SURV625 Applied Sampling

2025-04-14

SM 625: Week 4 Sampling Project Notes

For each of the three variables that will be the focus of the final course project, the Department of Education would like to generate estimates of means and proportions having a coefficient of variation of no more than 0.05. Using the numbers provided to you in the description of the final project, compute estimates of the element variances for each variable. Given these estimates, compute the desired level of precision (the desired sampling variance) for each estimate that corresponds to the desired coefficient of variation.

Now, given the desired levels of precision for each estimate, compute estimates of the necessary sample sizes for each of these three estimates (assuming simple random sampling), ignoring the finite population correction. These will be starting points for the eventual two-stage cluster sample design.

We first build a table to store our results for each week's assignments.

• We also add the expected averages for each outcome variable.

```
# build dataframe with inputs
MI_school_samples <- tibble(
   Outcome = c("smoked_cig", "smoked_mj", "age_approached_to_smoke"),
   type = c("prop", "prop", "mean"),
   desire_cv = rep(.05, 3),
   expect_mean = c(.25, .15, 12),
)
   # calculate element
MI_school_samples |> kable()
```

Outcome	type	desire_cv	expect_mean
smoked_cig smoked_mj	prop prop	$0.05 \\ 0.05$	0.25 0.15
$age_approached_to_smoke$	mean	0.05	12.00

Our process is to:

- 1st, calculate the estimated element variance.
 - For a proportion, to get the element variance we use $\hat{p}(1-\hat{p})$.
 - For a mean, to get the element variance we simply just square the estimated standard deviation $v(\bar{y}) = \sigma^2$.
- 2nd, we calculate the estimated standard error as $se(\hat{p}) = CV \times \hat{p}$.
- 3rd, we compute the desired sampling variance as: $var(\hat{p}) = se(\hat{p})^2$, where $se(\hat{p}) = \sqrt{var(\hat{p})}$

Outcome	desire_cv	expect_mean	var	sd	se	V
smoked_cig	0.05	0.25	0.1875	0.4330127	0.0125	0.0001563

Outcome	desire_cv	expect_mean	var	sd	se	V
smoked_mj	0.05	0.15	0.1275	0.3570714	0.0075	0.0000562
$age_approached_to_smo$	ke 0.05	12.00	1.0000	1.0000000	0.6000	0.3600000

We now estimate the desired sample sizes when we desire a CV =.05 as $n=\frac{s^2}{se^2}$

```
MI_school_samples <- MI_school_samples |>
  mutate(SRS_n = var / V)

MI_school_samples |> select(1, SRS_n) |> kable()
```

Outcome	SRS_n
smoked_cig	1200.000000
$smoked_mj$	2266.666667
$age_approached_to_smoke$	2.777778

SM 625: Week 5 Sampling Project Notes

For this week, we will consider the information available for stratified sampling of students. Eventually you are going to design a stratified cluster sample of students, where the clusters (or PSUs) are schools, but we aren't there yet.

Recall the regions of interest in the sampling project description:

```
school_frame <- read_xls(
   "~/work/d/SURV625project/data/MI_school_frame_head_counts.xls")</pre>
```

```
Region County_ID

1 07, 31, 66
2 22, 27, 36, 55
3 02, 21, 52
4 17, 48, 49, 77
5 01, 04, 06, 16, 20, 26, 35, 60, 65, 68, 69, 71, 72
6 05, 10, 15, 18, 24, 28, 40, 43, 45, 51, 53, 57, 67, 83
7 03, 08, 11, 12, 13, 14, 34, 39, 41, 54, 59, 61, 62, 64, 70, 75, 80
8 09, 19, 23, 25, 29, 30, 33, 37, 38, 46, 47, 56, 73, 78, 81
9 32, 44, 50, 58, 63, 74, 76, 79, 82
```

As "State officials are interested in providing, if at all possible, separate estimates for each of nine education regions in the state, where the regions are defined by groups of counties", we will use these nine regions as strata.

Prepare a table that includes the:

- Overall population counts in each of these nine strata (the total count of students in the target population at each school is in the tot_all column on the sampling frame).
- Given these counts, once you have the working overall sample size (unknown for now and will be decided by your team next week), what is the proportionate allocation plan of that sample of students across these nine strata?

```
# we will use Region, County_ID, and tot_all

# region counts
strata_Prop_allocate <- school_frame |>
group_by(Region) |>
```

Region	M_h	N_h	prop_allocation
1	3561	20	0.0042896
2	5474	30	0.0065941
3	8631	33	0.0103971
4	4855	31	0.0058484
5	18907	80	0.0227757
6	33133	133	0.0399126
7	191992	644	0.2312772
8	188830	549	0.2274682
9	374755	923	0.4514370

what is the proportionate allocation plan of that sample
of students across these nine strata?

SM 625: Week 6 Sampling Project Notes

From a previous study, you obtain estimates of the following design effects for each of these three estimates:

- proportion ever smoked one cigarette = 2.5;
- proportion ever smoked marijuana = 2.0; and
- mean age when first asked to smoke = 1.7.

This previous study featured a sample of size n=7,500 students between the ages of 13 and 19, selected from a total of a=150 clusters. Using this information, compute a synthetic estimate of roh for each of the three variables. These synthetic estimates of roh will be used to consider alternative cluster sample designs as you continue with your project work. Finally, budget and cost information is now available. The total budget for data collection for this project will be \$500,000. The client and the data collection organization estimate that the data collection will cost \$3,000 per primary stage cluster (school), and \$50 per completed questionnaire within a cluster. We will use this cost information moving forward for optimal subsample size calculations.

We can estimate the sample ICC or roh from the given design effect estimate as:

$$\hat{roh} = \frac{deff - 1}{m - 1}$$

We now that the sample total is nm = 7500 and the sample number of cluster is n = 150, which we can take the mean cluster size as m = nm/n = 7500/150 = 50 and use it to calculate roh.

MI_school_samples |> select(Outcome, desire_deff, roh) |> kable()

Outcome	desire_deff	roh
smoked_cig	2.5	0.0306122
$smoked_mj$	2.0	0.0204082
$age_approached_to_smoke$	1.7	0.0142857

SM 625: Week 7 Sampling Project Notes

Recall that the client and the data collection organization estimated that the data collection would cost \$3,000 per primary stage cluster (school), and \$50 per completed questionnaire within a cluster. We will now use this information for optimum subsample size calculations. Recall that the total budget for data collection will be \$500,000.

Given this cost information and your estimates of roh for the three different variables of primary interest from last week, compute the optimum subsample size (and the corresponding optimal number of first stage clusters, given the total budget above) for each of the variables.

- We now have budget constraints and denote the cost per cluster as $c_n = \$3,000$ and cost per element as $c_m = \$50$, with a total budget constraint of C = \$500,000. Since we know there are n = 150 clusters and a total sample size of 7,500 students.
- To compute the optimum m size we use the following equation:

$$m_{opt} = \sqrt{\frac{c_n}{c_m} \frac{1-roh}{roh}}$$

```
c_n = 3000 # cost per cluster
c_m = 50 # cost per element within cluster
C = 500000 # total budget

MI_school_samples <- MI_school_samples |>
    mutate(
        # compute optimum m size
        m_opt = sqrt( (c_n / c_m) * ( (1-roh)/roh) ),
        n_opt = C / (c_n + m_opt * c_m),
        # compute new deff
    deff_new = 1 + (m_opt-1) * roh,
        # compute total SSU
    total_nm = m_opt * n_opt)

MI_school_samples |> select(Outcome, roh, m_opt, n_opt) |>
    kable()
```

Outcome	roh	m_opt	n_opt
$smoked_cig$	0.0306122	43.58899	96.53536
$smoked_mj$	0.0204082	53.66563	87.97734
$age_approached_to_smoke$	0.0142857	64.34283	80.42281

How will you decide on a single overall optimum subsample size to use in your design?

- Above we estimated the new design effects which range from 2.3 to 1.9, which are almost in line with our desired design effects of 2.5, 1.7. Below we print the new design effects, optimum number of cluster and cluster size, total sample size total_nmfor our projected \$500,000 budget for all three outcome variables.
 - Finally, we compute the sampling cost as $n \times c_n + n \times m \times c_m$ which we defined these terms above.

Outcome	roh	m_opt	n_opt	$deff_new$	$total_nm$	$projected_cost$
smoked_cig	0.0306	43.5890	96.5354	2.3037	4207.879	\$500,000
$smoked_mj$	0.0204	53.6656	87.9773	2.0748	4721.359	\$500,000
age_approached_to	_smok@.0143	64.3428	80.4228	1.9049	5174.631	\$500,000

Think about a comparison of alternative cluster sample designs: under a fixed cost constraint, how would we decide which design would be best? What will be your overall sample size (n) under this new optimum subsample size?

As you make progress in writing up what you have done so far, provide some discussion of the rationale for your choices in this regard.

Next, given this optimum subsample size and treating the values of roh as portable, compute the new expected DEFF for each estimate given the new design (this can be specific to each variable / estimate, given the different optimum subsample sizes). In addition, compute a new expected SRS variance for each variable under the new design, using the new "optimum" overall sample size (remember that you can treat the element variances for each variable estimated last week as portable). Finally, compute the new expected sampling variance for each estimate under this new cluster sample design. Are you still meeting the client's precision requirements?

- Given that we have three outcome variables, we also have three optimum number of clusters and cluster size estimates. That is, we can design and examine three options of different optimum number of clusters and cluster sizes.
- We will use the portable roh estimate and calculate new design effects, SRS variance, and complex design variance for each outcome variable.

```
map(seq(1,3), function(x){
  MI school samples |>
  select(Outcome, roh, m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[x], # optimum m from first row
         n_{opt} = n_{opt}[x],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_{new} = 1 + (m_{opt-1}) * roh,
         # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance
         var_srs = var / (total_nm - 1),
         # calculate complex design variance
         var crs = var srs * deff new,
         #m_opt = round(m_opt)
         ) |>
    select(-roh, -var) |>
    mutate_at(2:3, floor) |>
    mutate(total_nm = m_opt*n_opt) |>
    mutate_at(5:7, round, 5) |>
  kable()
}) |>
  set_names(str_c(rep("Option ", 3), seq(1,3)))
```

\$`Option 1`

```
| m_opt| n_opt| total_nm| deff_new| var_srs| var_crs|
| Outcome
                      ---|----:|-----:|-----:|-----:|-----:|
                             43|
|smoked_cig
                                    961
                                            4128 | 2.30374 | 0.00006 | 0.00013 |
|smoked_mj
                              431
                                    961
                                            4128
                                                   1.86916 | 0.00003 | 0.00006 |
|age_approached_to_smoke |
                             43|
                                    961
                                            4128
                                                  1.60841 | 0.00214 | 0.00344 |
```

\$`Option 2`

```
| Outcome
                        | m_opt| n_opt| total_nm| deff_new| var_srs| var_crs|
                      --|----:|-----:|-----:|
|:----
|smoked_cig
                             53|
                                   87|
                                           4611 | 2.61221 | 0.00005 | 0.00013 |
|smoked_mj
                        1
                            53 l
                                   87 I
                                           4611 | 2.07481 | 0.00003 | 0.00006 |
                            53|
                                   87 l
                                           4611 | 1.75237 | 0.00191 | 0.00334 |
|age_approached_to_smoke |
```

\$`Option 3`

Outcome	m_opt	n_opt	total_nm	deff_new	var_srs	var_crs
:	:	:	:	:	:	:
smoked_cig	64	80	5120	2.93907	0.00005	0.00014
smoked_mj	64	80	5120	2.29271	0.00002	0.00006
age_approached_to_smoke	64	80	5120	1.90490	0.00174	0.00331

We print standard error for the complex design with 95% confidence intervals, and we also flag whether the sampling variance from the clustering is equal or smaller than the desired sampling variance.

```
map(seq(1,3), function(x){
  MI_school_samples |>
  select(Outcome, expect_mean, V, roh, m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[x], # optimum m from first row
         n_{opt} = n_{opt}[x],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_{new} = 1 + (m_{opt-1}) * roh,
        # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance
         var_srs = var / (total_nm - 1) ,
         # calculate complex design variance
         var_crs = var_srs * deff_new,
         # compute confidence intervals
         se = sqrt(var_crs),
         lower = expect_mean - 1.96*se,
```

```
upper = expect_mean + 1.96*se,
    # flag if var_crs is lower or = to desired sampling var
    var_ck = ifelse(var_crs <= V, "yes", "no")) |>
    mutate_at(3:5, round, 4) |>
    select(Outcome, expect_mean, se, lower, upper, var_ck) |>
    kable()
}) |>
    set_names(str_c(rep("Option ", 3), seq(1,3)))
```

\$`Option 1`

Outcome	expect_mean	se	lower	upper var_ck	
:	:	: -	: -	: :	-
smoked_cig	0.25	0.0114642	0.2275302	0.2724698 yes	
smoked_mj	0.15	0.0075266	0.1352479	0.1647521 no	
lage_approached_to_smoke	12.00	0.0586597	11.8850269	12.1149731 yes	-

\$`Option 2`

Outcome	expect_mean	se	lower	upper var_ck	
:	:	: -	: -	: :	۱.
smoked_cig	0.25	0.0115245	0.2274120	0.2725880 yes	1
smoked_mj	0.15	0.0074861	0.1353272	0.1646728 yes	1
age_approached_to_smoke	12.00	0.0578024	11.8867072	12.1132928 yes	1

\$`Option 3`

Outcome	expect_mean	se	lower	upper var_c	k
:	:	:	:	: :	
smoked_cig	0.25	0.0116765	0.2271141	0.2728859 yes	
smoked_mj	0.15	0.0075168	0.1352671	0.1647329 no	
age_approached_to_smoke	12.00	0.0575651	11.8871724	12.1128276 yes	-

• Option 2 with a number of cluster of 87 and cluster size of 53 is the design we will choose given that the total sample size of 4,611 is within the allocated budget (\$491,550).

We prefer this model because it stays close to the desired design effects we received from the customer. Additionally, the standard errors we estimate for this second option overall are the smallest resulting in tighter 95% confidence intervals for the expected estimates we were provided. This design in close to option 3, yet we prefer having a slightly smaller SSU if we can increase the number of PSUs sampled since this gives us a cost efficiency.

The client has also provided other new information: the estimated size of the target population is N = 830,138. Given this population size and your overall sample size (n) under the new optimum subsample size computed above, what is your overall working sampling fraction (f)? Does it seem like finite population corrections will be necessary in your sampling variances if you choose to perform SRSWOR at some point?

```
# total pop
N <- 830138

# optimum n
total_nm <- 4721
samp_frac <- total_nm / N; samp_frac</pre>
```

[1] 0.005687006

```
MI_school_samples |> select(Outcome, expect_mean, roh,
                             m_opt, n_opt, total_nm) |>
  # we can print projected total cost for n=50
  mutate(m_opt = m_opt[2], # optimum m from first row
         n_{opt} = n_{opt}[2],
         total_nm = m_opt*n_opt,
         # calculate new deff
         deff_{new} = 1 + (m_{opt}-1) * roh,
         # recalcualte element variance
         var = c(.24, .1275, 9),
         # calcualte SRS variance with sampl_fraction
         var_srs =(1 - samp_frac) * var / (total_nm - 1),
         # calculate complex design variance
         var_crs = var_srs * deff_new,
         # compute confidence intervals
         se = sqrt(var_crs),
         lower = expect_mean - 1.96*se,
         upper = expect_mean + 1.96*se ) |>
  select(Outcome, var_crs, se, lower, upper) |>
```

mutate_at(2:4, round, 5) |>
kable()

Outcome	var_crs	se	lower	upper
$smoked_cig$	0.00013	0.01149	0.22748	0.2725237
$smoked_mj$	0.00006	0.00746	0.13537	0.1646310
$age_approached_to_smoke$	0.00332	0.05764	11.88703	12.1129702

Our overall sampling fraction is .0057. In examining the complex design variances, and recalculating the expected standard error and 95% confidence interva given the sampling fractionl, it does not appear that accounting for a population correction makes a huge impact, and we suggest it will not be necessary in our sampling variance for an SRSWOR design

SM 625: Week 8 Sampling Project Notes

Assume that you will decide to allocate your final computed n_{opt} number of clusters to each of the nine project strata based on the proportions of the total number of students in the population in each stratum (i.e., if 20% of the population of students comes from Region 1, you would sample 20% of your clusters from that region). Describe the first-stage sampling fractions for each stratum, where the total number of schools to sample at the first stage in each stratum is defined by your proportionate allocation of the n_{opt} clusters.

Next your team should extend your design to consider stratified PPeS selection of schools from each of the nine strata at the first stage of your sample design.

You have been provided with a sampling frame that lists the schools within each region. Given the information on the sampling frame, how might you sort this list to achieve implicit stratification within the regions? You can treat the overall student count from a previous year (tot_all) as the measure of size for the PPeS sampling. Given this information, compute your zone size for systematic PPeS sampling within each of the nine strata (regions), and proceed with systematic selection based on fractional intervals to select the allocated number of schools within each stratum using PPeS sampling. What is your first-stage sampling fraction within each of the nine strata?

- Using the proportionate allocation by strata computed earlier, we assign and add cluster allocation by stratum by $n_{opt} \times prop-allocation$.
- nonresponse adjustment is achieved by taking our optimum values and adjusting them by the amount of respondents that are likely to complete the survey.
- We also calculate the zone size which we label as k h as:

$$k_h = \frac{nMOS_i}{\sum_t MOS_i}$$

Region	M_h	N_h	prop_allocation	n_h	f_h	k_h	RN
1	3561	20	0.0042896	1	0.0500000	3561	3168
2	5474	30	0.0065941	2	0.0666667	2737	2310
3	8631	33	0.0103971	3	0.0909091	2877	1321
4	4855	31	0.0058484	2	0.0645161	2428	131
5	18907	80	0.0227757	7	0.0875000	2701	2122
6	33133	133	0.0399126	12	0.0902256	2761	2114
7	191992	644	0.2312772	67	0.1040373	2866	374
8	188830	549	0.2274682	66	0.1202186	2861	380
9	374755	923	0.4514370	130	0.1408451	2883	1673



• To achieve implicit stratification we order the school list sorted by size of student in each region. To compute zone size we use

```
school_frame_sorted <- school_frame |>
   arrange(Region, desc(tot_all))

min_MOS <- m_opt</pre>
```

```
# we link the selected blocks
dat <- school_frame_sorted |>
    group_by(Region) |>
    mutate(
        # assing ids
        id = row_number(),
        # flag if minimum MOS not met
        min_m_req = ifelse(tot_all >= min_MOS, 1 , 0),
        # create links and convert to clusters
        linking = lead(min_m_req, default=1),
        # assign clustering
        cluster = cumsum(lag(linking, default=1)),
        # add cumulative counts
        cumulative_max = cumsum(tot_all),
        cumulative_min = 1 + lag(cumulative_max, default = 0)
        )
```

```
dat_linked <- dat_selected |>
  group_by(Region) |>
  mutate(
    # flag if minimum MOS not met
```

```
min_n_req = ifelse(tot_all >= min_MOS, 1 , 0),
# create links and convert to clusters
linking = lead(min_n_req, default=1),
# assign clustering
cluster = cumsum(lag(linking, default=1))) |>
ungroup()
```

```
# show cluster of blocks selected, total HUs
sample_selected <- map_dfr(1:9, function(x){</pre>
   linkage = dat_linked |>
    filter(Region %in% x, selected==1) |>
    select(Region, cluster) |>
    mutate(Selection = RN_sample[[x]]) |>
     pull(cluster)
  dat = dat_linked |>
  filter(Region %in% x,
         cluster %in% linkage) |>
  # now we link blocks within clusters
  group_by(cluster) |>
  # start clusters at 1
  mutate(cluster = cur_group_id()) |>
  # start with last id to link backwards
  arrange(desc(id))
  return(dat)
}) |>
  ungroup() |>
  # combine region 1 with 2
  mutate(Region = ifelse(Region==1, 2, Region))
write_xlsx(sample_selected,
           "~/work/d/SURV625project/data/sample_selected.xlsx")
```

SM 625: Week 10 Sampling Project Notes

There are four primary tasks for your team to consider over the next week:

- 1. Given your overall m_{opt} $n_o pt$ and N (based on the sampling frame), you've already computed the overall sampling fraction, . For each of the nine strata, compute the required number of students to subsample from each sampled school based on the stratified PPeS design in order to maintain epsem across all strata.
- Within strata, retain epsem for stratified PPS sampling across strata $f = f_h$ for all h.

$$f_h = \frac{n_h MOS_{hi}}{\sum_{i \in h} MOShi} \frac{m_h^*}{MOS_{hi}}$$

BELOW CANNOT BE RIGHT, WE GET THE SAME M SIZE FOR EACH STRATA

- 2. Do each of the schools that you sampled in a given region have the minimum sufficient size, given the stratum-specific subsample sizes computed in Task #1? Do subsequent schools on the list have the minimum sufficient size? If not, what will you do?
 - 2. School 07809 does not meet the requirement of 54 students. The next school in sequence does not meet the minimum requirement with only 29 student, but the next school in sequence does meet the requirement. We will need to link these two schools.
- 3. Begin to describe how you will physically select the subsample of students within a given sampled school (or set of linked schools). What will your second-stage sampling rate be for a given school within a given stratum? How will you acquire the updated rosters from each school? What technique will you use to select the sample at the specified second-stage rate?
 - 2. The linking is done by first selecting n_{opt} schools from each strata. When the next school on the list do not meet the sufficient size required of m_{opt} student size we move down the list until the first school that meets the minimum requirement is achieved. For all the schools that did not meet the requirement they are cumulated backwards until a linked school of minimum sufficient size is created. We do this process for all the schools initially selected.
- 4. Write down the overall sampling fraction based on the stratified PPeS design, indicating the overall probability of inclusion for a given student, from a given school (or linked set of schools), in a given stratum. Be careful with notation. Keep in mind that the MOS values used for the sampled schools at the first stage and the denominator at the second stage (Did you sample a single school? Or a linked set of schools?) will depend on your response to Task #2 above