Bayesian estimation of the interventional (in)direct effects in the LGCMM with the interaction model

Data used in JAGS:

X: treatment variable.

M: an N $\times$ TT matrix containing observed mediator scores at time point 1,2,...,TT for a sample of N individuals.

Y: an N $\times$ TT matrix containing observed outcome scores at time point 1,2,...,TT for a sample of N individuals.

TimeM: a column vector of length TT containing the mediator slope loadings at time point 1, 2, ..., TT.

TimeY: a column vector of length TT containing the outcome slope loadings at time point 1, 2, ..., TT.

Z: an N $\times$ numZ matrix containing a total of "numZ" observed pretreatment covariates, where "numZ" is the number of observed pretreatment covariates.

mZ: a column vector containing the mean of **Z**; if **Z** is centered, mZ = rep(0, numZ).

varZ: numZ by numZ diagonal matrix containing the variance-covariance matrix of **Z**; if numZ=1 and **Z** is standardized, varZ = diag(1, nrow=1).

For additional illustrative examples see https://github.com/xliu12/clgcmm.

JAGS model with the diffuse priors:

```
library(R2WinBUGS)
library(R2jags)
```

```
jagsm <- function(){

  for (i in 1:N) {
    for (j in 1:TT) {
      M[i, j] ~ dnorm(mM[i, j], prec_eM)
      mM[i, j] <- ISM[i, 1] + ISM[i, 2] * TimeM[j]
      Y[i, j] ~ dnorm(mY[i, j], prec_eY)
      mY[i, j] <- ISY[i, 1] + ISY[i, 2] * TimeY[j]
    }
  }
  for (i in whichX0) {
    ISM[i, 1:2] ~ dmnorm(mISM[i, 1:2], prec_vMctrl[1:2, 1:2])
  }
  for (i in whichX1) {
    ISM[i, 1:2] ~ dmnorm(mISM[i, 1:2], prec_vMtrt[1:2, 1:2])
  }
  for (i in 1:N) {
    mISM[i, 1] <- a0[1] + aZ[1:numZ, 1] * Z[i, 1:numZ] +
      aX[1] * X[i]
    mISM[i, 2] <- a0[2] + aZ[1:numZ, 2] * Z[i, 1:numZ] +
```

```
    aX[2] * X[i]
}
for(i in 1:N){

  ISY[i,1:2] ~ dmnorm( mISY[i,1:2], prec_vY[1:2,1:2])
  mISY[i,1]<-b0[1]+bZ[1:numZ, 1]*Z[i,1:numZ]+ bX[1]*X[i] +
    bIM[1]*ISM[i,1] + bSM[1]*ISM[i,2] +
    bIMSM[1]*ISM[i,1]*ISM[i,2] +
    bXIM[1]*X[i]*ISM[i,1] + bXSM[1]*X[i]*ISM[i,2] + bXIMSM[1]*X[i]*ISM[i,1]*ISM[i,2]

  mISY[i,2]<-b0[2]+bZ[1:numZ, 2]*Z[i,1:numZ]+ bX[2]*X[i] +
    bIM[2]*ISM[i,1] + bSM[2]*ISM[i,2] +
    bIMSM[2]*ISM[i,1]*ISM[i,2] +
    bXIM[2]*X[i]*ISM[i,1] + bXSM[2]*X[i]*ISM[i,2] + bXIMSM[2]*X[i]*ISM[i,1]*ISM[i,2]

}

prec_eM ~ dgamma(0.001, 0.001)
var_eM <- 1/prec_eM
prec_vMctrl[1:2, 1:2] ~ dwish(R, 3)
prec_vMtrt[1:2, 1:2] ~ dwish(R, 3)
Psi_vMtrt <- inverse(prec_vMtrt)
Psi_vMctrl <- inverse(prec_vMctrl)
prec_eY ~ dgamma(0.001, 0.001)
prec_vY[1:2, 1:2] ~ dwish(R, 3)
var_eY <- 1/prec_eY
Psi_vY <- inverse(prec_vY)
for (k in 1:2) {
  a0[k] ~ dnorm(0, 1e-06)
  aX[k] ~ dnorm(0, 1e-06)
  for (z in 1:numZ) {
    aZ[z, k] ~ dnorm(0, 1e-06)
  }
}
for (k in 1:2) {
  b0[k] ~ dnorm(0, 1e-06)
}
for (z in 1:numZ) {
  bZ[z, 1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
}
bX[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bIM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bIMSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXIM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])
bXIMSM[1:2] ~ dmnorm(zero, prec_vY[1:2, 1:2])

mIMt <- a0[1] + aX[1]+t(aZ[ ,1])%*%mZ
mIMc <- a0[1] +t(aZ[ ,1])%*%mZ
mSMt <- a0[2] + aX[2]+t(aZ[ ,2])%*%mZ
mSMc <- a0[2] +t(aZ[ ,2])%*%mZ
```

```
  # the estimators of the interventional indirect and direct effects

  # IIE due to the mutual dependence on IY and on SY
  X_mu[1:2] <- (bIMSM[1:2]+bXIMSM[1:2]) * (Psi_vMtrt[1, 2] - Psi_vMctrl[1, 2])
  # pure IIE due to the mutual dependence on IY and on SY
  pX_mu[1:2] <- (bIMSM[1:2]) * (Psi_vMtrt[1, 2] - Psi_vMctrl[1, 2])
  # IIE via IM on IY and on SY
  X_IM[1:2] <- aX[1] * (bIM[1:2] +bXIM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mSMc[1] )
  # pure IIE via IM on IY and on SY
  pX_IM[1:2] <- aX[1] * (bIM[1:2] + (bIMSM[1:2]) * mSMc[1] )
  # IIE via SM on IY and on SY
  X_SM[1:2] <- aX[2] * (bSM[1:2] +bXSM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mIMt[1] )
  # pure IIE via SM on IY and on SY
  pX_SM[1:2] <- aX[2] * (bSM[1:2] + (bIMSM[1:2]) * mIMt[1] )
  # IIE joint on IY and on SY
  X_jo[1:2] <- X_mu[1:2] + X_IM[1:2] + X_SM[1:2]
  # pure IIE joint on IY and on SY
  pX_jo[1:2] <- pX_mu[1:2] + pX_IM[1:2] + pX_SM[1:2]

  # IDE on IY and on SY
  Xde[1:2] <- bX[1:2] +bXIM[1:2]*mIMc + bXSM[1:2]*mSMc +
    bXIMSM[1:2]*( mIMc[1]*mSMc[1] + t(aZ[ ,1])%*%varZ%*%aZ[ ,2] + Psi_vMctrl[1,2] )
  # total IDE on IY and on SY
  tXde[1:2] <- bX[1:2] +bXIM[1:2]*mIMt + bXSM[1:2]*mSMt +
    bXIMSM[1:2]*( mIMt[1]*mSMt[1] + t(aZ[ ,1])%*%varZ%*%aZ[ ,2] + Psi_vMtrt[1,2] )

  # alternative IIE via IM on IY and on SY
  altX_IM[1:2] <- aX[1] * (bIM[1:2] +bXIM[1:2] + (bIMSM[1:2]+bXIMSM[1:2])  * mSMt[1] )
  # pure alternative IIE via IM on IY and on SY
  paltX_IM[1:2] <- aX[1] * (bIM[1:2] + (bIMSM[1:2])  * mSMt[1] )
  # alternative IIE via SM on IY and on SY
  altX_SM[1:2] <- aX[2] * (bSM[1:2] +bXSM[1:2] + (bIMSM[1:2]+bXIMSM[1:2]) * mIMc[1] )
  # pure alternative IIE via SM on IY and on SY
  paltX_SM[1:2] <- aX[2] * (bSM[1:2] + (bIMSM[1:2]) * mIMc[1] )

  # difference between two IIE versions
  diffX_IM[1:2] <- altX_IM[1:2] - X_IM[1:2]
  diffX_SM[1:2] <- altX_SM[1:2] - X_SM[1:2]


}

# write the JAGS model to a file
write.model(jagsm, "jagsmod_interaction_XIMSM.txt")
```

Fitting the interaction model to example data

Example data “dat.RData” can be found at https://github.com/xliu12/clgcmm.

```
load("dat.RData")
# observed mediator scores
M <- dat[ , grep("M",colnames(dat)) ]
# observed outcome scores
Y <- dat[ , grep("Y",colnames(dat)) ]
```

```r
# treatment
X <- dat[, "X"]
# covariates
Z <- dat[ , grep("Z",colnames(dat)) ]


jagsdata <- list(
  M=M, Y=Y, X=X,
  N = nrow(M),
  TT = ncol(M),
  TimeM = ((1:ncol(M))-2),
  TimeY = ((1:ncol(M))-ncol(M)),
  R=diag(1,nrow = 2),
  zero=c(0,0),
  Z=as.matrix(Z), numZ=1,
  mZ=rep(0, 1), # column vector containing the mean of Z
  varZ=diag(1, nrow = 1), # matrix containing the variance-covariance matrix of Z
  whichX0=which(X==0), # row numbers of control group
  whichX1=which(X==1) # row numbers of treatment group
)

jagsout <- jags(data = jagsdata, jags.seed = 123,
      model.file = "jagsmod_interaction_XIMSM.txt",
      parameters.to.save = c(
      "aX", "a0","aZ", "Psi_vMctrl","Psi_vMtrt","var_eM",
      # mediator model parameters
      "bX","bIM","bSM","bIMSM","bXIM","bXSM","bXIMSM","b0","bZ", "Psi_vY", "var_eY",
      # outcome model parameters
      "X_IM","altX_IM","X_SM", "altX_SM","X_mu", "Xde",
      "pX_IM","paltX_IM","pX_SM", "paltX_SM","pX_mu", "tXde",
      # the estimators of the interventional (in)direct effects
      "diffX_IM", "diffX_SM"
      # differences between the alternative and orignal versions
      #of the interventional indirect effects via IM alone and those via SM alone
      ),
      n.chains = 2,
      n.iter = 1e5,
      # could increase the number of iterations if convergence were not achieved
      n.thin = 2
)
```

```r
# jagsout

# if convergence is achieved,
#posterior means and 0.95 percential intervals can be saved as follows
jagsres <- data.frame(jagsout$BUGSoutput$summary[ , c(1,3,7, 8)])
jagsres[c(grep("X_", rownames(jagsres), value = TRUE),
          grep("pX_", rownames(jagsres), value = TRUE),
          grep("tX_", rownames(jagsres), value = TRUE)
          ), ]
```

```
##                     mean        X2.5.       X97.5.     Rhat
## X_IM[1]     -0.383545256 -0.641497945 -0.138274237 1.001332
## X_IM[2]      0.002472751 -0.135785361  0.140677527 1.001027
```

```
## X_SM[1]       -0.232616283 -0.561915166  0.101209227 1.001352
## X_SM[2]       -0.156098096 -0.338661085  0.026993776 1.001289
## X_mu[1]       -0.067597434 -0.114706461 -0.028890169 1.001000
## X_mu[2]       -0.036782789 -0.063315648 -0.014741241 1.000986
## altX_IM[1]    -0.764702358 -0.958482023 -0.589065463 1.001336
## altX_IM[2]    -0.204855486 -0.291417441 -0.125686996 1.001281
## altX_SM[1]     0.148540818 -0.230443531  0.535247291 1.001161
## altX_SM[2]     0.051230141 -0.162097740  0.268406437 1.001273
## diffX_IM[1]   -0.381157101 -0.606345051 -0.175155488 1.001010
## diffX_IM[2]   -0.207328237 -0.334340022 -0.089975820 1.000994
## diffX_SM[1]    0.381157101  0.175155488  0.606345051 1.001010
## diffX_SM[2]    0.207328237  0.089975820  0.334340022 1.000994
## pX_IM[1]      -0.366782503 -0.481936728 -0.259910612 1.001065
## pX_IM[2]       0.051333810 -0.002412748  0.107285786 1.000985
## pX_SM[1]      -0.024819414 -0.362163166  0.308845609 1.001017
## pX_SM[2]      -0.201786561 -0.392722626 -0.014863219 1.001034
## pX_mu[1]      -0.036805200 -0.087959747  0.008460058 1.000981
## pX_mu[2]      -0.019518940 -0.047846821  0.005558717 1.000979
## paltX_IM[1]   -0.574563341 -0.878923226 -0.296977266 1.001004
## paltX_IM[2]   -0.058500321 -0.215156637  0.093971655 1.000982
## paltX_SM[1]    0.182961424 -0.059717157  0.428267289 1.001085
## paltX_SM[2]   -0.091952430 -0.231261946  0.046408614 1.001099
## pX_IM[1].1    -0.366782503 -0.481936728 -0.259910612 1.001065
## pX_IM[2].1     0.051333810 -0.002412748  0.107285786 1.000985
## pX_SM[1].1    -0.024819414 -0.362163166  0.308845609 1.001017
## pX_SM[2].1    -0.201786561 -0.392722626 -0.014863219 1.001034
## pX_mu[1].1    -0.036805200 -0.087959747  0.008460058 1.000981
## pX_mu[2].1    -0.019518940 -0.047846821  0.005558717 1.000979
## altX_IM[1].1  -0.764702358 -0.958482023 -0.589065463 1.001336
## altX_IM[2].1  -0.204855486 -0.291417441 -0.125686996 1.001281
## altX_SM[1].1   0.148540818 -0.230443531  0.535247291 1.001161
## altX_SM[2].1   0.051230141 -0.162097740  0.268406437 1.001273
## paltX_IM[1].1 -0.574563341 -0.878923226 -0.296977266 1.001004
## paltX_IM[2].1 -0.058500321 -0.215156637  0.093971655 1.000982
## paltX_SM[1].1  0.182961424 -0.059717157  0.428267289 1.001085
## paltX_SM[2].1 -0.091952430 -0.231261946  0.046408614 1.001099
```

Outputs

The estimation and inference results for the interventional indirect effects (IIEs) and interventional direct effects (IDEs) can be found from the following:

X_IM[1:2]: the estimated IIEs of treatment on outcome intercept and slope, via mediator intercept (under the original definition)

altX_IM[1:2]: the estimated IIEs of treatment on outcome intercept and slope, via mediator intercept (under the alternative definition)

pX_IM[1:2] and paltX_IM[1:2]: estimates of the pure versions of X_IM[1:2] and altX_IM[1:2] (defined by fixing the treatment at x=0), respectively

X_SM[1:2]: the estimated IIEs of treatment on outcome intercept and slope, via mediator slope (under the original definition)

altX_SM[1:2]: the estimated IIEs of treatment on outcome intercept and slope, via mediator slope (under the alternative definition) pX_SM[1:2] and paltX_SM[1:2]: estimates of the pure versions of X_SM[1:2] and altX_SM[1:2] (defined by fixing the treatment at x=0), respectively

X_mu[1:2]: the estimated IIEs of treatment on outcome intercept and slope, due to the mutual dependent between mediator intercept and mediator slope

pX_mu[1:2]: estimates of the pure version of X_mu[1:2]

Xde[1:2]: the estimated IDEs of treatment on outcome intercept and slope

tXde[1:2]: estimates of the total version of Xde[1:2] (defined by fixing the potential joint distribution of mediator intercept and slope at that under treatment)