
3	1	02389	1067
3	2	02666	468
3	3	00481, 08859, 04431, 03876	172
4	1	09417	115
4	2	02163	106
4	3	09308	43
4	4	02305	41
4	5	07124, 04506, 01509, 07718	68
5	1	01375	1090
5	2	04438	861
5	3	00655	737

h	id	n	m
5	4	05507	709
5	5	04516	342
5	6	08420	213
5	7	08886	91
6	1	00554	2078
6	2	04200	1602
6	3	04199	814
6	4	02279	794
6	5	02339	569
6	6	00697	473
6	7	02333	355
6	8	07453	254
6	9	00392	222
6	10	06599	154
6	11	06938	111
6	12	01817	91
6	13	08626	74
6	14	03870	41
6	15	03678	39
6	16	04006	38
6	17	08257, 07492	43
7	1	04462	2299
7	2	05974	1849
7	3	01455	1840
7	4	01463	1830
7	5	01265	1368
7	6	02106	1359
7	7	03175	1326
7	8	01697	1317
7	9	03097	1304
7	10	00491	1304
7	11	01498	1212
7	12	01785	1138
7	13	04623	1044
7	14	03793	992
7	15	06294	966
7	16	00744	950
7	17	03253	946
7	18	02095	929
7	19	04176	914
7	20	01576	886

h	id	n	m
7	21	00601	813
7	22	00062	802
7	23	02847	788
7	24	07994	769
7	25	02887	743
7	26	03440	743
7	27	00765	705
7	28	01475	676
7	29	04253	659
7	30	01264	645
7	31	06022	640
7	32	05156	615
7	33	01497	612
7	34	05529	563
7	35	06357	559
7	36	01757	550
7	37	04651	535
7	38	05220	528
7	39	06296	487
7	40	02651	484
7	41	00604	479
7	42	02019	468
7	43	01519	457
7	44	01575	435
7	45	00387	428
7	46	03562	422
7	47	01596	406
7	48	06426	357
7	49	03218	341
7	50	01061	326
7	51	03406	326
7	52	01474	318
7	53	06018	317
7	54	06310	306
7	55	09403	292
7	56	00467	281
7	57	03409	261
7	58	09555	216
7	59	08642	184
7	60	08229	178
7	61	06953	171

h	id	n	m
7	62	08246	142
7	63	03004	139
7	64	08362	139
7	65	05491	130
7	66	08910	128
7	67	05927	113
7	68	07698	86
7	69	00576	76
7	70	09913	74
7	71	08161	63
7	72	03624	61
7	73	09922	61
7	74	08919	58
7	75	05839	57
7	76	07389	56
7	77	07765	47
7	78	07943	47
7	79	03713	46
7	80	09531	45
7	81	08923	41
7	82	09525	40
7	83	04929	39
7	84	04020	37
7	85	03185	36
7	86	09471	34
7	87	03881	32
7	88	05106	31
7	89	03360	30
7	90	09149	29
7	91	03885	29
7	92	07935	29
7	93	05387	29
7	94	02916	28
7	95	02904	26
7	96	05342	26
7	97	05470	25
7	98	05480	25
7	99	09068	24
7	100	04014, 07005, 08239, 01829, 09766, 04703, 06976, 09176, 09903, 00722, 03217, 06711, 01743, 03783, 08236, 04691, 06473, 05041, 09523, 06602, 06461, 08883, 08955	206

h	id	n	m
8	1	02436	2580
8	2	01453	2562
8	3	04882	2488
8	4	00402	2243
8	5	05671	1946
8	6	05158	1779
8	7	06203	1772
8	8	01870	1732
8	9	01457	1725
8	10	02187	1549
8	11	00227	1529
8	12	05157	1431
8	13	00125	1408
8	14	06257	1366
8	15	06273	1210
8	16	01025	1131
8	17	05708	1110
8	18	02924	1088
8	19	00732	1046
8	20	01286	984
8	21	05625	979
8	22	04143	975
8	23	03336	951
8	24	02777	914
8	25	06207	902
8	26	02231	898
8	27	08606	886
8	28	09891	861
8	29	03554	845
8	30	05763	765
8	31	01441	750
8	32	05166	742
8	33	02354	690
8	34	01775	673
8	35	01257	651
8	36	03040	598
8	37	08039	593
8	38	05333	585
8	39	03013	578
8	40	01391	526
8	41	04144	523

h	id	n	m
8	42	01671	500
8	43	02774	495
8	44	06304	484
8	45	05814	477
8	46	05138	475
8	47	01561	453
8	48	04598	444
8	49	06184	401
8	50	05691	392
8	51	01973	366
8	52	02685	361
8	53	01782	346
8	54	00138	318
8	55	04546	304
8	56	06358	267
8	57	06656	263
8	58	09296	210
8	59	07055	190
8	60	07770	172
8	61	05252	169
8	62	07757	140
8	63	09450	109
8	64	08771	102
8	65	03949	96
8	66	07776	95
8	67	07431	91
8	68	03694	75
8	69	09752	56
8	70	01283	55
8	71	07973	53
8	72	09317	45
8	73	02900	44
8	74	08314	39
8	75	03611	39
8	76	09779	39
8	77	07585	38
8	78	09923	38
8	79	08570	37
8	80	04717	37
8	81	08659	35
8	82	04029	35

h	id	n	m
8	83	08655	33
8	84	04018	31
8	85	03956	31
8	86	00339	30
8	87	03846	30
8	88	05434	29
8	89	03991	28
8	90	08580, 05391, 09374, 04960, 08755, 05984, 09668, 09478, 09725, 08700, 08792, 03965, 03062	178
9	1	02088	2426
9	2	03540	2297
9	3	02772	2195
9	4	00679	2181
9	5	01950	2119
9	6	04226	2108
9	7	00025	2104
9	8	04848	2091
9	9	04931	2079
9	10	05959	2063
9	11	08997	2027
9	12	06276	2014
9	13	05770	2013
9	14	06503	1999
9	15	06265	1989
9	16	06487	1988
9	17	05315	1961
9	18	06171	1953
9	19	04407	1897
9	20	03256	1868
9	21	00814	1860
9	22	00645	1809
9	23	03242	1768
9	24	02034	1732
9	25	01801	1691
9	26	04340	1685
9	27	06428	1680
9	28	03092	1673
9	29	01512	1641
9	30	09050	1620
9	31	05705	1610
9	32	01840	1593

h	id	n	m
9	33	01003	1543
9	34	05419	1528
9	35	05596	1517
9	36	01634	1487
9	37	01359	1475
9	38	07680	1419
9	39	06019	1419
9	40	02729	1396
9	41	02130	1385
9	42	02207	1352
9	43	02798	1337
9	44	02437	1322
9	45	06288	1320
9	46	01944	1311
9	47	00291	1250
9	48	05880	1234
9	49	00385	1233
9	50	03015	1230
9	51	00065	1206
9	52	00833	1191
9	53	02149	1175
9	54	02105	1154
9	55	02089	1152
9	56	00525	1113
9	57	00739	1079
9	58	02030	1067
9	59	01222	1056
9	60	04069	1029
9	61	07786	1018
9	62	03664	985
9	63	01666	926
9	64	08376	892
9	65	03084	890
9	66	05674	885
9	67	02507	872
9	68	07024	860
9	69	06683	843
9	70	06177	834
9	71	01137	809
9	72	05957	803
9	73	00054	728

h	id	n	m
9	74	05879	691
9	75	00721	685
9	76	03505	681
9	77	09415	663
9	78	00119	631
9	79	09481	625
9	80	06591	614
9	81	08456	602
9	82	04458	593
9	83	03216	587
9	84	06681	583
9	85	04071	570
9	86	05788	549
9	87	05847	522
9	88	09372	513
9	89	03079	506
9	90	00421	488
9	91	08471	485
9	92	05887	478
9	93	02414	458
9	94	07912	458
9	95	04818	457
9	96	04575	452
9	97	01069	449
9	98	04088	445
9	99	07833	442
9	100	01979	441
9	101	09154	436
9	102	03031	419
9	103	02683	412
9	104	05762	387
9	105	02009	378
9	106	07728	364
9	107	06862	355
9	108	05594	344
9	109	05071	338
9	110	09649	331
9	111	08454	324
9	112	02206	317
9	113	03261	297
9	114	04554	294

h	id	n	m
9	115	09624	289
9	116	04237	284
9	117	06450	224
9	118	04846	223
9	119	08472	218
9	120	06693	207
9	121	08455	206
9	122	08422	163
9	123	01244	155
9	124	09306	141
9	125	07575	139
9	126	00103	135
9	127	02758	122
9	128	06724	120
9	129	07599	107
9	130	09606	89
9	131	08487	83
9	132	05970	83
9	133	05006	76
9	134	00964	70
9	135	03080	57
9	136	08670	52
9	137	09396	50
9	138	07745	47
9	139	02620	40
9	140	03969	39
9	141	06011, 09354, 06324, 06168, 09929, 05512, 03381, 01066, 06995, 04761, 08686, 03912, 09330, 09033	146

Selection Technique:

Systematic sampling is a suitable technique. For school h_i

- Calculate the sampling interval $k_h = MOS_{hi}/n_h$.
- Choose a random starting number between 1 and k_{hi}.
- Select the student at the random start position and every $k'_{hi}th$ student thereafter from the ordered roster.
- If schools are linked due to insufficient numbers, the rosters need to be combined and sampled uniformly.

- Record unresponsive students and report adjusted weights.

4. Write down the overall sampling fraction based on the stratified PPeS design, indicating the overall probability of inclusion for a given student, from a given school (or linked set of schools), in a given stratum. Be careful with notation. Keep in mind that the MOS values used for the sampled schools at the first stage and the denominator at the second stage (Did you sample a single school? Or a linked set of schools?) will depend on your response to Task #2 above

- The overall sampling fraction is $f = \frac{n}{N} = \frac{4{,}721}{830138} = .0057$
- The inclusion probability for a given student is $P_{hi} = \frac{n_h \times MOS_{hi}}{MOS_h} \times \frac{m_h}{MOS_{hi}} = \frac{n_h \times m_h}{MOS_h}$.

```
linked_schools<- linked_schools%>%
  group_by(Region)%>%
  mutate(P_h = n_h*total_mos/M_h,
         P_i=m_h_star/total_mos,
         Prob=P_h*P_i,
         epsem_check = abs(Prob - mean(Prob)) < 1e-6)

stopifnot(all(linked_schools$epsem_check))

# check for false
linked_schools |>
  group_by(Region) |>
  reframe(Schools=n(), Students = sum(m_h_star )) |>
  kable()
```

Region	Schools	Students
1	2	40.51031
2	5	77.84100
3	6	98.18716
4	8	110.48473
5	7	107.54401
6	18	282.68483
7	122	1958.99939
8	102	1634.95104
9	154	2486.85066

SM 625: Week 11 Sampling Project Notes

```r
# Step 1:  Calculate how many students to sample
f_overall <- samp_frac
oneschool <- school_frame
MOS_7 <- 242
ACT_MOS_7 <- nrow(oneschool)
M_h_7 <- 191992
m_h_start_7 <- ceiling(16.29896)
# Step 2: Sampling rate
sam_rate <- m_h_start_7/ACT_MOS_7


# Step 2: Calculate sampling interval
k_interval <- ACT_MOS_7 / m_h_start_7
round_k_interval <- k_interval*100000
round_mos <- 219*100000+99999



# Step 3: Random start between 1 and interval
set.seed(123)
start <- sample(1:round_k_interval, 1)

# Step 4: Select every `interval`-th student starting from `start`
indices <- seq(start, by = round_k_interval, length.out = m_h_start_7)
true_indices <- floor(indices/100000)
sampled_students <- oneschool[true_indices, ]

# View sampled students
sampled_students |> kable()
```

BCODE	BNAME	DCODE	District_Name	Region	County_ID	County_Name	Public/nonpublic	tot_gr7	tot_gr8	tot_gr9	tot_gr10	tot_gr11	tot_gr12	tot_all
06812	Malcolm High School	17010	Sault Ste. Marie Area Schools	4	17	Chippewa	Public	0	0	0	9	17	38	64
03568	SPRING VALE ACADEMY	78110	NA	6	83	Wexford	nonpublic	0	0	7	14	13	17	51

BCODE	BNAME	DCODE	District_Name	Region	County_ID	County_Name	Public/nonpublic	g7_tot	g8_tot	g9_tot	g10_tot	g11_tot	g12_tot	tot_all
00921	DIVINE CHILD ELEMENTARY SCH	82030	NA	7	41	Kent	nonpublic	93	98	0	0	0	0	191
03217	Riverside School	11670	Hagar Township S/D #6	7	11	Berrien	Public	3	6	0	0	0	0	9
05487	ST STEPHEN PARISH SCHOOL	73010	NA	7	13	Calhoun	nonpublic	32	32	0	0	0	0	64
08007	REFORMED HERITAGE CHRISTIAN	39010	NA	7	11	Berrien	nonpublic	7	3	4	3	3	4	24
09903	Alpha House	64080	Shelby Public Schools	7	64	Oceana	Public	3	2	2	3	0	0	10
02436	Howell High School	47070	Howell Public Schools	8	47	Livingston	Public	0	1	678	633	636	632	2580
05150	Alternative Education	47060	Hartland Consolidated Schools	8	47	Livingston	Public	0	0	6	15	33	43	97
08061	LOYOLA HIGH SCHOOL	82010	NA	8	78	Shiawassee	nonpublic	0	0	43	43	42	31	159
00165	Bad Axe High School	32010	Bad Axe Public Schools	9	32	Huron	Public	0	0	102	96	131	114	443
01799	Huron High School	82340	Huron School District	9	82	Wayne	Public	0	0	235	242	185	225	887
03232	COWDEN LAKE BIBLE ACADEMY	59090	NA	9	82	Wayne	nonpublic	0	3	2	2	1	1	9

BCODE	BNAME	DCODE	District_Name	Region	County_ID	County_Name	Public/nonpublic	tot_g7	tot_g8	tot_g9	tot_g10	tot_g11	tot_g12	total_all
04950	Annapolis High School	82040	Dearborn Heights School District #7	9	82	Wayne	Public	0	0	240	316	148	120	824
06613	STURGIS CHRISTIAN SCHOOL	75010	NA	9	63	Oakland	nonpublic	1	2	1	1	1	3	9
08560	Cesar Chavez Middle School	82918	Cesar Chavez Academy	9	82	Wayne	Public	175	163	0	0	0	0	338
09612	Blanche Kelso Bruce Academy-St. Jude's	82971	Blanche Kelso Bruce Academy	9	82	Wayne	Public	1	4	4	1	0	0	10

SM 625: Week 11 Sampling Project Notes

By now, you should have noted from the sampling frame that one approach for sorting the schools within a region is by grade level of the schools (middle, generally including grades 7 and 8, and high, generally including grades 9 through 12). We would want to reduce the chance of a random sample of schools within a region only including students from grades 7 and 8 by sorting our list in this fashion.

This week, you have been provided with the actual classroom rosters from a randomly sampled middle school according to your design (see the file "sample_school_student_list.xls" on Canvas). Suppose that the randomly sampled middle school was from Region 7, and the MOS for this school was 242. At this point, you have determined the m_h needed from Region 7 to maintain epsem overall (see last week's project notes). Given the actual classroom rosters, what is the actual size of this school? Assuming that this school was not linked with any other schools, what is the sampling rate that you would apply to this school to achieve epsem? And what would your expected actual sample size be, once you apply this rate to the actual roster?

Given your plan for within-school sampling developed last week, describe your approach to selecting the sample at your specified rate, and then implement that technique to actually select the sample. You can provide the resulting sample as an Appendix for your final project, but the selection technique needs to be clearly described in the body of your report. Ultimately, your description of this process should enable readers to understand what would happen to select the sample of students within each sampled school.

- Selection Technique: Systematic sampling is a suitable technique. For school hi: • Calculate the sampling interval $k_h i = Mos_{hi}/n_h$ Choose a random starting number between 1 and k_hi.
- Select the student at the random start position and every k_hi-th student thereafter from the ordered roster.
- If schools are linked due to insufficient numbers, the rosters need to be combined and sampled uniformly.
- Record unresponsive students and report adjusted weights.

The overall sampling fraction is

$$f = \frac{n}{N} = \frac{4,721}{830138} = .0057$$

The inclusion probability for a given student is

$$P_{hi} = \frac{a_h \times MOS_{hi}}{MOS_h} \times \frac{m_h}{MOS_{hi}} = \frac{a_h \times m_h}{MOS_h}$$

The number of students to sample from this school (based on MOS) is:

$$m_{hi} = f \cdot M_{hi} = 0.0057 \cdot 242$$

$$\text{Sampling Rate} = \frac{m_{hi}}{N_{hi}} = \frac{2}{219}$$

The actual size is 219. The sampling rate should be 0.07762557. The expected actual expected sample size is 17.

```r
# Step 1:  Calculate how many students to sample
f_overall <- 0.0057
oneschool <- school_frame
MOS_7 <- 242
ACT_MOS_7 <- nrow(oneschool)
M_h_7 <- 191992
m_h_start_7 <- ceiling(16.29896)
# Step 2: Sampling rate
sam_rate <- m_h_start_7/ACT_MOS_7


# Step 2: Calculate sampling interval
k_interval <- ACT_MOS_7 / m_h_start_7
round_k_interval <- k_interval*100000
round_mos <- 219*100000+99999



# Step 3: Random start between 1 and interval
set.seed(123)
start <- sample(1:round_k_interval, 1)

# Step 4: Select every `interval`-th student starting from `start`
indices <- seq(start, by = round_k_interval, length.out = m_h_start_7)
true_indices <- floor(indices/100000)
sampled_students <- oneschool[true_indices, ]

# View sampled students
sampled_students |>
  kable()
```

BCODE	BNAME	DCODE	District_Name	Region	County	County_Name	Public_Nonpublic	tot_g7	tot_g8	tot_g9	tot_g10	tot_g11	tot_g12	tot_all
06812	Malcolm High School	17010	Sault Ste. Marie Area Schools	4	17	Chippewa	Public	0	0	0	9	17	38	64
03568	SPRING VALE ACADEMY	78110	NA	6	83	Wexford	nonpublic	0	0	7	14	13	17	51
00921	DIVINE CHILD ELEMENTARY SCH	82030	NA	7	41	Kent	nonpublic	93	98	0	0	0	0	191
03217	Riverside School	11670	Hagar Township S/D #6	7	11	Berrien	Public	3	6	0	0	0	0	9
05487	ST STEPHEN PARISH SCHOOL	73010	NA	7	13	Calhoun	nonpublic	32	32	0	0	0	0	64
08007	REFORMED HERITAGE CHRISTIAN	39010	NA	7	11	Berrien	nonpublic	7	3	4	3	3	4	24
09903	Alpha House	64080	Shelby Public Schools	7	64	Oceana	Public	3	2	2	3	0	0	10
02436	Howell High School	47070	Howell Public Schools	8	47	Livingston	Public	0	1	678	633	636	632	2580
05150	Alternative Education	47060	Hartland Consolidated Schools	8	47	Livingston	Public	0	0	6	15	33	43	97
08061	LOYOLA HIGH SCHOOL	82010	NA	8	78	Shiawassee	nonpublic	0	0	43	43	42	31	159
00165	Bad Axe High School	32010	Bad Axe Public Schools	9	32	Huron	Public	0	0	102	96	131	114	443

BCODE	BNAME	DCODE	District_Name	Region	County_ID	County_Name	Public/nonpublic	tot_g7	tot_g8	tot_g9	tot_g10	tot_g11	tot_g12	tot_all
01799	Huron High School	82340	Huron School District	9	82	Wayne	Public	0	0	235	242	185	225	887
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06613	STURGIS CHRISTIAN SCHOOL	75010	NA	9	63	Oakland	nonpublic	1	2	1	1	1	3	9
08569	Cesar Chavez Middle School	82918	Cesar Chavez Academy	9	82	Wayne	Public	175	163	0	0	0	0	338
09612	Blanche Kelso Bruce Academy-St. Jude's	82971	Blanche Kelso Bruce Academy	9	82	Wayne	Public	1	4	4	1	0	0	10

Week 13

1. Based on the final sample design that your team has developed, formulate a sampling error calculation model that users of your data will be able to employ to estimate sampling variance. That is, what stratum codes will you provide to users? How will you form sampling error computation units (SECUs)? How many SECUs will there be per stratum? What are expected sample sizes per SECU?

```
# Week 13 - Q1: Sampling Error Calculation Model


# Inputs based on project design
a_select <- n_opt # n_opt
b_star <- m_opt # m_opt
```

```r
n_strata_var <- a_select / 2
n_regions <- 9

school_id <- 1:a_select

var_stratum_id <- rep(1:n_strata_var, each = 2)
# drop last
var_stratum_id <- var_stratum_id[1:length(school_id)]

SECU_id <-  1:a_select

expected_sample_size <- b_star

# Each pair of schools forms one variance stratum
# Each school is one SECU
variance_strata <- tibble(
  school_id,
  var_stratum_id,
  SECU_id,
  expected_sample_size,
)

# Summary table for documentation
summary_table <- tibble(
  Description = c(
    "Total Schools Selected (a_select)",
    "Explicit Strata (Regions)",
    "Variance Estimation Strata (Paired PSUs)",
    "SECUs per Variance Stratum",
    "Expected Sample Size per SECU (b*)"
  ),
  Value = c(
    a_select,
    n_regions,
    n_strata_var,
    2,
    b_star
  )
)

kable(summary_table, align = "lc")
```

Description	Value
Total Schools Selected (a_select)	293.22953
Explicit Strata (Regions)	9.00000
Variance Estimation Strata (Paired PSUs)	146.61476
SECUs per Variance Stratum	2.00000
Expected Sample Size per SECU (b*)	76.68084

```
kable(head(variance_strata))
```

school_id	var_stratum_id	SECU_id	expected_sample_size
1	1	1	76.68084
2	1	2	76.68084
3	2	3	76.68084
4	2	4	76.68084
5	3	5	76.68084
6	3	6	76.68084

```
#kable(variance_strata)
```

2. Describe the variance estimation procedures that one would employ to form a confidence interval for one of the three key descriptive parameters. This should build on your proposed SECUs from the first task. How many degrees of freedom will your sampling error calculation model afford? In addition, write the formula for one of the estimated proportions or means; are weights necessary in forming this estimator, given your sample design? That is, is your design epsem, or will weights be needed to compensate for unequal probabilities of selection?

```
df <- a_select / 2           # Degrees of freedom = number of variance strata
num_strata <- df


# Simulate SECU-level estimates for a descriptive proportion (e.g., smoked a cigarette)
set.seed(9999)
cig_lower <- MI_school_samples_table |>
  slice(1) |>
  pull(lower)

cig_upper <- MI_school_samples_table |>
  slice(1) |>
```

```r
  pull(upper)

p_secu_1 <- runif(num_strata, cig_lower, cig_upper)  # SECU 1 estimates

mj_lower <- MI_school_samples_table |>
  slice(2) |>
  pull(lower)

mj_upper <- MI_school_samples_table |>
  slice(2) |>
  pull(upper)


p_secu_2 <- runif(num_strata, mj_lower, mj_upper)  # SECU 2 estimates


# Paired Difference Variance Estimation
diffs <- p_secu_1 - p_secu_2
var_estimate <- mean(diffs^2) / 2         # Variance across pairs
se_estimate <- sqrt(var_estimate)

# Confidence interval (95%)
t_crit <- qt(0.975, df = df)
estimate_mean <- mean(c(p_secu_1, p_secu_2))
CI_lower <- estimate_mean - t_crit * se_estimate
CI_upper <- estimate_mean + t_crit * se_estimate
```

Degrees of Freedom (df): 146.6148

Standard Error (SE): 0.07179

95% Confidence Interval for estimated proportion:

[0.0575 , 0.3413]

Estimator Formula for Proportion:

p̂ = sum(w_hij * y_hij) / sum(w_hij)

where:

y_hij = 1 if student j in school i of stratum h has the trait (e.g., smoked), 0 otherwise

w_hij = final weight for student hij (includes selection probability, nonresponse, etc.)

Are weights needed? YES.

1. Although the design aimed for EPSEM, weights are necessary in practice.

2. Adjustments are needed for:

   - School-level nonresponse (30%)

   - Student-level nonresponse (70%)

3. Weights also adjust for second-stage linking or other deviations during implementation.

3. Keep in mind the client's request for estimates and inference related to a 20% subclass. Will confidence intervals for the subclass be formed in the same way? Are your SECUs large enough to accommodate this request?

```r
total_secus <- a_select            # Number of SECUs (schools)
expected_b_star <- b_star          # Expected completes per SECU
subclass_pct <- 0.20          # Subclass proportion (20%)
df <- total_secus / 2         # Degrees of freedom remains the same

# Estimate expected subclass size per SECU
expected_subclass_per_secu <- expected_b_star * subclass_pct
```

```
 Subclass Estimation for 20% Group

 Confidence Intervals:

CI for subclass estimates can be formed using the same paired difference method.

Degrees of Freedom remains: 146.6148

 SECU Size Check:

Expected sample size per SECU (b*): 76.68084

Estimated subclass members per SECU: 15.3

 This is generally sufficient for stable variance estimation at the subclass level.
```