## Model fit

Now, let's fit the model. Let $R_j$ = tons of production during regular time in month j, $O_j$ = tons of production during overtime in month j, $S_j$ = tons sold in month j , $ST_j$ = storage tons carried over from end of month j to beginning of month j + 1, $L_j$= the limit of raw material availability and Dj= the demand in month j.

The objective function is to maximize the profit and the profit equals to the revenue minus the labor cost and storage cost.

The revenue is $\mathbf{18} \sum_{j=1}^{3} Sj + \mathbf{25} \sum_{j=4}^{9} Sj + \mathbf{30} \sum_{j=10}^{12} Sj$ **(1.1.1)**.

The labor cost is the sum of regular time labor cost and overtime labor cost.

The regular time labor cost is $\mathbf{4} \sum_{j=1}^{3} Rj + \mathbf{6} \sum_{j=4}^{9} Rj + \mathbf{4} \sum_{j=10}^{12} Rj$ **(1.1.2)** and the overtime labor cost is $\mathbf{6} \sum_{j=1}^{3} Oj + \mathbf{9} \sum_{j=4}^{9} Oj + \mathbf{6} \sum_{j=10}^{12} Oj$ **(1.1.3)**.

The storage cost is $\mathbf{1*} \sum_{j=1}^{12} STj$ **(1.1.4)**.

So by combining **(1.1.1)-(1.1.4)**, we get the objective function

$$\begin{aligned} &\mathbf{18} \sum_{j=1}^{3} Sj + \mathbf{25} \sum_{j=4}^{9} Sj + \mathbf{30} \sum_{j=10}^{12} Sj \\ &- (\mathbf{4} \sum_{j=1}^{3} Rj + \mathbf{6} \sum_{j=4}^{9} Rj + \mathbf{4} \sum_{j=10}^{12} Rj) \\ &- (\mathbf{6} \sum_{j=1}^{3} Oj + \mathbf{9} \sum_{j=4}^{9} Oj + \mathbf{6} \sum_{j=10}^{12} Oj) \\ &-( \mathbf{1*} \sum_{j=1}^{12} STj) \end{aligned} \tag{1.1}$$

The coefficient of 18, 25, 30 is the selling price corresponding to each month and other coefficient is the cost price corresponding to each month.

Now, let's consider the constraints. The sum of regular time production in month j and overtime production in month j and the storage in month j-1 is equal to the sold amount in month j plus the storage in month j. So, the equation is:

$$\mathbf{R_j+O_j+ST_{j-1}- (S_j+ST_j) =0 \text{ for } j=2 \ldots 12} \tag{1.2}$$

For j=1, the storage amount before the month 1 is 50, so the equation is:

$$\mathbf{R_j+O_j+50- (S_j+ST_j) =0 \text{ for } j=1} \tag{1.3}$$

According to the problem, at the end of month 12 the company should have a stock of at least 50 tons of the product. So

$$\mathbf{ST_{12}>=50} \tag{1.4}$$

For the limit of raw material, the production in month j can't exceed to the limitation in month j, so the equation is:

$$\mathbf{R_j+O_j<=L_j, \text{ for } j=1\ldots12} \tag{1.5}$$

Considering the demand of each month, we get:

$$\mathbf{S_j<=D_j, \text{ for } j =1\ldots12} \tag{1.6}$$

Considering the limitation of labor capacity, we get:

$$\mathbf{R_j<=1200, \text{ for } j=1,2,3,10,11,12} \tag{1.7}$$

$$\mathbf{R_j<=800, \text{ for } j=4,5,6,7,8,9} \tag{1.8}$$

$$\mathbf{O_j<=600, \text{ for } j=1,2,3,10,11,12} \tag{1.9}$$

$$\mathbf{O_j<=500, \text{ for } j=4,5,6,7,8,9} \tag{1.10}$$

Finally, all the coefficients should be non-negative. To combine the objective function **(1.1)** and constraints **(1.2)** to **(1.10)**, our linear programming model is:

**Max**

$$\mathbf{18}\sum_{j=1}^{3} Sj \mathbf{+25}\sum_{j=4}^{9} Sj \mathbf{+30}\sum_{j=10}^{12} Sj$$

$$\mathbf{-(4}\sum_{j=1}^{3} Rj \mathbf{+6}\sum_{j=4}^{9} Rj \mathbf{+4}\sum_{j=10}^{12} Rj\mathbf{)}$$

$$\mathbf{-(6}\sum_{j=1}^{3} Oj \mathbf{+9}\sum_{j=4}^{9} Oj \mathbf{+6}\sum_{j=10}^{12} Oj\mathbf{)}$$

$$\mathbf{-(1*}\sum_{j=1}^{12} STj\mathbf{)}$$

**Subject to:**

$$R_j+O_j+ST_{j-1}-(S_j+ST_j)=0 \text{ for } j=2 \ldots 12$$

$$R_j+O_j+50-(S_j+ST_j)=0 \text{ for } j=1$$

$$ST_{12}>=50$$

$$R_j+O_j<=L_j, \text{ for } j=1\ldots 12$$

$$S_j<=D_j, \text{ for } j=1\ldots 12$$

$$R_j<=1200, \text{ for } j=1,2,3,10,11,12$$

$$R_j<=800, \text{ for } j=4,5,6,7,8,9$$

$$O_j<=600, \text{ for } j=1,2,3,10,11,12$$

$$O_j<=500, \text{ for } j=4,5,6,7,8,9$$

$$R_j, O_j, S_j, ST_j>=0 \text{ for } j=1\ldots 12$$