# Package 'hdtrd'

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dimensional linear regression with applications to detect transferability

Type Package

Title Testing relevant difference in high-

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Author Xu Liu [aut,cre]
Maintainer Xu Liu <liu.xu@sufe.edu.cn></liu.xu@sufe.edu.cn>
<b>Description</b> Provide the p-value of the test statistic for high-dimensional relevant difference in the generalized linear regression models and its application to transfer learning. In the paper Liu(2024), we propose novel statistics to test relevant difference of two high-dimensional coefficients in the generalized linear regression models. The proposed method can serve as a means for high-dimensional transfer learning the generalized linear regression models.
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R topics documented:
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hdtrd-package	Testing relevant difference in high-dimensional linear regression with applications to detect transferability
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#### **Description**

Provide the p-value of the test statistic for high-dimensional relevant difference in the generalized linear regression models and its application to transfer learning. In the paper Liu(2024), we propose novel statistics to test relevant difference of two high-dimensional coefficients in the generalized linear regression models. The proposed method can serve as a means for high-dimensional transfer learning the generalized linear regression models.

### **Details**

Package: hdtrd
Type: Package
Version: 1.0.1
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## References

Cui, H., Guo, W. and Zhong, W. (2018). Test for high-dimensional regression coefficients using refitted cross-validation variance estimation. The Annals of Statistics, 46, 958-988.

Chen, Z., Cheng, V. X. and Liu, X. (2024). Hypothesis testing on high dimensional quantile regression. Journal of Econometrics.

Chen, J., Li, Q., and Chen, H. Y. (2022). Testing generalized linear models with highdimensional nuisance parameters. Biometrika, 110. 83-99.

Guo, B.and Chen, S. X. (2016). Tests for high dimensional generalized linear models. Journal of the Royal Statistical Society, Series B, 78, 1079-1102.

Karoui, N, E. (2008) Spectrum estimation for large dimensional covariance matrices using random matrix theory. The Annals of Statistics, 36(6), 2757-2790.

Kong, W. and Valiant, G. (2017). Spectrum estimation from samples. Annals of Statistics. 45, 2218-2247.

Liu, S. (2024). Unified Transfer Learning Models for High-Dimensional Linear Regression. Proceedings of The 27th International Conference on Artificial Intelligence and Statistics, PMLR. 238, 1036-1044.

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

Liu, X., Zheng, S. and Feng, X. (2020). Estimation of error variance via ridge regression. Biometrika. 107, 481-488.

Tian, Y. and Feng, Y. (2023) Transfer Learning Under High-Dimensional Generalized Linear Models. Journal of the American Statistical Association, 118, 2684-2697.

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Yang, W., Guo, X. and Zhu, L. (2023). Score function-based tests for ultrahigh-dimensional linear models. arXiv:2212.08446.

Zhang, X. and Cheng, G. (2017). Simultaneous inference for high-dimensional linear models. Journal of the American Statistical Association, 112, 757-768.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

bandmatrix

Construct a sparse banded matrix.

## **Description**

Provide a sparse banded matrix.

## Usage

```
bandmatrix(rho, p, T = 5)
```

## **Arguments**

rho	A vector with length $T$ .
p	The dimension of the banded matrix.
T	The width of band. Default is $T = 5$ .

#### Value

```
sighalf The matrix \Gamma \in \mathcal{R}^{(p+T) \times p} satisfying \Sigma = \Gamma^T \Gamma.
sigma The sparse banded matrix \Sigma \in \mathcal{R}^{p \times p}.
```

## References

Chen, Z., Cheng, V. X. and Liu, X. (2024). Hypothesis testing on high dimensional quantile regression. Journal of Econometrics.

## **Examples**

```
p <- 6
T <- 3
rho <- seq(T)/(T+1)
fit <- bandmatrix(rho, p, T)
fit$sigma</pre>
```

4 eigmax

eigmax	Estimation of the largest eigenvalue of covariance of a high-dimensional vector

# Description

Provide the estimator of the largest eigenvalue of covariance of a high-dimensional vector (Liu (2024)), as well as all estimated eigenvalues.

## Usage

```
eigmax(X, zK = NULL, tJ = NULL, K = 1000, J = 1000, method = 'mpmo', nmoms = NULL, timeout = 0L)
```

## **Arguments**

Χ	A data matrix with dimension $n \times p$ .
zK	A matrix with dimension $K \times 2$ , a given complex number, where the first column is the real part and the second column is the imaginary part. Default is $zK = NULL$ , where $zK[,1] = rnorm(K)$ is generated from standard normal distribution, and $zK[,2] = rep(1,K)/sqrt(n)$ .
tJ	A $J$ -vector. Default is tJ = NULL, where tJ is a grid of points in the interval $[\lambda_{\min}(\Sigma), \lambda_{\max}(\Sigma)]$ .
K	A positive integer, which is the number of complex numbers zK. Default is K = 1000.
J	A positive integer, which is the length of $tJ$ . Default is $J = 1000$ .
method	There are three methods, 'mpmo', 'mplp' and 'empi', to estimate the largest eigenvalue of $\Sigma$ , see details in Liu (2024). Default is method = 'mpmo'.
nmoms	The number of moments. Default is nmoms = NULL, where nmoms = 7 if method = 'mpmo', nmoms = 4 if method = 'mplp', and nmoms is useless if method = 'empi'.
timeout	An integer: timeout variable in seconds, defaults to 0L which means no limit is set, see details in the function 1inp of R package "limSolve".

## **Details**

See details in the paper Liu (2024).

Here, for the methods to estimate the largest eigenvalue of  $\Sigma$ , 'mpmo' denotes the MPMO method; 'mplp' denotes the MPLP method; and 'empi' denotes the EMPI method.

## Value

Lammax Estimator of the largest eigenvalue o	1 2	2	ì.
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lamest All estimated eigenvalues of  $\Sigma$ .

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#### References

Karoui, N, E. (2008) Spectrum estimation for large dimensional covariance matrices using random matrix theory. The Annals of Statistics, 36(6), 2757-2790.

Kong, W. and Valiant, G. (2017). Spectrum estimation from samples. Annals of Statistics. 45, 2218-2247.

Liu, X. (2024). High-dimensional test of relevant difference and its application totransfer learning. Manuscript.

Tian, X., Lu, Y., and Li, W. (2015). A robust test for sphericity of high-dimensional covariance matrices. Journal of Multivariate Analysis, 141, 217-227.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

## **Examples**

```
p = 300
n = 200
sig = toeplitz(0.5^(c(1:p)-1))
sighalf = chol( sig )
X = matrix(rnorm(n*p), nrow = n)
eigens = eigmax(X = X, method = 'mpmo')
eigens$lammax
```

predict\_utr

Prediction of a new predictor

## **Description**

Provide the prediction for a new predictor.

## Usage

```
predict_utr(fittrans, X, type = "response")
```

## **Arguments**

fittrans An object from fitting utrans.

X A new predictor, a matrix with dimension  $n \times p$ .

type The type of prediction, including "response" (Default) and "class". Here "re-

sponse" provides the predicted probability when family = "binomial". "class"

predict 0/1 response for logistic regression. Applies only when family = "binomial".

## Details

See details in the paper Liu (2024).

# Value

yhat

The new response  $\hat{y}$  based on the new predictor x.

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#### References

Liu, S. (2024). Unified Transfer Learning Models for High-Dimensional Linear Regression. Proceedings of The 27th International Conference on Artificial Intelligence and Statistics, PMLR. 238, 1036-1044.

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

Tian, X., Lu, Y., and Li, W. (2015). A robust test for sphericity of high-dimensional covariance matrices. Journal of Multivariate Analysis, 141, 217-227.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

## **Examples**

```
data(simulData_trans_gauss)
fittrans <- utrans(target = dataset[[1]], source = dataset[-1], idtrans = seq(5))

p = ncol(dataset[[1]]$X)
n = 5
sig = toeplitz(0.5^(c(1:p)-1))
sighalf = chol( sig )
x = matrix(rnorm(n*p), nrow = n)

predict_utr(fittrans, x)</pre>
```

projection

Projection of y onto the closure of covariates x

## **Description**

Provide the projection of y onto the closure of covariates x.

#### Usage

```
projection(x, y, family = "gaussian", method = 'lasso', isresid = TRUE)
```

## **Arguments**

X	Covariates, a $n \times p$ -matrix.
У	Response, a <i>n</i> -vector.
family	Family for the generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian".
method	There are two methods, "qfabs" and "lasso", to estimate the nuisance parameter $\alpha$ in quantile regression. Default is method = 'lasso'.
isresid	logical. Projected residual $\hat{\eta} = x - \hat{H}z$ is output if is resid = TRUE. Coefficient matrix $\hat{H}$ is calculated if is resid = FALSE. Default is resids = TRUE.

## **Details**

High-dimensional test of relevant difference and its application to transferability test in the generalized Linear regression models (see details in the paper Liu (2024))

$$y_i = H\boldsymbol{X}_i^T.$$

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### Value

proj Projection.

### References

Cheng, C., Feng, X., Huang, J. and Liu, X. (2022). Regularized projection score estimation of treatment effects in high-dimensional quantile regression. Statistica Sinica. 32, 23-41.

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

## **Examples**

```
data(simulData_test_gauss)
x <- datahb$X
y <- datahb$Y
proj <- projection(x, y)</pre>
```

pvalclc

*P-value for high-dimensional test in the generalized linear regression models* 

## Description

Provide p-value for high-dimensional test in the generalized linear regression models when the nuisance parameter is high-dimensional, see Chen et. al. (2022) for details.

# Usage

```
pvalclc(data, family = 'gaussian', method = 'lasso', resids = NULL, psi = NULL)
```

## Arguments

data	A list, including $Y$ (response), $X$ , $Z$ , where $Z$ can be NULL.
family	Family for the generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian".
method	There are two methods, "gfabs" and "lasso", to estimate the nuisance parameter $\alpha$ in GLMs. Default is method = 'lasso', which calls glmnet.
resids	An $n$ -vector, which is residual of the GLM. Default is resids = NULL. The canonical link function is used if resids and psi are NULL.
psi	An <i>n</i> -vector, which is $\psi(X_i, \beta_0, \phi) = g'(X_i^{\top}\beta_0)/V(\mu_i(\beta_0); \phi)$ , see Guo and Chen (2016) for the details. Default is psi = NULL. The canonical link function is used if both resids and psi are NULL. Here, psi = rep(1, n) if psi = NULL.

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### **Details**

The generalized linear regression models (see details in the paper Guo and Chen (2016))

$$\mu_i = \boldsymbol{X}_i^T \boldsymbol{\beta} + \boldsymbol{Z}_i^T \boldsymbol{\gamma},$$

where  $\boldsymbol{Z}_{i}^{T}\boldsymbol{\gamma}$  is the control mean function.

The hypothesis test problem is

$$H_0: \boldsymbol{\beta} = \mathbf{0} \quad versus \quad H_1: \boldsymbol{\beta} \neq \mathbf{0}.$$

One can input estimated residual resids =  $y_i - \hat{\mu}_i$  and psi = NULL which produces the test statistic and p-value given by Chen et. al. (2022), where  $\hat{\mu}_i$  is an estimator of  $\mu_i$  according to Chen et. al. (2023).

### Value

pvals P-value of the corresponding test statistic.

Tn Test statistic  $|\hat{U}_n|/\sqrt{2\hat{R}_n}$ . Reject  $H_0$  if  $|\hat{U}_n|/\sqrt{2\hat{R}_n} > z_{1-\alpha/2}$ .

### References

Guo, B.and Chen, S. X. (2016). Tests for high dimensional generalized linear models. Journal of the Royal Statistical Society, Series B, 78, 1079-1102.

Chen, J., Li, Q., and Chen, H. Y. (2023). Testing generalized linear models with highdimensional nuisance parameters. Biometrika. 110, 83-99.

## **Examples**

```
data(simulData_test_gauss)
pvals <- pvalclc(data = datahb, family = "gaussian")
pvals$pvals</pre>
```

pvalgc

P-value for high-dimensional test in the generalized linear regression models

# Description

Provide p-value for high-dimensional test in the generalized linear regression models, see Guo and Chen (2016) for details.

## Usage

```
pvalgc(data, family = "gaussian", resids = NULL, psi = NULL)
```

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# Arguments

data	A list, including $Y$ (response), $X$ , $Z$ , where $Z$ can be NULL.
family	Family for the generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian".
resids	An $n$ -vector, which is residual of the GLM. Default is resids = NULL. The canonical link function is used if resids and psi are NULL.
psi	An <i>n</i> -vector, which is $\psi(X_i, \beta_0, \phi) = g'(X_i^{\top}\beta_0)/V(\mu_i(\beta_0); \phi)$ , see Guo and Chen (2016) for the details. Default is psi = NULL. The canonical link function is used if resids and psi are NULL. psi = rep(1,n) if psi = NULL.

#### **Details**

The generalized Linear regression models (see details in the paper Guo and Chen (2016))

$$\mu_i = \boldsymbol{X}_i^T \boldsymbol{\beta} + \boldsymbol{Z}_i^T \boldsymbol{\gamma},$$

where  $Z^T \gamma$  is the control mean function.

The hypothesis test problem is

$$H_0: \boldsymbol{\beta} = \mathbf{0} \quad versus \quad H_1: \boldsymbol{\beta} \neq \mathbf{0}.$$

One can input estimated residual resids =  $y_i - \hat{\mu}_i$  and psi = NULL which produces the test statistic and p-value given by Chen et. al. (2022), where  $\hat{\mu}_i$  is an estimator of  $\mu_i$  according to Chen et. al. (2023).

#### Value

pvals	P-value of the corresponding test statistic.
Tn	test statistic $\hat{U}_n/\sqrt{2\hat{R}_n}$ . Reject $H_0$ if $\hat{U}_n/\sqrt{2\hat{R}_n}>z_{1-\alpha}$ .

### References

Guo, B.and Chen, S. X. (2016). Tests for high dimensional generalized linear models. Journal of the Royal Statistical Society, Series B, 78, 1079-1102.

Chen, J., Li, Q., and Chen, H. Y. (2023). Testing generalized linear models with highdimensional nuisance parameters. Biometrika. 110, 83-99.

### **Examples**

```
data(simulData_test_gauss)
pvals <- pvalgc(data = datahb, family = "gaussian")
pvals$pvals</pre>
```

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pvalrd	P-value for high-dimensional testing of relevant difference in the generalized linear regression models when the nuisance parameter is
	high-dimensional

## **Description**

Provide p-value for high-dimensional testing of relevant difference in generalized linear regression models (Liu (2024)) when the nuisance parameter is high-dimensional.

## Usage

## **Arguments**

data	A list, including $Y$ (response), $X$ , $Z$ , where $Z$ is high-dimensional.
family	Family for the generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian".
delta0	Relevant difference, a given value by hypothesis test problem $H_0: \ \beta\ _2 \leq \delta_0$ . Default is delta0 = 0.1.
method	There are two methods, "qfabs" and "lasso", to estimate the nuisance parameter $\alpha$ in quantile regression. Default is method = 'lasso'.
resids	An $n$ -vector, which is residual of the GLM under $H_0$ . Default is resids = NULL, where the canonical link function is used if resids and psi are NULL.
sigma2	Estimator of error's variance if family = "gaussian". Default is sigma2 = NULL, where sigma2 = 1.
lammax	Estimator of the largest eigenvalue $\sup_{\ \beta\ _2 \le \delta_0} \beta^T \Sigma^2 \beta$ . Default is lammax = NULL, which is estimated empirically by $\lambda_{\max}(S_n)/(1+\sqrt{p/n})$ , see details in Liu (2024).

# **Details**

High-dimensional test of relevant difference and its application to transferability test in the generalized Linear regression models (see details in the paper Liu (2024))

$$\mu_i = \boldsymbol{X}_i^T \boldsymbol{\beta} + \boldsymbol{Z}_i^T \boldsymbol{\gamma},$$

where  $Z^T \gamma$  is the control mean function, and X is high-dimensional.

The hypothesis test problem is

$$H_0: \|\boldsymbol{\beta}\| \leq \delta_0 \quad versus \quad H_1: \|\boldsymbol{\beta}\| > \delta_0.$$

## Value

pvals P-value of the corresponding test statistic.

Tn Standardized test statistic.

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#### References

Karoui, N, E. (2008) Spectrum estimation for large dimensional covariance matrices using random matrix theory. The Annals of Statistics, 36(6), 2757-2790.

Kong, W. and Valiant, G. (2017). Spectrum estimation from samples. Annals of Statistics. 45, 2218-2247.

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

Tian, X., Lu, Y., and Li, W. (2015). A robust test for sphericity of high-dimensional covariance matrices. Journal of Multivariate Analysis, 141, 217-227.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

## **Examples**

```
data(simulData_test_gauss)
pvals <- pvalrd(data = datahb)
pvals$pvals</pre>
```

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P-value for high-dimensional testing of relevant difference in high-dimensional transfer learning in the generalized linear regression models.

# Description

Provide p-value for high-dimensional testing of relevant difference in high-dimensional transfer learning in the generalized linear regression models (Liu (2024)).

## Usage

## **Arguments**

target	The target dataset, a list, including $Y$ (response), $\boldsymbol{X}$ (covariates).
source	The source dataset, a list with sublist. Each sublist includes $Y$ (response), $\boldsymbol{X}$ (covariates).
family	Family for generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian".
delta0	Relevant difference, a given value by hypothesis test problem $H_0: \ \beta\ _2 \le \delta_0$ . Default is delta0 = 0.1.
nsource	The number of source datasets. Default is nsource = 10.
testmethd	There are two methods, "pvalrd" and "pvalclc", to calculate the p-value. Default is testmethd = 'pvalrd', see details in Liu (2024).

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method	There are two methods, "glm" and "lasso", to estimate the nuisance parameter $\alpha$ under the null hypothesis in the generalized linear regression models, where "glm" method estimates nuisance parameter for classic low-dimensional setting, and "lasso" for high-dimensional setting. Default is method = 'lasso' for high-dimensional setting.
resids	An $n$ -vector, which is residual of GLM under $H_0$ . Default is resids = NULL, where the canonical link function is used if resids and psi are NULL.
isproj	logical. Projection score method is applied if isproj = TRUE. Default is isproj = FALSE, which means that no projection score is applied.
proj	The estimated residual of projection score, a list, where each element is a $n \times p$ -matrix, $\hat{\eta} = x - \hat{H}z$ . Default is proj = NULL, which means that projection score is calculated.
sigma2	Estimator of error's variance if family = "gaussian". Default is sigma2 = NULL, where sigma2 = 1.
lammax	Esimator of the largest eigenvalue $\sup_{\ \beta\ _2 \le \delta_0} \beta^T \Sigma^2 \beta$ , see details in eigmax. Default is lammax = NULL, where lammax is estimated by EMPI method, see eigmax. If testmethd = 'pvalrd', there are two choices lammax = 'mpmo' or lammax = 'mplp'. It is useless if testmethd = 'pvalclc'.
nmoms	The number of moments. Default is nmoms = NULL, where nmoms = 7 if method = 'mpmo', nmoms = 4 if method = 'mplp', and nmoms is useless if method = 'empi'.
zK	A matrix with dimension $K \times 2$ , a given complex number, where the first column is the real part and the second column is the imaginary part. Default is zK = NULL, where zK[,1] = rnorm(K) is generated from standard normal distribution, and zK[,2] = rep(1,K)/sqrt(n).
J	A positive integer, which is the length of tJ. Default is $J = NULL$ , which means $J = max(500, 3*n, 2*p) + 200$ .
K	A positive integer, which is the number of complex numbers zK. Default is $K = NULL$ , which means $K = max(500, 3*n, 2*p)+200$ .
timeout	An integer: timeout variable in seconds, defaults to 0L which means no limit is set, see details in the function linp of R package "limSolve".

### **Details**

High-dimensional test of relevant difference and its application to transferability test in the generalized Linear regression models (see details in the paper Liu (2024)).

Linear regression model for target data:

$$Y_{0i} = \boldsymbol{X}_{0i}^T \boldsymbol{\beta}_0 + \epsilon_{0i},$$

and

linear regression model for the kth source data:

$$Y_{ki} = \boldsymbol{X}_{ki}^T \boldsymbol{\beta}_k + \epsilon_{ki},$$

where  $X^T \beta$  is a baseline mean function, and X is high-dimensional.

The hypothesis test problem is

$$H_0: \|\boldsymbol{\beta} - \boldsymbol{\beta}_0\| \le \delta_0 \quad versus \quad H_1: \|\boldsymbol{\beta} - \boldsymbol{\beta}_0\| > \delta_0.$$

Here, for the methods to estimate the largest eigenvalue of  $\Sigma$ , 'mpmo' denotes the MPMO method; 'mplp' denotes the MPLP method; and 'empi' denotes the EMPI method.

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#### Value

pvals

P-value of the corresponding test statistic, which is a vector with length nsource.

#### References

Chen, Z., Cheng, V. X. and Liu, X. (2024). Hypothesis testing on high dimensional quantile regression. Journal of Econometrics.

Karoui, N, E. (2008) Spectrum estimation for large dimensional covariance matrices using random matrix theory. The Annals of Statistics, 36(6), 2757-2790.

Kong, W. and Valiant, G. (2017). Spectrum estimation from samples. Annals of Statistics. 45, 2218-2247.

Liu, S. (2024). Unified Transfer Learning Models for High-Dimensional Linear Regression. Proceedings of The 27th International Conference on Artificial Intelligence and Statistics, PMLR. 238, 1036-1044.

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

Liu, X., Zheng, S. and Feng, X. (2020). Estimation of error variance via ridge regression. Biometrika. 107, 481-488.

Tian, Y. and Feng, Y. (2023) Transfer Learning Under High-Dimensional Generalized Linear Models. Journal of the American Statistical Association, 118, 2684-2697.

Yang, W., Guo, X. and Zhu, L. (2023). Score function-based tests for ultrahigh-dimensional linear models. arXiv:2212.08446.

Zhang, X. and Cheng, G. (2017). Simultaneous inference for high-dimensional linear models. Journal of the American Statistical Association, 112, 757-768.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

## Examples

```
data(simulData_trans_gauss)
pvals <- pvaltrans(target = dataset[[1]], source = dataset[-1])
pvals</pre>
```

pvaltrans\_cv

P-value for high-dimensional testing of relevant difference in high-dimensional transfer learning in the generalized linear regression models via cross validation method.

## **Description**

Provide p-value for high-dimensional testing of relevant difference in high-dimensional transfer learning in the generalized linear regression models via cross validation method (Liu (2024)).

# Usage

```
pvaltrans_cv(target, source, family = "gaussian", delta0 = 0.1, nsource = 10,
    method = 'lasso', ncv = 10, alpha = 0.05, resids = NULL,
    isproj = FALSE, proj = NULL, sigma2 = NULL, lammax = NULL,
    nmoms = NULL, zK = NULL, J = NULL, timeout = 0)
```

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## **Arguments**

target The target dataset, a list, including Y (response), X (covariates). source The source dataset, a list with sublist. Each sublist includes Y (response), X (covariates). family Family for generalized linear models, including 'gaussian', 'binomial', and 'poisson'. Default is family = "gaussian". delta0 Relevant difference, a given value by hypothesis test problem  $H_0: \|\beta\|_2 \leq \delta_0$ . Default is delta0 = 0.1. nsource The number of source datasets. Default is nsource = 10. method There are two methods, "glm" and "lasso", to estimate the nuisance parameter  $\alpha$  under the null hypothesis in the generalized linear regression models, where "glm" method estimates nuisance parameter for classic low-dimensional setting, and "lasso" for high-dimensional setting. Default is method = 'lasso' for highdimensional setting. Number of folds in the cross-validation, which is used to select transferable level ncv  $\delta_0$ . Default is ncv = 10. alpha logical. Projection score method is applied if isproj = TRUE. Default is isproj = FALSE, which means that no projection score is applied. resids An *n*-vector, which is residual of GLM under  $H_0$ . Default is resids = NULL, where the canonical link function is used if resids and psi are NULL. isproj logical. Projection score method is applied if isproj = TRUE. Default is isproj = FALSE, which means that no projection score is applied. The estimated residual of projection score, a list, where each element is a  $n \times p$ proj matrix,  $\hat{\eta} = x - Hz$ . Default is proj = NULL, which means that projection score is calculated. sigma2 Estimator of error's variance if family = "gaussian". Default is sigma2 = NULL, where sigma2 = 1. Esimator of the largest eigenvalue  $\sup_{\|\beta\|_2 \le \delta_0} \beta^T \Sigma^2 \beta$ , see details in eigmax. lammax Default is lammax = NULL, where lammax is estimated by EMPI method, see eigmax. If testmethd = 'pvalrd', there are two choices lammax = 'mpmo' or lammax = 'mplp'. It is useless if testmethd = 'pvalclc'. nmoms The number of moments. Default is nmoms = NULL, where nmoms = 7 if method = 'mpmo', nmoms = 4 if method = 'mplp', and nmoms is useless if method = 'empi'. zΚ A matrix with dimension  $K \times 2$ , a given complex number, where the first column is the real part and the second column is the imaginary part. Default is zK = NULL, where zK[,1] = rnorm(K) is generated from standard normal distribution, and zK[,2] = rep(1,K)/sqrt(n). J A positive integer, which is the length of tJ. Default is J = NULL, which means J = max(500, 3\*n, 2\*p) + 200.Κ A positive integer, which is the number of complex numbers zK. Default is K = NULL, which means K = max(500, 3\*n, 2\*p)+200. timeout An integer: timeout variable in seconds, defaults to 0L which means no limit is

set, see details in the function linp of R package "limSolve".

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#### **Details**

High-dimensional test of relevant difference and its application to transferability test in the generalized Linear regression models (see details in the paper Liu (2024)).

Linear regression model for target data:

$$Y_{0i} = \boldsymbol{X}_{0i}^T \boldsymbol{\beta}_0 + \epsilon_{0i},$$

and

linear regression model for the kth source data:

$$Y_{ki} = \boldsymbol{X}_{ki}^T \boldsymbol{\beta}_k + \epsilon_{ki},$$

where  $X^T\beta$  is a baseline mean function, and X is high-dimensional.

The hypothesis test problem is

$$H_0: \|\boldsymbol{\beta} - \boldsymbol{\beta}_0\| \le \delta_0 \quad versus \quad H_1: \|\boldsymbol{\beta} - \boldsymbol{\beta}_0\| > \delta_0.$$

Here, for the methods to estimate the largest eigenvalue of  $\Sigma$ , 'mpmo' denotes the MPMO method; 'mplp' denotes the MPLP method; and 'empi' denotes the EMPI method.

### Value

pvals P-value of the corresponding test statistic, which is a vector with length nsource.

s\_opt The s\_optth  $\delta_0$  is Selected.

## References

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Zhang, X. and Cheng, G. (2017). Simultaneous inference for high-dimensional linear models. Journal of the American Statistical Association, 112, 757-768.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

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#### **Examples**

```
data(simulData_trans_gauss)
np <- dim(dataset[[1]]$X)
delta0 <- c(1:10)*log(np[1])/np[2]
## pvals <- pvaltrans_cv(target = dataset[[1]], source = dataset[-1], delta0 = delta0, nsource = 1)</pre>
```

simulData

Simulated data for generalized linear regression models

## **Description**

Simulated data for generalized linear regression models.

- 'Linear regression' for testing relevant difference (simulData\_test\_gauss),
- 'Poisson regression' for testing relevant difference (simulData\_test\_poiss),
- 'Logistic regression' for testing relevant difference (simulData\_test\_binom).
- 'Linear regression' for transfer learning (simulData\_trans\_gauss),
- 'Poisson regression' for transfer learning (simulData\_trans\_poiss), and
- 'Logistic regression' for transfer learning (simulData\_trans\_binom).

Each dataset includes a list entitled

- datahb in simulData\_test for linear regression models,
- data\_binom in simulatedData\_Binom for logistic regression models,
- data\_poiss in simulatedData\_Poiss for Poisson regression models,
- dataset in simulData\_trans\_gauss, simulData\_trans\_binom and simulData\_trans\_poiss for linear regression, logistic regression and Poisson regression models, respectively. dataset[[1]] is the target dataset, and dataset[-1] is the 10 source datasets.

# Usage

```
data(simulData_test_gauss)
```

#### **Details**

For simulData\_test\_gauss, we simulated data generated from linear regression models

$$Y_i = \boldsymbol{X}_i^T \boldsymbol{\alpha} + \boldsymbol{Z}_i^T \boldsymbol{\beta} + \epsilon_i,$$

where  $X^T \alpha$  is a baseline mean function.

- Y: the response, an n-vector,
- X: the baseline variable with dimension  $n \times p$ ,
- Z: the interested variable with dimension  $n \times q$ .

For simulData\_trans\_gauss, we simulated data generated from linear regression models

$$Y_{0i} = \boldsymbol{X}_{0i}^T \boldsymbol{\beta}_0 + \epsilon_{0i},$$

and

Linear regression model for the kth source data:

$$Y_{ki} = \boldsymbol{X}_{ki}^T \boldsymbol{\beta}_k + \epsilon_{ki},$$

where  $X^T\beta$  is a baseline mean function, and X is high-dimensional.

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#### References

Liu, X. (2024). Testing relevant difference in high-dimensional linear regression with applications to detect transferability. Manuscript.

## **Examples**

```
data(simulData_test_gauss)
y <- datahb$Y[1:5]</pre>
dim(datahb$X)
dim(datahb$Z)
data(simulData_trans_gauss)
y <- dataset[[1]]$Y
dim(dataset[[1]]$X)
dim(dataset[[2]]$X)
```

utrans

Estimation of coefficient for the target data by transfer learning from the source data

## **Description**

Provide the estimator of coefficient for the target data by transfer learning from the source data (Liu (2024)).

## Usage

```
utrans(target, source, family = "gaussian", idtrans = NULL)
```

### **Arguments**

The target dataset, a list, including Y (response), X (covariates). target The source dataset, a list with sublist. Each sublist includes Y (response), Xsource (covariates). source could be NULL, in which case utrans only fits the target data by glmnet. Family for generalized linear models, including 'gaussian', 'binomial', and 'poisfamily son'. Default is family = "gaussian". The transferable source indices. It can be either a subset of 1,..., length(source). idtrans

Default is idtrans = NULL, which is idtrans = seq(length(source)).

#### **Details**

See details in the paper Liu (2024)

## Value

The object from fitting cv.glmnet by CV method, see details in R package fitglmnet

"glmnet".

beta The coefficient (including intercept term) of the GLMs to fit target data by trans-

fer learning.

family The response type. 18 utrans

### References

Liu, S. (2024). Unified Transfer Learning Models for High-Dimensional Linear Regression. Proceedings of The 27th International Conference on Artificial Intelligence and Statistics, PMLR. 238, 1036-1044.

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Tian, X., Lu, Y., and Li, W. (2015). A robust test for sphericity of high-dimensional covariance matrices. Journal of Multivariate Analysis, 141, 217-227.

Van den Meersche, K., Soetaert, K., and Van Oevelen, D. (2009). xsample(): An R Function for Sampling Linear Inverse Problems. Journal of Statistical Software, Code Snippets, 30, 1-15.

## **Examples**

```
data(simulData_trans_gauss)
fit <- utrans(target = dataset[[1]], source = dataset[-1], idtrans = seq(5))
fit$beta[1:10]</pre>
```

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