(*Calculations for Lemma 4.2 Case 1*)
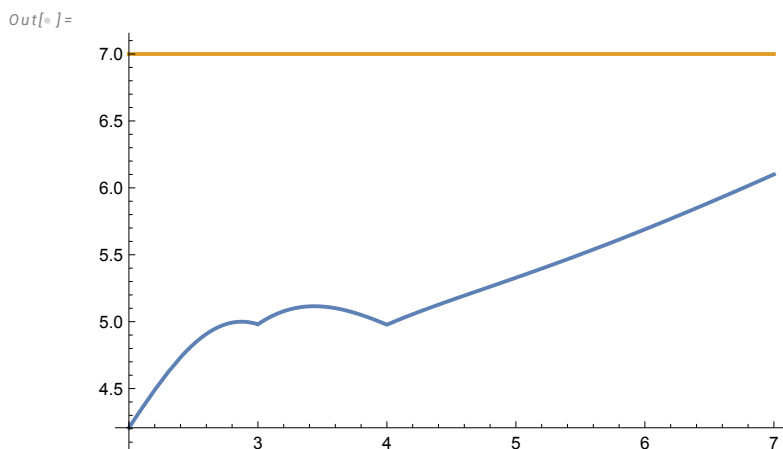
In[ ]:= `Simplify[`

$$\left(\frac{1}{2+p}\right) * \left(\frac{1}{2+p}\right) * \left(\frac{p}{2+p}\right)^p + \left(\frac{1}{2+p}\right) * \left(\frac{1}{2+p}\right)^p * \left(\frac{p}{2+p}\right) + \left(\frac{1}{2+p}\right)^p * \left(\frac{1}{2+p}\right) * \left(\frac{p}{2+p}\right)]$$

Out[ ]=

$$\frac{2 p \left(\frac{1}{2+p}\right)^p + \left(\frac{p}{2+p}\right)^p}{(2+p)^2}$$

In[ ]:= `Plot[`$\left\{\left(\frac{3}{2*E} * \text{Max}\left[\left\{\frac{1}{2^{p-1}}, \frac{1}{2^3}\left(\frac{p-1}{p}\right)^{p-4}\right\}\right] + \text{Max}\left[\left\{\frac{1}{2^p}, \frac{1}{2^3}\left(\frac{p}{p+1}\right)^{p-3}\right\}\right]\right) \Big/$

$\left((p+2) * \frac{2 p \left(\frac{1}{2+p}\right)^p + \left(\frac{p}{2+p}\right)^p}{(2+p)^2}\right), 7\right\}, \{p, 2, 7\}]$

Out[ ]=

In[ ]:= `NMaximize[`$\left(\frac{3}{2*E} * \text{Max}\left[\left\{\frac{1}{2^{p-1}}, \frac{1}{2^3}\left(\frac{p-1}{p}\right)^{p-4}\right\}\right] + \text{Max}\left[\left\{\frac{1}{2^p}, \frac{1}{2^3}\left(\frac{p}{p+1}\right)^{p-3}\right\}\right]\right) \Big/$

$\left((p+2) * \frac{2 p \left(\frac{1}{2+p}\right)^p + \left(\frac{p}{2+p}\right)^p}{(2+p)^2}\right), p \geq 2 \,\&\&\, p \leq 7, \{p\}]$

Out[ ]=

$\{6.10037, \{p \to 7.\}\}$

(*Calculations for Lemma 4.2 Case 2*)

In[ ]:= `Block[`$\{p = 8\}, \text{Simplify}\Big[\frac{3 E}{2} * \frac{(p-1)^2+3}{(p-1)^2-1}\left(\frac{(p+2)(p-3)}{(p-2)^p} + \frac{p+2}{p-2} * \frac{1}{E}\right) +$

$\frac{2 - p + p^2}{(-2+p)(1+p)}\left(\frac{(p+2)(p-2)}{(p-1)^{p+1}} + \frac{p+2}{p-1} * \frac{1}{E}\right) * E^2\Big]\Big]$

Out[ ]=

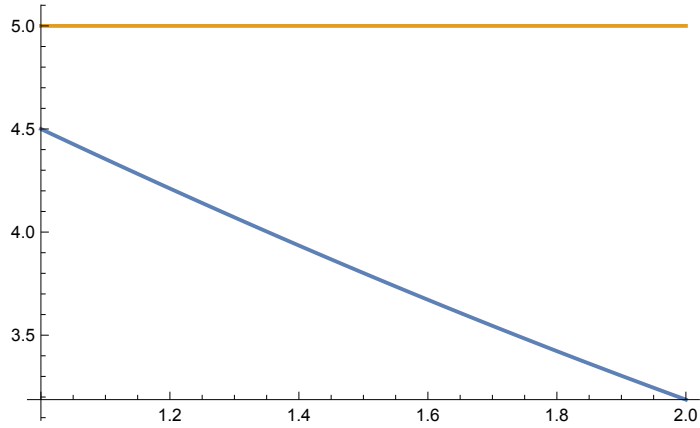$$\frac{65}{24} + \frac{72\,163\,555\,e}{47\,029\,248} + \frac{580\,e^2}{363\,182\,463}$$

In[ ]:= `N[`$\frac{65}{24} + \frac{72\,163\,555\,e}{47\,029\,248} + \frac{580\,e^2}{363\,182\,463}]$

Out[ ]=

6.87939

(*Calculations for Lemma 4.4 Base case*)

In[ ]:= Plot$\left[\left\{\dfrac{p\left(\frac{1}{2}\right)^{2-p}\left(\frac{1}{9}\right)^{p-1}+\frac{1}{4^{p-1}}}{(p+2)\left(\frac{1}{3}\right)^{p+1}}, 5\right\}, \{p, 1, 2\}\right]$

Out[ ]=

In[ ]:= NMaximize$\left[\dfrac{p\left(\frac{1}{2}\right)^{2-p}\left(\frac{1}{9}\right)^{p-1}+\frac{1}{4^{p-1}}}{(p+2)\left(\frac{1}{3}\right)^{p+1}}, p \geq 1 \,\&\&\, p \leq 2, \{p\}\right]$

Out[ ]=

$\{4.5, \{p \to 1.\}\}$