There're 48 decision variables, which are Rtons (R in description), Otons (O in description), Soldtons(S in description) and Storagetons(ST in description) in 12 months. The optimal value is 190550 by producing 600, 450, 425, 800, 800, 800, 800, 800, 800, 500, 500, 500 tons of production in regular time corresponding to each month, by producing 0, 0, 0, 100, 250, 500, 500, 500, 500, 0, 0, 0 tons of production in overtime corresponding to each month, by selling 400, 700, 425, 900, 900, 900, 800, 600, 800, 1200, 1100, 1400 tons of production corresponding each month, and by storing 250, 0, 0, 0, 150, 550, 1050, 1750, 2250, 1550, 950, 50 corresponding to each month.

Slack Variables

The slack variables for the first, second, and third constraints (Row 2 - 14) are equal to zero, since it's an equality in the equation.

Then, we will look at the slack variables for the constraints of limited raw materials. The slack variables of month 1, 2, 3, 9, 10, 11, and 12 (Row 15, 16, 17, 23, 24, 25, and 26) are equal to zero. It means that the factory need to use all of the available raw materials to produce and no raw materials should be left. However, the slack variables for month 4, 5, 6, 7, and 8 (Row 18 - 22) are not equal to zero. For example, for month 4 (Row 18), the slack variables equal to 300, which means the factory has 300 tons of raw materials left. Similarly, for month 5, 6, 7, and 8 (Row 19 - 22), there are 250, 300, 300, and 200 tons of raw materials left.

Then, we will look at the slack variables for the constraints of demand. Only the slack variable for month 3 (Row 29) is not equal to zero. It equals 175, which means the demand will not fulfilled and the factory only needs to produce 175 tons less than the demand. The slack variables for all of the other months (Row 27, 28, 30 - 38) are equal to zero. It implies the factory needs to produce the exact amount of demands.

Then, we will look at the labor restrictions during regular time for month 1, 2, 3, 10, 11, and 12 (Row 39 - 44). None of the slack variables equal to zero. For instance, the slack variable for month 1 (Row 39) is 600. It means the factory should hire the amount of labor which produces 600 tons less than the total available labor which produces 1200 tons. Hence, the factory should only hire the labor which produces 600 (= 1200 - 600) tons of products. Similarly, the factory should hire the labor which produces 450 (= 1200 - 750), 425 (= 1200 - 775), 500 (= 1200 - 700), 500 (= 1200 - 700), and 500 (= 1200 - 700) tons of products for month 2, 3, 10, 11, and 12 (Row 40 - 44).

Then, we will look at the labor restrictions during regular time for month 4, 5, 6, 7, 8 and 9 (Row 45 - 50). All of the slack variables equal to zero. It means the factory needs to hire all the available labors which can produce 800 tons of products.

Then, we will look at the overtime labor restrictions for month 1, 2, 3, 10, 11, and 12 (Row 51 - 56). All of the slack variables are equal to 600, which is equal to the total available labors. It means the factory doesn't need to hire any labors to produce anything during overtime for month 1, 2, 3, 10, 11, and 12.

Then, we will look at the overtime labor restrictions for month 4, 5, 6, 7, 8 and 9 (Row 57 - 62). The slack variables for month 6, 7, 8, and 9 (Row 59 - 62) are equal to zero. The factory needs to hire all of the available overtime labors which can produce 500 tons of products. However, the slack variables for month 4 and 5 (Row 57 and 58) are equal to 400 and 250. Hence, the factory should hire the amount of labor which produces 100 (= 500 - 400) and 250 (= 500 - 250) tons of products.

Dual Price

The dual prices for the first constraints of month 2 to 12 (Row 2 - 12) are -17, -18, -9, -9, -10, -11, -12, -13, -14, -15, and -16. For instance, dual price for month 2 (Row 2) is -17. Hence, the profit will decrease $17 if an extra waste of the products for month 2. Similarly, the profit wll decrease $18, $9, $9, $10, $11, $12, $13, $14, $15, and $16 if an extra waste of the products for month 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

The dual prices for the second and third constraints are -16 and -17. The dual price for month 1 (Row 13) means for one less ton of stock for month 1, the profit will decrease $16. The dual price for month 12 (Row 14) means for one extra ton of stock for month 12, the profit will decrease $17.

The dual prices of constraints of limited raw materials for month 4, 5, 6, 7, 8 and 9 (Row 18 - 23) are equal to 0. It means the increase of availability of raw materials will not affect the profit, since the factory should not use all the raw materials in the optimal plan. The dual prices for month 1, 2, 3, 10, 11, and 12 (Row 15, 16, 17, 24, 25, and 26) are 12, 13, 14, 10, 11, and 12. Take month 1 (Row 15) as an

example, the profit will increase $12 for an extra unit of available raw materials. Similarly, the profit will increase $13, $14, $10, $11, and $12. for an extra unit of available raw materials for month 1, 2, 3, 10, 11, and 12 (Row 16, 17, 24, 25, and 26) .

The dual prices of constraints of demands for all months except month 3 (Row 29) isn't equal to zero. The zero dual price of month 3 means for the increase of demands will not influence the profit. However, the others aren't equal to 0. Take month 1 (Row 27) as an example, the dual price for month 1 is 2. It means for one extra unit of demand, the profit will increase $2. Similarly, for month 2 and 4 to 12 (Row 2 and Row 4 to 12), the profit will increase $1, $16, $16, $15, $14, $13, $12, $16, $15, and $14 for one extra unit of demand.

The dual prices of labor restrictions during regular time for month 1, 2, 3, 10, 11, and 12 (Row 39 - 44) are zero. The increase of labor availability during regular time will not change the profit since the factory shouldn't use all the labor available in the optimal plan.

The dual prices of labor restrictions during regular time for month 4, 5, 6, 7, 8 and 9 (Row 45 - 50) are 3, 3, 4, 5, 6, and 7. Take month 4 as an example (Row 45), for one more unit of available labor, the profit will increase $3. Similarly, for one more unit of available labor, the profit will increase $3, $4, $5, $6, and $7 for month 5, 6, 7, 8 and 9 (Row 46 - 50).

The dual prices of the overtime labor restrictions during for month 1, 2, 3, 10, 11, and 12 (Row 51 - 56) are zero. The increase of labor availability during overtime will not affect the profit since the factory won't use all the labor available in the optimal plan.

The dual prices of the overtime labor restrictions for month 6, 7, 8 and 9 (Row 59 - 62) are 1, 2, 3, and 4. It means for one more unit of available labor, the profit will increase $1, $2, $3, and $4 for month 6, 7, 8 and 9 (Row 59 - 62). The dual prices for month 4 and 5 (Row 57 - 58) are zero. It mean the increase of labor availability during overtime will not change the profit since the factory shouldn't use all the labor available in the optimal plan.

Reduced Cost

The reduced costs for OTONS(1), OTONS(2), OTONS(3), OTONS(10), OTONS(11),and OTONS(12) are 2. It means we need to increase the 2 tons of contribution of production for month 1, 2, 3, 10, 11, and 12 during overtime to make it attractive from an economic point of view.

The reduced costs for STORAGETONS(3) and STORAGETONS(4) are 10 and 1. It means we need to increase the 10 and 1 tons of storage for month 3 and 4 to make it contribute to the profit.

Recommendation

The optimal plan is the following:

The optimal value is 190550 by producing 600, 450, 425, 800, 800, 800, 800, 800, 800, 500, 500, 500 tons of production in regular time corresponding to each month, by producing 0, 0, 0, 100, 250, 500, 500, 500, 500, 0, 0, 0 tons of production in overtime corresponding to each month, by selling 400, 700, 425, 900, 900, 900, 800, 600, 800, 1200, 1100, 1400 tons of production corresponding each month, and by storing 250, 0, 0, 0, 150, 550, 1050, 1750, 2250, 1550, 950, 50 corresponding to each month.

MONTHS	REGULAR TIME PRODUCTION	OVERTIME PRODUCTION	SELLING	STORING
1	600	0	400	250
2	450	0	700	0
3	425	0	425	0
4	800	100	900	0
5	800	250	900	150
6	800	500	900	550

7	800	500	800	1050
8	800	500	600	1750
9	800	500	800	2250
10	500	0	1200	1550
11	500	0	1100	950
12	500	0	1400	50
TOTAL	7775	2350	10125	8550