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(*Calculatiosn for Proposition 2.3 in "Tiling H in dense graphs"*)

(*First, we define the function Phi*)

Phi[y0_, y1_, y2_, y3_] := (y1 + 2 y2 + 3 y3) (1 - 6 α) + 18 (y0)^2 + 12 (y1)^2 + 12 (y2)^2 +
   9 (y3)^2 + 30 * y0 * y1 + 24 * y0 * y2 + 24 y0 * y3 + 24 * y1 * y2 + 24 * y1 * y3 + 21 * y2 * y3;
(*Since Phi is increasing in y3, we can replace y3 by α-(y0+y1+y2)*)
Block[{y3 = α - (y0 + y1 + y2)}, Expand[Phi[y0, y1, y2, y3]]]
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$$-3\,y0+3\,y0^{2}-2\,y1-3\,y1^{2}-y2-3\,y0\,y2-3\,y1\,y2+3\,\alpha+24\,y0\,\alpha+18\,y1\,\alpha+9\,y2\,\alpha-9\,\alpha^{2}$$

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Psi[y0_, y1_, y2_] :=
  -3 y0 + 3 y0^2 - 2 y1 - 3 y1^2 - y2 - 3 y0 y2 - 3 y1 y2 + 3 α + 24 y0 α + 18 y1 α + 9 y2 α - 9 α^2;
Simplify[Maximize[{Psi[y0, y1, y2],
   y0 ≥ 0 && y1 ≥ 0 && y2 ≥ 0 && y1 + y2 + y0 ≤ α && α ≥ 0 && α ≤ 1/6}, {y0, y1, y2}]]
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$$\left\{\begin{cases} 18\,\alpha^{2} & \alpha == 0 \,||\, \frac{1}{9}\le\alpha\le\frac{1}{6} \\ 3\,(1-3\,\alpha)\,\alpha & 0<\alpha<\frac{1}{9} \\ -\infty & \text{True}\end{cases},\ \left\{y0\to\begin{cases}\alpha & \frac{1}{9}<\alpha\le\frac{1}{6}\\ 0 & 0\le\alpha\le\frac{1}{9}\\ \text{Indeterminate} & \text{True}\end{cases},\ y1\to\begin{cases}0 & 0\le\alpha\le\frac{1}{6}\\ \text{Indeterminate} & \text{True}\end{cases},\ y2\to\begin{cases}\frac{1}{18} & \alpha==\frac{1}{9}\\ 0 & 0\le\alpha<\frac{1}{9}\,||\,\frac{1}{9}<\alpha\le\frac{1}{6}\\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$