Bayesian estimation of the interventional (in)direct effects with the interaction model

Data used in JAGS:

X: treatment variable.

M: an N × TT matrix containing observed mediator scores at time point 1,2,...,TT for a sample of N individuals.

Y: an N × TT matrix containing observed outcome scores at time point 1,2,...,TT for a sample of N individuals.

TimeM: a column vector of length TT containing the mediator slope loadings at time point 1, 2, ..., TT.

TimeY: a column vector of length TT containing the outcome slope loadings at time point 1, 2, ..., TT.

Z: an N × numZ matrix containing a total of "numZ" observed pretreatment covariates, where "numZ" is the number of observed pretreatment covariates.

mZ: a column vector containing the mean of **Z**; if **Z** is centered, mZ = rep(0, numZ).

varZ: numZ by numZ diagonal matrix containing the variance-covariance matrix of **Z**; if numZ=1 and **Z** is standardized, varZ = diag(1, nrow=1).

For additional illustrative examples see https://github.com/xliu12/clgcmm.

JAGS model with the diffuse priors:

```
library(R2WinBUGS)
library(R2jags)
```

```
jagsm <- function(){

  for (i in 1:N) {
    for (j in 1:TT) {
      M[i, j] ~ dnorm(mM[i, j], prec_eM)
      mM[i, j] <- ISM[i, 1] + ISM[i, 2] * TimeM[j]
      Y[i, j] ~ dnorm(mY[i, j], prec_eY)
      mY[i, j] <- ISY[i, 1] + ISY[i, 2] * TimeY[j]
    }
  }
  for (i in whichX0) {
    ISM[i, 1:2] ~ dmnorm(mISM[i, 1:2], prec_vMctrl[1:2, 1:2])
  }
  for (i in whichX1) {
    ISM[i, 1:2] ~ dmnorm(mISM[i, 1:2], prec_vMtrt[1:2, 1:2])
  }
  for (i in 1:N) {
    mISM[i, 1] <- a0[1] + aZ[1:numZ, 1] %*% Z[i, 1:numZ] +
      aX[1] * X[i]
    mISM[i, 2] <- a0[2] + aZ[1:numZ, 2] %*% Z[i, 1:numZ] +
```

```
    aX[2] * X[i]
}
for(i in 1:N){

  ISY[i,1:2] ~ dmnorm( mISY[i,1:2], prec_vY[1:2,1:2])
  mISY[i,1]<-b0[1]+bZ[1:numZ, 1] %*% Z[i,1:numZ]+ bX[1]*X[i] +
    bIM[1]*ISM[i,1] + bSM[1]*ISM[i,2] +
    bIMSM[1]*ISM[i,1]*ISM[i,2] +
    bXIM[1]*X[i]*ISM[i,1] + bXSM[1]*X[i]*ISM[i,2] + bXIMSM[1]*X[i]*ISM[i,1]*ISM[i,2]

  mISY[i,2]<-b0[2]+bZ[1:numZ, 2] %*% Z[i,1:numZ]+ bX[2]*X[i] +
    bIM[2]*ISM[i,1] + bSM[2]*ISM[i,2] +
    bIMSM[2]*ISM[i,1]*ISM[i,2] +
    bXIM[2]*X[i]*ISM[i,1] + bXSM[2]*X[i]*ISM[i,2] + bXIMSM[2]*X[i]*ISM[i,1]*ISM[i,2]

}

prec_eM ~ dgamma(0.001, 0.001)
var_eM <- 1/prec_eM
prec_vMctrl[1:2, 1:2] ~ dwish(R, 3)
prec_vMtrt[1:2, 1:2] ~ dwish(R, 3)
Psi_vMtrt <- inverse(prec_vMtrt)
Psi_vMctrl <- inverse(prec_vMctrl)
prec_eY ~ dgamma(0.001, 0.001)
prec_vY[1:2, 1:2] ~ dwish(R, 3)
var_eY <- 1/prec_eY
Psi_vY <- inverse(prec_vY)
for (k in 1:2) {
  a0[k] ~ dnorm(0, 1e-06)
  aX[k] ~ dnorm(0, 1e-06)
  for (z in 1:numZ) {
    aZ[z, k] ~ dnorm(0, 1e-06)
  }
}
for (k in 1:2) {
  b0[k] ~ dnorm(0, 1e-06)
}
for (z in 1:numZ) {
  bZ[z, 1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
}
bX[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bIM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bIMSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXIM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXIMSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])

mIMt <- a0[1] + aX[1]+t(aZ[ ,1])%*%mZ
mIMc <- a0[1] +t(aZ[ ,1])%*%mZ
mSMt <- a0[2] + aX[2]+t(aZ[ ,2])%*%mZ
mSMc <- a0[2] +t(aZ[ ,2])%*%mZ
```

```
  # the estimators of the interventional indirect and direct effects

  # IIE due to the mutual dependence on IY and on SY
  X_mu[1:2] <- (bIMSM[1:2]+bXIMSM[1:2]) * (Psi_vMtrt[1, 2] - Psi_vMctrl[1, 2])
  # pure IIE due to the mutual dependence on IY and on SY
  pX_mu[1:2] <- (bIMSM[1:2]) * (Psi_vMtrt[1, 2] - Psi_vMctrl[1, 2])
  # IIE via IM on IY and on SY
  X_IM[1:2] <- aX[1] * (bIM[1:2] +bXIM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mSMc[1] )
  # pure IIE via IM on IY and on SY
  pX_IM[1:2] <- aX[1] * (bIM[1:2] + (bIMSM[1:2]) * mSMc[1] )
  # IIE via SM on IY and on SY
  X_SM[1:2] <- aX[2] * (bSM[1:2] +bXSM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mIMt[1] )
  # pure IIE via SM on IY and on SY
  pX_SM[1:2] <- aX[2] * (bSM[1:2] + (bIMSM[1:2]) * mIMt[1] )
  # IIE joint on IY and on SY
  X_jo[1:2] <- X_mu[1:2] + X_IM[1:2] + X_SM[1:2]
  # pure IIE joint on IY and on SY
  pX_jo[1:2] <- pX_mu[1:2] + pX_IM[1:2] + pX_SM[1:2]

  # IDE on IY and on SY
  Xde[1:2] <- bX[1:2] +bXIM[1:2]*mIMc + bXSM[1:2]*mSMc +
    bXIMSM[1:2]*( mIMc[1]*mSMc[1] + t(aZ[ ,1])%*%varZ%*%aZ[ ,2] + Psi_vMctrl[1,2] )
  # total IDE on IY and on SY
  tXde[1:2] <- bX[1:2] +bXIM[1:2]*mIMt + bXSM[1:2]*mSMt +
    bXIMSM[1:2]*( mIMt[1]*mSMt[1] + t(aZ[ ,1])%*%varZ%*%aZ[ ,2] + Psi_vMtrt[1,2] )

  # alternative IIE via IM on IY and on SY
  altX_IM[1:2] <- aX[1] * (bIM[1:2] +bXIM[1:2] + (bIMSM[1:2]+bXIMSM[1:2])  * mSMt[1] )
  # pure alternative IIE via IM on IY and on SY
  paltX_IM[1:2] <- aX[1] * (bIM[1:2] + (bIMSM[1:2])  * mSMt[1] )
  # alternative IIE via SM on IY and on SY
  altX_SM[1:2] <- aX[2] * (bSM[1:2] +bXSM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mIMc[1] )
  # pure alternative IIE via SM on IY and on SY
  paltX_SM[1:2] <- aX[2] * (bSM[1:2] + (bIMSM[1:2]) * mIMc[1] )

  # difference between two IIE versions
  diffX_IM[1:2] <- altX_IM[1:2] - X_IM[1:2]
  diffX_SM[1:2] <- altX_SM[1:2] - X_SM[1:2]


}

# write the JAGS model to a file
write.model(jagsm, "jagsmod_interaction_XIMSM.txt")
```

Fitting the interaction model to example data

Example data "dat.RData" can be found at https://github.com/xliu12/clgcmm.

```
load("dat.RData")
# observed mediator scores
M <- dat[ , grep("M",colnames(dat)) ]
# observed outcome scores
Y <- dat[ , grep("Y",colnames(dat)) ]
```

```r
# treatment
X <- dat[, "X"]
# covariates
Z <- dat[ , grep("Z",colnames(dat)) ]


jagsdata <- list(
  M=M, Y=Y, X=X,
  N = nrow(M),
  TT = ncol(M),
  TimeM = ((1:ncol(M))-2),
  TimeY = ((1:ncol(M))-ncol(M)),
  R=diag(1,nrow = 2),
  zero=c(0,0),
  Z=as.matrix(Z), numZ=1,
  mZ=rep(0, 1), # column vector containing the mean of Z
  varZ=diag(1, nrow = 1), # matrix containing the variance-covariance matrix of Z
  whichX0=which(X==0), # row numbers of control group
  whichX1=which(X==1) # row numbers of treatment group
)

jagsout <- jags(data = jagsdata, jags.seed = 123,
      model.file = "jagsmod_interaction_XIMSM.txt",
      parameters.to.save = c(
      "aX", "a0","aZ", "Psi_vMctrl","Psi_vMtrt","var_eM",
      # mediator model parameters
      "bX","bIM","bSM","bIMSM","bXIM","bXSM","bXIMSM","b0","bZ", "Psi_vY", "var_eY",
      # outcome model parameters
      "X_IM","altX_IM","X_SM", "altX_SM","X_mu", "Xde",
      "pX_IM","paltX_IM","pX_SM", "paltX_SM","pX_mu", "tXde",
      # the estimators of the interventional (in)direct effects
      "diffX_IM", "diffX_SM"
      # differences between the alternative and orignal versions
      #of the interventional indirect effects via IM alone and those via SM alone
      ),
      n.chains = 2,
      n.iter = 1e5,
      # could increase the number of iterations if convergence were not achieved
      n.thin = 2
)
```

```r
# jagsout

# if convergence is achieved,
#posterior means and 0.95 percential intervals can be saved as follows
jagsres <- data.frame(jagsout$BUGSoutput$summary[ , c(1,3,7, 8)])
jagsres[grep("X_", rownames(jagsres), value = TRUE), ]
```

```
##                    mean       X2.5.      X97.5.     Rhat
## X_IM[1]     -0.384257526 -0.63917159 -0.14172997 1.001296
## X_IM[2]      0.001308668 -0.13479728  0.13984188 1.000980
## X_SM[1]     -0.234725038 -0.56435113  0.09974685 1.001060
## X_SM[2]     -0.155707214 -0.33816949  0.02590739 1.001033
## X_mu[1]     -0.067477304 -0.11440960 -0.02892757 1.000980
```

```
## X_mu[2]     -0.036660500 -0.06333983 -0.01483453 1.000979
## altX_IM[1]  -0.765734229 -0.95938545 -0.58891548 1.001129
## altX_IM[2]  -0.205815454 -0.29325771 -0.12587087 1.001055
## altX_SM[1]   0.146751665 -0.23154226  0.53743217 1.000983
## altX_SM[2]   0.051416908 -0.16218002  0.26781467 1.001089
## diffX_IM[1] -0.381476703 -0.60817992 -0.17545326 1.001080
## diffX_IM[2] -0.207124122 -0.33363220 -0.08951769 1.001035
## diffX_SM[1]  0.381476703  0.17545326  0.60817992 1.001080
## diffX_SM[2]  0.207124122  0.08951769  0.33363220 1.001035
```

Outputs

The estimation and inference results for the interventional indirect effects (IIEs) and interventional direct effects (IDEs) can be found from the following:

X_IM[1]: the estimated IIE of treatment on outcome intercept via mediator intercept (under the original definition)

altX_IM[1]: the estimated IIE of treatment on outcome intercept via mediator intercept (under the alternative definition)

diffX_IM[1]: the estimated difference between the original (IIE) and alternative (alt.IIE) versions of the IIE of treatment on outcome intercept via mediator intercept

X_SM[1]: the estimated IIE of treatment on outcome intercept via mediator slope (under the original definition)

altX_SM[1]: the estimated IIE of treatment on outcome intercept via mediator slope (under the alternative definition)

diffX_SM[1]: the estimated difference between the original (IIE) and alternative (alt.IIE) versions of the IIE of treatment on outcome intercept via mediator slope

X_mu[1]: the estimated IIE of treatment on outcome intercept due to the mutual dependent between mediator intercept and mediator slope

Xde[1]: the estimated IDE of treatment on outcome intercept

X_IM[2]: the estimated IIE of treatment on outcome slope via mediator intercept (under the original definition)

altX_IM[2]: the estimated IIE of treatment on outcome slope via mediator intercept (under the alternative definition)

diffX_IM[2]: the estimated difference between the original (IIE) and alternative (alt.IIE) versions of the IIE of treatment on outcome slope via mediator intercept

X_SM[2]: the estimated IIE of treatment on outcome slope via mediator slope (under the original definition)

altX_SM[2]: the estimated IIE of treatment on outcome slope via mediator slope (under the alternative definition)

diffX_SM[2]: the estimated difference between the original (IIE) and alternative (alt.IIE) versions of the IIE of treatment on outcome slope via mediator slope

X_mu[2]: the estimated IIE of treatment on outcome slope due to the mutual dependent between mediator intercept and mediator slope

Xde[2]: the estimated IDE of treatment on outcome slope