# Package 'tensorApp'

February 16, 2020

Type Package	
Title tensorApp	
Version 0.1.0	
Author Xu Liu [aut,cre].	
Maintainer Xu Liu <liu.xu@sufe.edu.cn></liu.xu@sufe.edu.cn>	
<b>Description</b> High-order SVD approximation of a tensor by Tucker or CP decomposition and rank selection.	
License GPL (>= 2)	
<b>Imports</b> Rcpp (>= 0.11.15), RcppEigen (>= 0.3.2.3.0)	
LinkingTo Rcpp, RcppEigen	
RoxygenNote 6.0.1	
NeedsCompilation yes	
Repository github	
<pre>URL https://github.com/xliusufe/tensorApp</pre>	
Encoding UTF-8	
R topics documented:	
tensorApp-package	2
cpals	2
cpsym2	4
gtcp	5
gtcpsem	7
gtt	8
gttsem	9
hosvd	0
hosvd_dr 1	1
tmp	3
ttu	4
Index 1	6

2 cpals

tensorApp-package High-order SVD approximation of a tensor Y by Tucker or CP decomposition and selection of ranks

#### **Description**

High-order SVD approximation of a tensor Y by Tucker decomposition or CANDECOMP/PARAFAC (CP) decomposition and selection of ranks. Alternating Least Squares algorithm is applied to Tuchker decomposition, and both Alternating Least Squares algorithm or Tensor Power Method are applied to CP decomposition. This package provides several generator functions, which generate low-rank tensor or low-rank semi-symmetric tensor.

#### **Details**

High-order SVD approximation of a tensor Y by Tucker decomposition or CANDECOMP/PARAFAC (CP) decomposition and selection of ranks.

#### Author(s)

Xu Liu

Maintainer: Xu Liu liu.xu@sufe.edu.cn>

#### References

Allen, G., (2012). Sparse higher-order principal components analysis, in: International Conference on Artificial Intelligence and Statistics, pp. 27-36.

De Lathauwer, L., De Moor, B., Vandewalle, J., (2000). A multilinear singular value decomposition. SIAM Journal on Matrix Analysis and Applications, **21**, 1253-1278.

De Lathauwer, L., De Moor, B., Vandewalle, J., (2000). On the best rank-1 and rank- $(r_1, r_2, \dots, r_n)$  approximation of higher-order tensors. SIAM Journal on Matrix Analysis and Applications, **21**, 1324-1342.

Kolda, T.G., (2001). Orthogonal tensor decompositions. SIAM Journal on Matrix Analysis and Applications, **23**, 243-255.

Kolda, T., Bader, B., (2009). Tensor decompositions and applications. SIAM Review, 51, 455-500.

cpals

High-order SVD approximation of a tensor Y by CP decomposition

#### Description

High-order SVD approximation of a tensor Y by CANDECOMP/PARAFAC (CP) decomposition with preset rank or rank to be selected. The Alternating Least Squares (als) algorithm is applied.

## Usage

cpals 3

#### **Arguments**

Υ	An array with dimension dims, or a $n_{d0} \times N/n_{d0}$ numeric matrix of responses that is the mode d0-unfolding of tensor in $\mathcal{R}^{n_1 \times \cdots \times n_d}$ , where $N = n_1 \times \cdots \times n_d$ .
d0	${\tt d0}$ is the mode. Y is the mode- ${\tt d0}$ unfolding of the tensor. ${\tt d0}$ can be NULL (the default) if Y is an array with dimension dims.
dims	The size of tensor Y, which is a $d$ -vector $(n_1, \dots, n_d)$ . dims can be NULL (the default) if Y is an array with dimension dims. If the length of dims is 2, it is the ordinary SVD decomposition of a matrix.
dr	The user-specified rank for CP decomposition. Default is 10.
isfixr	A logical value indicating whether the rank is fixed. The rank is selected automatically if it is FALSE. Default is FALSE.
DO	A user-specified list of initial matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , D0=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ . By default, initial matrices are provided by random.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-4.
max_step	Maximum number of iterations. Default is 50.
thresh	Convergence threshhold in the outer loop. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.

## **Details**

This function gives a  $n_{d0} \times N/n_{d0}$  matrix, which is the mode-d0 unfolding, and approximates Y.

## Value

Tnew	Approximation of Y.
Tn	A list of estimated matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , Tn=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ .
ranks	The ranks of estimated tensor Tnew. It is an integer.

## See Also

hosvd\_dr

```
dims <- c(8,8,10,10,6)
N <- length(dims)
ranks <- rep(2,N)
S0 <- matrix(runif(prod(ranks),3,7),ranks[N])
T1 <- matrix(rnorm(dims[1]*ranks[1]),nrow = dims[1])
tmp <- qr.Q(qr(T1))
for(k in 2:(N-1)){
   T1 <- matrix(rnorm(dims[k]*ranks[k]),nrow = dims[k])
   tmp <- kronecker(qr.Q(qr(T1)),tmp)
}
T1 <- matrix(rnorm(dims[N]*ranks[N]),nrow = dims[N])
U <- qr.Q(qr(T1))</pre>
```

4 cpsym2

```
Y <- U%*%S0%*%t(tmp)

fit <- cpals(Y,N,dims)
Tnew <- fit$Tnew
ranks1 <- fit$ranks
U1 <- fit$Tn[[1]]
TNew1 <- ttu(Tnew,N,1,dims)</pre>
```

cpsym2

High-order SVD approximation of a tensor Y by CP decomposition

## **Description**

High-order SVD approximation of a semi-symmetric tensor Y by CANDECOMP/PARAFAC (CP) decomposition with preset rank or rank to be selected. The Tensor Power Method is applied. The semi-symmetric tensor means that both the mode-r1 and mode-r2 unfoldings are equal. For semi-symmetric tensor approximation, the r1 and r2 dimensions must be small than others.

## Usage

```
cpsym2(Y=NULL, r1=1, r2=2, d0=NULL, dims=NULL, dr=10, D0=NULL, isfixr=FALSE, isOrth=FALSE, eps=1e-4, max_step=50, thresh=1e-6)
```

#### **Arguments**

Υ	An array with dimension dims, or a $n_{d0} \times N/n_{d0}$ numeric matrix of responses that is the mode d0-unfolding of tensor in $\mathbb{R}^{n_1 \times \cdots \times n_d}$ , where $N = n_1 \times \cdots \times n_d$ .
r1	Both r1 and r2 are the user-specified modes, which means that both the mode- r1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
r2	Both r1 and r2 are the user-specified modes, which means that both the mode- r1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
d0	d0 is the mode. Y is the mode- $d0$ unfolding of the tensor. $d0$ can be NULL (the default) if Y is an array with dimension dims.
dims	The size of tensor Y, which is a $d$ -vector $(n_1, \cdots, n_d)$ . dims can be NULL (the default) if Y is an array with dimension dims. If the length of dims is 2, it is the ordinary SVD decomposition of a matrix.
dr	The user-specified rank for CP decomposition. Default is 10.
D0	A user-specified list of initial matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , D0=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ . By default, initial matrices are provided by random.
isfixr	A logical value indicating whether the rank is fixed. The rank is selected automatically if it is FALSE. Default is FALSE.
isOrth	A logical value indicating whether it outputs orthonognal PCs if CP decomposition is used. Default is FALSE.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-4.
max_step	Maximum number of iterations. Default is 50.
thresh	Convergence threshhold in the outer loop. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.

gtcp 5

#### **Details**

This function gives a  $n_{d0} \times N/n_{d0}$  matrix, which is the mode-d0 unfolding, and approximates Y.

#### Value

Tnew Approximation of Y.  $\text{Tn} \qquad \text{A list of estimated matrices of } U_1, U_2, \cdots, U_d \text{ and core tensor } S, \text{Tn=list}(U_1 = U_1, \cdots, U_d = U_d, S = S).$   $\text{ranks} \qquad \text{The ranks of estimated tensor Tnew. It is an integer.}$ 

#### See Also

hosvd\_dr

#### **Examples**

```
dims <- c(6,6,8,7,7)
N <- length(dims)
ranks <- rep(2,N)</pre>
dm <- prod(ranks)</pre>
S1 <- matrix(runif(dm,3,7),nrow = ranks[1])</pre>
S2 <- ttu(S1,1,2,ranks)
S1 <- (S1+S2)/2
S0 <- ttu(S1,1,N,ranks)</pre>
T1 <- matrix(rnorm(dims[1]*ranks[1]),nrow = dims[1])
tmp \leftarrow qr.Q(qr(T1))
Uj <- kronecker(tmp,tmp)</pre>
for(k in 3:(N-1)){
  T1 <- matrix(rnorm(dims[k]*ranks[k]),nrow = dims[k])
  tmp \leftarrow qr.Q(qr(T1))
  Uj <- kronecker(tmp,Uj)</pre>
T1 <- matrix(rnorm(dims[N]*ranks[N]),nrow = dims[N])</pre>
U \leftarrow qr.Q(qr(T1))
Y <- U%*%S0%*%t(Uj)
fit <- cpsym2(Y,r1=1,r2=2,d0=N,dims=dims)</pre>
Tnew <- fit$Tnew</pre>
ranks1 <- fit$ranks
U1 <- fit$Tn[[r1]]
U2 <- fit$Tn[[r2]]
TNew1 <- ttu(Tnew,N,r1,dims)</pre>
TNew2 <- ttu(Tnew,N,r2,dims)
```

Generate a low-rank tensor characterizing the form of CP decomposition

6 gtcp

## Description

This function generates a low-rank tensor characterizing the form of CP decomposition with dimension dims and factors lambda.

#### Usage

```
gtcp(dims, lambda=NULL, d0=NULL, dr=NULL, seed_id=2)
```

## **Arguments**

dims	The size of the tensor, which is a vector $(n_1, \dots, n_d)$ . dims must be specified.
lambda	The factors of CP decomposition. It is an vector. Factors lambda will be given randomly if it is $NULL$ .
d0	${\tt d0}$ is the mode. The output tensor is the mode- ${\tt d0}$ unfolding of the tensor. ${\tt d0}$ can be NULL (the default) if the output tensor is an array with dimension dims.
dr	The user-specified rank. Default is 10.
seed_id	A positive integer, the seed for generating the random numbers. Default is 2.

#### **Details**

This function generates a low-rank tensor characterizing the form of CP decomposition with dimension dims and factors lambda.

#### Value

Dn the output mode-d0-unfolding,  $D_{(d_0)}$ . Or an array with dimesion dims if d0 is NULL.

#### See Also

```
gtcpsem, gtt
```

```
dims <- c(8,8,10,10,6)
N <- length(dims)
lambda <- seq(6,1,by=-1)
dr <- 5

T1 <- gtcp(dims=dims,lambda=lambda,d0=1,dr=dr)
T2 <- ttu(T1,1,2,dims)</pre>
```

gtcpsem 7

gtcpsem	Generate a low-rank semi-symmetric tensor characterizing the form of CP decomposition

## Description

This function generates a low-rank semi-symmetric tensor characterizing the form of CANDE-COMP/PARAFAC (CP) decomposition with dimension dims and factors lambda. The semi-symmetric tensor means that both the mode-r1 and mode-r2 unfoldings are equal.

## Usage

```
gtcpsem(dims, lambda=NULL, r1=1, r2=2, d0=NULL, dr=NULL, seed_id=2)
```

## Arguments

dims	The size of the tensor, which is a vector $(n_1, \dots, n_d)$ . dims must be specified.
lambda	The factors of CP decomposition. It is an vector. Factors lambda will be given randomly if it is NULL.
r1	Both r1 and r2 are the user-specified modes, which means that both the moder1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
r2	Both r1 and r2 are the user-specified modes, which means that both the moder1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
d0	d0 is the mode. The output tensor is the mode-d0 unfolding of the tensor. d0 can be NULL (the default) if the output tensor is an array with dimension dims.
dr	The user-specified rank. Default is 10.
seed_id	A positive integer, the seed for generating the random numbers. Default is 2.

#### **Details**

This function generates a low-rank semi-symmetric tensor characterizing the form of CP decomposition with dimension dims and factors lambda.

#### Value

Dn the output mode-d0-unfolding,  $D_{(d_0)}$ . Or an array with dimesion dims if d0 is NULL.

## See Also

```
gtcp, gttsem
```

```
dims <- c(8,6,10,6,7)
N <- length(dims)
lambda <- seq(6,1,by=-1)
r1 <- 2
r2 <- 4
dr <- 5</pre>
```

8 gtt

```
T1 <- gtcpsem(dims=dims,lambda=lambda,r1=r1,r2=r2,d0=r1,dr=dr)
T2 <- ttu(T1,r1,r2,dims)
```

gtt Generate a low-rank tensor characterizing the form of Tucker decomposition

#### **Description**

This function generates a low-rank tensor characterizing the form of Tucker decomposition with dimension dims and core tensor S.

#### Usage

```
gtt(dims, S=NULL, d0=NULL, ranks=NULL, seed_id=2)
```

#### **Arguments**

dims	The size of the tensor, which is a vector $(n_1, \dots, n_d)$ . dims must be specified.
S	The core tensor. An array with dimension dims, or a mode-d1-unfolding of core tensor with size $n_1 \times \cdots \times n_d$ . Core tensor S will be given randomly if it is NULL.
d0	d0 is the mode. The output tensor is the mode-d0 unfolding of the tensor. d0 can be NULL (the default) if the output tensor is an array with dimension dims.
ranks	The user-specified ranks. It is a vector with length $d$ . If ranks is NULL (the default), this function outputs a tensor without low-rank.
seed_id	A positive integer, the seed for generating the random numbers. Default is 2.

#### **Details**

This function generates a low-rank tensor characterizing the form of Tucker decomposition with dimension dims and core tensor S.

## Value

Dn the output mode-d0-unfolding,  $D_{(d_0)}$ . Or an array with dimesion dims if d0 is NULL.

#### See Also

```
gtcp, gttsem
```

```
dims <- c(8,8,10,10,6)
N <- length(dims)
ranks <- rep(2,N)

T1 = gtt(dims=dims,d0=1,ranks=ranks)
T2 <- ttu(T1,1,2,dims)</pre>
```

gttsem 9

gttsem	Generate a low-rank semi-symmetric tensor characterizing the form of Tucker decomposition

## Description

This function generates a low-rank semi-symmetric tensor characterizing the form of Tucker decomposition with dimension dims and core tensor S. The semi-symmetric tensor means that both the mode-r1 and mode-r2 unfoldings are equal, where the absolute difference of r1 and r2 is restricted to no more than 3.

## Usage

```
gttsem(dims, S=NULL, r1=1, r2=2, d0=NULL, ranks=NULL, seed_id=2)
```

## Arguments

dims	The size of the tensor, which is a vector $(n_1, \cdots, n_d)$ . dims must be specified.
S	The core tensor. An array with dimension dims, or a mode-d1-unfolding of core tensor with size $n_1 \times \cdots \times n_d$ . Core tensor S will be given randomly if it is NULL.
r1	Both r1 and r2 are the user-specified modes, which means that both the moder1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
r2	Both r1 and r2 are the user-specified modes, which means that both the moder1 and mode-r2 unfoldings are equal. Default is $r1 = 1$ and $r2 = 2$ .
d0	d0 is the mode. The output tensor is the mode-d0 unfolding of the tensor. d0 can be NULL (the default) if the output tensor is an array with dimension dims.
ranks	The user-specified ranks. It is a vector with length $d$ . If ranks is NULL (the default), this function outputs a tensor without low-rank.
seed_id	A positive integer, the seed for generating the random numbers. Default is 2.

## **Details**

This function generates a low-rank semi-symmetric tensor characterizing the form of Tucker decomposition with dimension dims and core tensor S.

#### Value

Dn	the output mode-d0-unfolding, $D_{(d_0)}$ . Or an array with dimesion dims	if d0 is
	NULL.	

## See Also

gtt, gtcpsem

10 hosvd

#### **Examples**

```
dims <- c(8,6,8,6,7)
N <- length(dims)
ranks <- rep(2,N)
r1 <- 2
r2 <- 4

T1 <- gttsem(dims=dims,r1=r1,r2=r2,d0=r1,ranks=ranks)
T2 <- ttu(T1,r1,r2,dims)</pre>
```

hosvd

High-order SVD approximation of a tensor Y by Tucker or CP decomposition

## **Description**

High-order SVD approximation of a tensor Y by Tucker decomposition or CANDECOMP/PARAFAC (CP) decomposition with preset rank. Alternating Least Squares algorithm is applied to Tuchker decomposition, and Tensor Power Method is applied to CP decomposition.

## Usage

```
\label{eq:loss_norm} $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \  \  \  \  \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \, $$ \  \  \  \, \  \  \, $$ \  \  \, \  \  \, $$ \  \  \, \  \  \  \, \  \  \
```

#### **Arguments**

Υ	An array with dimension dims, or a $n_{d0} \times N/n_{d0}$ numeric matrix of responses that is the mode d0-unfolding of tensor in $\mathbb{R}^{n_1 \times \cdots \times n_d}$ , where $N = n_1 \times \cdots \times n_d$ .
d0	${\tt d0}$ is the mode. Y is the mode- ${\tt d0}$ unfolding of the tensor. ${\tt d0}$ can be NULL (the default) if Y is an array with dimension dims.
dims	The size of tensor Y, which is a $d$ -vector $(n_1, \dots, n_d)$ . dims can be NULL (the default) if Y is an array with dimension dims. If the length of dims is 2, it is the ordinary SVD decomposition of a matrix.
isCP	A logical value indicating whether CP decomposition will be used. Default is TRUE.
ranks	The user-specified ranks. It is a vector with length $d$ . Default is $(2, \cdots, 2)$ .
dr	The user-specified rank for CP decomposition. It is useless if Tucker decomposition is used. Default is 20.
DO	A user-specified list of initial matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , D0=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ . By default, initial matrices are provided by random.
isOrth	A logical value indicating whether it outputs orthonognal PCs if CP decomposition is used. Default is FALSE.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.
max_step	Maximum number of iterations. Default is 100.

hosvd\_dr 11

#### **Details**

This function gives a  $n_{d0} \times N/n_{d0}$  matrix, which is the mode-d0 unfolding, and approximates Y.

#### Value

Tnew	Approximation of Y.
Tn	A list of estimated matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , Tn=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ .
ranks	The ranks of estimated tensor Tnew. It is a vector with the same length as dims if Tucker decomposition is used, or an integer if CP decomposition is used.

#### See Also

hosvd dr

#### **Examples**

```
dims <-c(8,8,10,10,6)
N <- length(dims)
ranks <- rep(2,N)
S0 <- matrix(runif(prod(ranks),3,7),ranks[N])</pre>
T1 <- matrix(rnorm(dims[1]*ranks[1]),nrow = dims[1])</pre>
tmp \leftarrow qr.Q(qr(T1))
for(k in 2:(N-1)){
  T1 <- matrix(rnorm(dims[k]*ranks[k]),nrow = dims[k])
  tmp <- kronecker(qr.Q(qr(T1)),tmp)</pre>
T1 <- matrix(rnorm(dims[N]*ranks[N]),nrow = dims[N])</pre>
U \leftarrow qr.Q(qr(T1))
Y <- U%*%S0%*%t(tmp)
fit <- hosvd(Y,N,dims,isCP=TRUE)</pre>
Tnew <- fit$Tnew
ranks1 <- fit$ranks</pre>
lambda <- fit$Tn[[N+1]]</pre>
U1 <- fit$Tn[[1]]
TNew1 <- ttu(Tnew,N,1,dims)
```

hosvd\_dr

High-order SVD approximation of a tensor Y by Tucker or CP decomposition and selection of ranks

## Description

High-order SVD approximation of a tensor Y by Tucker decomposition or CANDECOMP/PARAFAC (CP) decomposition and selection of ranks. Alternating Least Squares algorithm is applied to Tucker decomposition, and Tensor Power Method is applied to CP decomposition.

hosvd\_dr

## Usage

hosvd\_dr(Y=NULL, d0=NULL, dims=NULL, isCP=TRUE, ranks=NULL, dr=100, D0=NULL, isOrth=FALSE, eps=1e-6, max\_step=100, thresh=1e-6)

## Arguments

Υ	An array with dimension dims, or a $n_{d0} \times N/n_{d0}$ numeric matrix of responses that is the mode d0-unfolding of tensor in $\mathcal{R}^{n_1 \times \cdots \times n_d}$ , where $N = n_1 \times \cdots \times n_d$ .
d0	d0 is the mode. Y is the mode-d0 unfolding of the tensor. d0 can be NULL (the default) if Y is an array with dimension dims.
dims	The size of tensor Y, which is a $d$ -vector $(n_1, \dots, n_d)$ . dims can be NULL (the default) if Y is an array with dimension dims. If the length of dims is 2, it is the ordinary SVD decomposition of a matrix.
isCP	A logical value indicating whether CP decomposition will be used. Default is TRUE.
ranks	The user-specified ranks. It is a vector with length d. Default is $(2,\cdots,2)$ .
dr	The user-specified rank for CP decomposition. It is useless if Tucker decomposition is used. Default is 100.
DO	A user-specified list of initial matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , D0=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ . By default, initial matrices are provided by random.
isOrth	A logical value indicating whether it outputs orthonognal PCs if CP decomposition is used. Default is FALSE.
eps	Convergence threshhold in the inner loop. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.
max_step	Maximum number of iterations. Default is 100.
thresh	Convergence threshhold in the outer loop. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.

## **Details**

This function gives a  $n_{d0} \times N/n_{d0}$  matrix, which is the mode-d0 unfolding, and approximates Y.

## Value

Tnew	Approximation of Y.
Tn	A list of estimated matrices of $U_1,U_2,\cdots,U_d$ and core tensor $S$ , Tn=list $(U_1=U_1,\cdots,U_d=U_d,S=S)$ .
ranks	The ranks of estimated tensor Tnew. It is a vector with the same length as dims if Tucker decomposition is used, or an integer if CP decomposition is used.

## See Also

hosvd

tmp 13

#### **Examples**

```
dims <-c(8,8,10,10,6)
N <- length(dims)
ranks <- rep(2,N)
S0 <- matrix(runif(prod(ranks),3,7),ranks[N])</pre>
T1 <- matrix(rnorm(dims[1]*ranks[1]),nrow = dims[1])</pre>
tmp \leftarrow qr.Q(qr(T1))
for(k in 2:(N-1)){
  T1 <- matrix(rnorm(dims[k]*ranks[k]),nrow = dims[k])
  tmp <- kronecker(qr.Q(qr(T1)),tmp)</pre>
T1 <- matrix(rnorm(dims[N]*ranks[N]),nrow = dims[N])</pre>
U \leftarrow qr.Q(qr(T1))
Y <- U%*%S0%*%t(tmp)
fit_dr <- hosvd_dr(Y,N,dims,isCP=TRUE)</pre>
Tnew <- fit_dr$Tnew</pre>
ranks1 \leftarrow fit_dr$ranks
lambda <- fit_dr$Tn[[N+1]]</pre>
U1 <- fit_dr$Tn[[1]]
TNew1 <- ttu(Tnew,N,1,dims)</pre>
```

tmp

Modal Product

#### **Description**

Modal product calculate the product an order d tensor S and a matrix M, that is  $T = S \times_{d0} M$ ,  $T_{(d0)} = M \cdot S_{(d0)}$ , where  $S_{(d0)}$  is the mode-d0 unfloding of the tensor S.

#### Usage

```
tmp(S=NULL, M=NULL, d0=NULL, dims=NULL)
```

#### **Arguments**

S	An order $d$ tensor with dimension dims, where dims is $(n_1, \dots, n_d)$ .
М	A matrix with $n_{d0}$ columns and $K$ rows. The modal product produce a new order $d$ tensor with dimension dims replaced the d0 dimnesion by $K$ .
dims	The dimension of tensor Y, which is a $d$ -vector $(n_1, \dots, n_d)$ .
d0	$\ensuremath{\mathrm{d0}}$ is the mode. $M$ multiplies the mode- $\ensuremath{\mathrm{d0}}$ unfoldings of S.

#### **Details**

This function gives the modal product of tensor S and matrix M, which is the product of M and the mode-d0 unfolding of tensor S, that is  $S \times_{d0} M = MS_{(d0)}$ , where  $S_{(d0)}$  is the mode-d0 unfloding of the tensor S. The modal product produce a new order d tensor with dimension dims replaced the d0 dimnesion by K.

14 ttu

#### Value

Tnew

Product of S and M.

#### See Also

TransUnfoldingsT

#### **Examples**

```
dims <- c(8,8,10,10,6)
N <- length(dims)
ranks <- rep(2,N)
S0 <- matrix(runif(prod(ranks),3,7),ranks[N])</pre>
T1 <- matrix(rnorm(dims[1]*ranks[1]),nrow = dims[1])</pre>
tmp <- qr.Q(qr(T1))
for(k in 2:(N-1)){
  T1 <- matrix(rnorm(dims[k]*ranks[k]),nrow = dims[k])
  tmp \leftarrow kronecker(qr.Q(qr(T1)),tmp)
T1 <- matrix(rnorm(dims[N]*ranks[N]),nrow = dims[N])</pre>
U \leftarrow qr.Q(qr(T1))
Y <- U%*%S0%*%t(tmp)
Y1 <- array(ttu(Y,N,1,dims),dims)
M <- matrix(1:(4*dims[3]),4)</pre>
X1 <- tmp(Y1,M,3,dims)</pre>
X <- ttu(matrix(X1,dims[1]),1,N,dims)</pre>
dim(Y1)
dim(X1)
print(Y[,1])
print(X[,1])
```

ttu

Transfer a tensor's modal unfoldings to another.

#### **Description**

Transfer a tensor's modal unfoldings to another.

#### Usage

```
ttu(S=NULL, d1=NULL, d2=0, dims=NULL)
```

#### **Arguments**

d1

S An array with dimension dims, or a mode-d1-unfolding of a tensor with size  $n_1 \times \cdots \times n_d$ .

An integer, the mode of unfolding  $S_{(d_1)}$ . d1 can be NULL (the default) if S is an array with dimension dims.

ttu 15

d2	An integer, the mode of output unfolding $S_{(d_2)}$ . It transfers S to an array with
	dimension dims if d2=0. The default is 0.
dims	The size of tensor $S$ , which is a vector $(n_1, \dots, n_d)$ . dims can be NULL (the
	default) if S is an array with dimension dims.

## **Details**

This function transfers an input mode-d1-unfolding  $S_{(d_1)}$  to mode-d2-unfolding  $S_{(d_2)}$ 

## Value

Td2 the output mode-d2-unfolding,  $S_{(d_2)}$ .

```
T1 <- matrix(1:24,nrow = 4) # A tensor unfolding with size 4*6
T2 <- ttu(T1,1,2,c(4,3,2))

T0 <- ttu(T2,2,dims=c(4,3,2))
```

## **Index**

```
*Topic CP decomposition; HOSVD;
        Tucker decomposition.
    cpals, 2
    cpsym2, 4
    hosvd, 10
    tmp, 13
*Topic CP decomposition; HOSVD;
        Tucker decomposition
    gtcp, 5
    gtcpsem, 7
    gtt, 8
    gttsem, 9
    hosvd_dr, 11
    tensorApp-package, 2
    ttu, 14
cpals, 2
cpals-function (cpals), 2
cpsym2, 4
cpsym2-function(cpsym2), 4
gtcp, 5
gtcp-function (gtcp), 5
gtcpsem, 7
gtcpsem-function (gtcpsem), 7
gtt, 8
gtt-function (gtt), 8
gttsem, 9
gttsem-function(gttsem), 9
hosvd, 10
hosvd-function (hosvd), 10
hosvd_dr, 11
hosvd_dr-function (hosvd_dr), 11
tensorApp (tensorApp-package), 2
tensorApp-package, 2
tmp, 13
tmp-function(tmp), 13
ttu, 14
ttu-function (ttu), 14
```